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Dust Acoustic Shock Waves in Coupled Dusty Plasmas with Kappa-Distributed Ions

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Abstract**—**We have considered an unmagnetized dusty plasma system consisting of ions obeying superthermal distribution and strongly coupled negatively charged dust. We have used reductive perturbation method and derived the Kordeweg-de Vries-Burgers (KdV-Burgers) equation. The behavior of the shock waves in the plasma has been investigated.

Keywords**—**Shock, Soliton, Coupling, Superthermal ions.

I. Introduction

DUSTY plasmas are ordinary plasmas with embedded solid microparticles. In recent years, the study of nonlinear waves in plasmas has become one of the most important topic in plasma physics. Rao et al. [1] theoretically predicted the existence of dust-acoustic waves (DAWs), in which the inertia is provided by the dust particle mass and the restoring force is provided by the pressures of the inertialess electrons and ions. There has been a rapidly growing interest in understanding the physics of strongly coupled dusty plasma and associated low-frequency dust modes because of their vital role in space and astroplasmas (such as white dwarf matter, interior of heavy planets, etc.), laboratory plasmas (for example, plasma crystals, plasmas produced by laser compression of matter, etc.), and industrial plasma processing. It was first pointed out by Ikezi [2] that a classical Coulomb plasma with micron-sized dust particles can readily go into the strongly coupled regime. The laboratory experiments [3–5] as well as a number of theoretical analysis [6–9] conclusively verified the prediction of Ikezi [2] and demonstrated that the dust particles organize themselves into crystalline patterns in such a dusty plasma. It is also observed by experiments that as the coupling is increased, the dust crystals melt and then vaporize so that one encounters the usual weakly coupled ideal Coulomb plasma. Thus, laboratory experiments in such a dusty plasma system provide an excellent opportunity for the study of transitions from the strongly coupled to weakly coupled regimes. A number of authors in the recent years have studied the behavior of dust acoustic shock waves in coupled dusty plasmas. Shukla and Mamun [10] have derived Korteweg de Vries-Burgers (KdV-Burgers) equation by reductive perturbation method and they have studied the properties of the solitons and shock waves for strongly coupled unmagnetized dusty plasmas. Also Mamun et al [11] have studied dusty plasma with a Boltzmann electron distribution, a nonisothermal vortex-like ion distribution and strongly correlated grains in a liquid-like state and discussed about the properties of shock wave structures.

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Ghosh and Gupta have investigated the nonlinear propagation of shock wave in strongly coupled collisional dusty plasma using the GH model incorporating a charging-delay effect [12]. More recently, it had been found that the electrons and ions distributions play a crucial role in characterizing the physics of the nonlinear waves [13-16]. The nonlinear features of dust acoustic shock waves in a strongly coupled unmagnetized dusty plasma containing Boltzmann electron distribution, nonisothermal ions and negatively charged dust has been studied in [17]. The effect of nonthermal ions on dust acoustic shock waves in dusty plasma was investigated in [18]. Numerous observations of space plasmas [19-21] indicate clearly the presence of superthermal electron and ion structures as ubiquitous in a variety of astrophysical plasma environments. We consider a fluid model of dusty plasma in which the electron number density is assumed to be sufficiently depleted, i.e., $n_e << n_d$. The aim of the present paper is study the effect of superthermal ions on the dust acoustic shock waves in coupled dusty plasmas. The manuscript is organized as follows. The GH equations are presented in Section 2. In Section 3 we derive the Korteweg de Vries Burgers equation using the reductive perturbation method. We study the solitary and shock waves solutions in Section 4. Finally the main results from this investigation have been given in Section 5.

II. Basic Equations

We consider an unmagnetized strongly coupled dusty plasma with superthermal distributed ions and negatively charged dust grains. We assume that the ions are weakly coupled compared to the dust grains. The dynamics of the DAW in our coupled dusty plasma are given by GH equations [22-24] as follows

$$\begin{cases} \dfrac{\partial n_d}{\partial t}+\dfrac{\partial}{\partial x}(n_d u_d) = 0 \\ \left[1+\tau_m \dfrac{\partial}{\partial t}\right]\left[n_d(\dfrac{\partial u_d}{\partial t}+u_d\dfrac{\partial u_d}{\partial x}-\dfrac{\partial \phi}{\partial x})\right] = \eta_l \dfrac{\partial^2 u_d}{\partial x^2} \\ \dfrac{\partial^2 \phi}{\partial x^2}+\dfrac{\partial^2 \phi}{\partial y^2} = n_d/-n_i \end{cases} \tag{1}$$

