

Dispersion of a Solute in Peristaltic Motion of a Couple Stress Fluid through a Porous Medium with Slip Condition

Habtu Alemayehu and G. Radhakrishnamacharya

Abstract—The paper presents an analytical