n_d (n_i) is the dust (ion) number density normalized by its equilibrium value $n_{\circ d}$ ($n_{\circ i}$), u_d is the dust fluid velocity normalized by the dust-acoustic speed $C_d = \sqrt{\frac{Z_d T_i}{m_d}}$ (Z_d is the number of electrons residing on the dust grain, T_i is the ion temperature, and m_d is the dust mass), ϕ is the electrostatic wave potential normalized by $\frac{T_i}{e}$ (e is the magnitude of the electron charge). The time and space variables are normalized by the dust plasma period $\omega_{Pd}^{-1} = \sqrt{\frac{m_d}{4\pi n_{d0} Z_d^2 e^2}}$ and the Debye length $\lambda_D = \sqrt{\frac{T_i}{4\pi n_{d0} Z_d e^2}}$, respectively. Furthermore, η_1 is the normalized longitudinal viscosity coefficient and is given by

$$\eta_1 = \frac{\tau_d}{m_d n_{d0} \lambda_D^2} \left[s + \frac{4}{3} b \right] \tag{2}$$

where ***s*** and ***b*** are the transport coefficients of shear and bulk viscosities, respectively. The viscoelastic relaxation time τ_m is normalized by the dust plasma period τ_d and is given by

$$\tau_m = \eta_1 \frac{T_e}{T_d} \left[1 - \mu_d + \frac{4}{15} u(\Gamma) \right]^{-1} \tag{3}$$

where

$$\mu_d = 1 + \frac{1}{3} u_{(\Gamma)} + \frac{\Gamma}{9} \frac{\partial u_{(\Gamma)}}{\partial \Gamma} \tag{4}$$

is the compressibility [25,26] and $u_{(\Gamma)}$ is a measure of the excess internal energy of the system and is calculated for weakly coupled plasma (Γ<1) as [25,27] $u(\Gamma) \cong -(\sqrt{3}/2)\Gamma^{3/2}$. To express $u_{(\Gamma)}$ in terms of Γ for a range of 1< Γ <100, Slattery *et al.* [28] analytically derived a relation

$$u_{(\Gamma)} \approx -0.89\Gamma + 0.95\Gamma^4 + 0.19\Gamma^{-1/4} - 0.81 \tag{5}$$

where a small correction term due to finite number particles is neglected.
We adopt a superthermal distribution for the ions, and by integrating over velocity space obtain the ions number density [29]

$$n_i = n_{i0} (1 + \frac{\phi}{k - 3/2})^{-(k-1/2)} \tag{6}$$

k is a spectral index. The spectral index is a measure of the slope of the energy spectrum of the suprathermal particles forming the tail of the velocity distribution function; the smaller the value of *k* the more suprathermal particles in the distribution function tail and the harder the energy spectrum. Kappa distributions approach the Maxwellian as $k \to \infty$. The normalization has been provided by n_{i0} for any value of the *k*>3/2.

$$n_i = (1 + \frac{\phi}{k - 3/2})^{-(k-1/2)} \tag{7}$$

III. Derivation of KdV-Burger equation

According to the general method of reductive perturbation theory [30], we choose the independent variables as

$$\xi = \varepsilon^{1/4} (x - \lambda t) \quad , \quad \tau = \varepsilon^{3/4} t \quad , \\ \eta_l = \varepsilon^{1/4} \eta_\circ \quad , \quad \tau_m = \varepsilon^{1/4} \tau_{m\circ} \tag{8}$$

where ε is a small dimensionless expansion parameter which characterizes the strength of nonlinearity in the system and λ is the phase velocity of the wave along the x direction and normalized by dust acoustic velocity. Now we expand dependent variables as follows

$$\begin{cases} n_d = 1 + \varepsilon n_{1d} + \varepsilon^2 n_{2d} + ... \\ u_d = \varepsilon u_{1d} + \varepsilon^2 u_{2d} + ... \\ \phi = \varepsilon \phi_1 + \varepsilon^2 \phi_2 + ... \end{cases} \tag{9}$$

Substituting (9) into (1) and collecting the terms in different powers of ε the following equations can be obtained at the lower order of ε

$$n_{1d} = \frac{-\phi_1}{\lambda^2} \quad , \quad u_{1d} = \frac{-\phi_1}{\lambda} \quad , \quad \lambda = \sqrt{\frac{2k-3}{2k-1}} \tag{10}$$

At the higher order of ε , we have

$$\begin{vmatrix} \frac{\partial n_{1d}}{\partial \tau} - \lambda \frac{\partial n_{2d}}{\partial \xi} + \frac{\partial}{\partial \xi}(u_{2d} + n_{1d} u_{1d}) = 0 \\ \frac{\partial u_{1d}}{\partial \tau} - \lambda \frac{\partial u_{2d}}{\partial \xi} + u_{1d} \frac{\partial u_{1d}}{\partial \xi} - \frac{\partial \phi_2}{\partial \xi} = \eta_\circ \frac{\partial^2 u_{1d}}{\partial \xi^2} \\ \frac{\partial^2 \phi_1}{\partial \xi^2} = n_{2d} + \left[(\frac{2k-1}{2k-3})\phi_2 - (\frac{4k^2-1}{(2k-3)^2})\phi_1^2 \right] \end{vmatrix} \tag{11}$$

Finally from (10) and (11) KdV-Burgers equation yields

$$\frac{\partial \phi_1}{\partial \tau} + A\phi_1 \frac{\partial \phi_1}{\partial \xi} + B\frac{\partial^3 \phi_1}{\partial \xi^3} - C\frac{\partial^2 \phi_1}{\partial \xi^2} = 0 \tag{12}$$

where the coefficients are

$$A = \lambda^3 \left[\frac{4k^2 - 1}{(2k-3)^2}\right] - \frac{3}{2\lambda}, B = \frac{\lambda^3}{2}, C = \frac{\eta_\circ}{2} \tag{13}$$

Eq. (12) is a KdV-Burger equation which includes the effects of superthermal ions (k) and strongly coupled dust ($\eta_\circ$). The Burger term is proportional to the dissipation due to dust viscosity through dust-dust correlation.

IV. Discussion and Conclusion

Equation (12) is the well known KdV-Burgers equation describing the nonlinear propagation of the dust acoustic shock waves in a coupled dusty plasma with superthermal ions. In this equation A and B are the nonlinear coefficient and dispersive term and the Burger term (C) arises due to the coupling of dust particles. The KdV-Burgers equation is widely used in plasma physics and theoretical physics. The tangent hyperbolic method is a powerful method for the computation of exact traveling wave solutions. More recently, Asif Shah et al. [30] have derived the monotonic shock waves solution theoretically by employing the tangent hyperbolic method [31]. They used the transformation $\chi = \kappa(\xi - v\tau)$ (where κ and v are wave number and wave velocity, respectively) and presented the solution in terms of independent variable χ as

$$\phi_1 = \frac{12B}{A}\left[1 - \tanh^2 \chi\right] - \frac{36C}{15A}\tanh \chi \tag{14}$$

Now, using this stationary solution, we have numerically solved (14), and have studied the effects of the coupling and superthermal ions on the waves. Figs. 1 and 2 show how the coupling force effect covert the soliton profile to shock-like structures. Fig1. Shows the variation of ϕ_1 with respect to χ for k=2 and different values of $\eta_\circ$. If the dissipative term C is negligible, in comparison with the nonlinear A and dispersive B terms, the solitary structure will be established by balancing the effects of dispersive and nonlinear terms. On the other hand, if the coupling becomes very strong the shock waves will appear. It is also seen in Fig. 1 that both strength and steepness of the shock structure increase by increasing the coupling. Figure 2 shows the variation in the ϕ_1 with respect to χ for k=2 and different values of $\eta_\circ$. It is clear that there are the same results for both Figs. 1 and 2. However, it seems that the behavior of the shock waves changes from a kink wave structure to an anti-kink type with k=2 in Fig.1 and k=5 in Fig.2.

Variation of the shock amplitude as a function of k can be studied by plotting the amplitude respect to k for the case of χ = 0 using Eq. (14). In fig. 3 the amplitude (ϕ_0) has been plotted respect to k for $\eta_\circ$=8. As one can see, when k increases in the interval 1.6<k<2.413, the amplitude is positive, but it is negative for k>2.413. Thus k=2.413 is a special value of spectral index for the amplitude in which the shock waves have different behavior which separates regions k>2.413 and k<2.413. In fact, a kink (an anti kink) of shock structure appear for k<2.413 (k>2.413). On the other hand, it is clear that there exists a singularity in the shock amplitude for $k = k_c = 2.5$. Since Eq. (14) shows that the amplitude of shock waves depends on A and B and also since B is always positive, so it can be concluded that A is important to indicate of the shock structure.

The amplitude of shock waves becomes zero for a critical value of k (k_c) which can be determined by A = 0. So we have

$$A = \lambda^3 \left(\frac{4k^2 - 1}{(2k-3)^2}\right) - \frac{3}{2\lambda} = 0 \quad \Rightarrow \quad k_c = 2.5 \tag{15}$$

This value of spectral index has been shown in Fig. 3. We can see that A becomes zero when $k \to k_c$ and thus the amplitude of shock waves increases sharply to infinity. On the other hand distribution function approaches to the Maxwellian distribution for $k \to \infty$ and in this situation we have shock waves with decreasing negative amplitude.

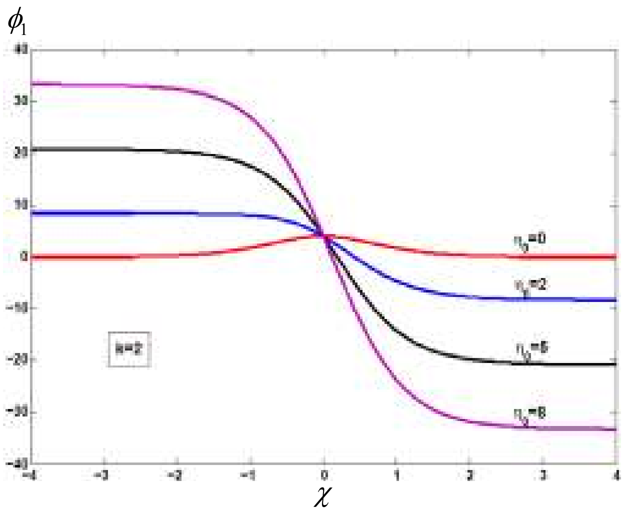

Fig. 1 Variation of ϕ_1 with χ for different values of $\eta_\circ$

ϕ_1

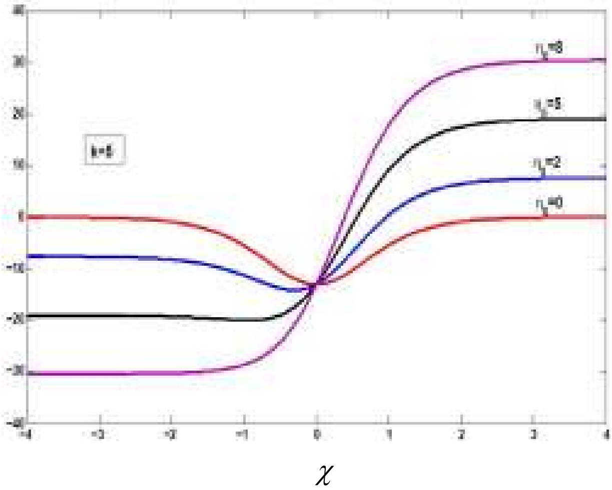

Fig .2 Variation of ϕ_1 with χ for different values of $\eta_\circ$

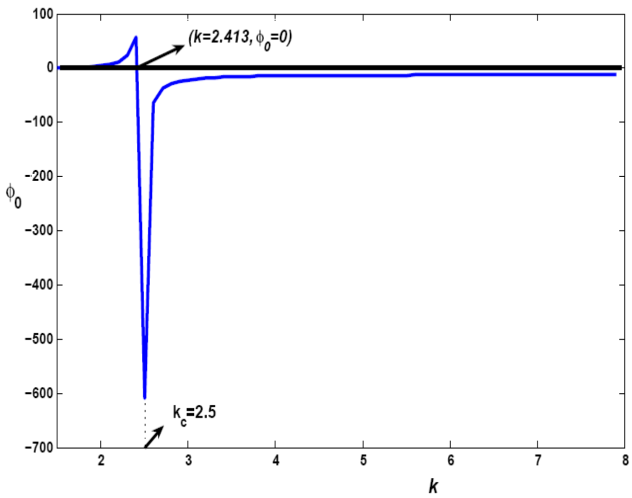

Fig. 3 Variation of the amplitude of shock wave with respect to *k* for $\eta_\circ$ =8.

IV. Conclusion

We have studied the properties of dust acoustic solitary and shock waves by deriving the KdV-Burgers equation in coupled dusty plasma with superthermal ions**.** We have considered a fluid model of dusty plasma in which the electron number density is assumed to be sufficiently depleted, ($n_e << n_d$). The dissipative Burger term in the nonlinear KdVB equation arises by considering the coupling viscosity through dust-dust correlation. Our results show that in such plasmas, solitary and shock structures can be created. We have shown that soliton profile is converted into shock structure when the coupling force increases. The increase of spectral index *k* is significant as it leads to shift from a kink shock wave with positive amplitude to an anti kink shock wave with negative amplitude (see Fig. 3). It is also shown that the large amplitude of solitary and shock waves appear when $k \to k_c = 2.5$. The shock wave solution cannot be established when $k = k_c$ (A is zero). This situation can be investigated in further works. The limitation of the present analysis is that the plasma has been considered as a medium which is not containing electrons and also the dust charge has been taken constant. It is clear that plasmas with variable dust charge and distributed electron, has a new scenario with very different behavior.

In view of the observations of superthermal distributed ions in Saturn's magnetosphere [32], the results of this investigation can help in the interpretation of nonlinear electrostatic shock waves that may be observed in that region.

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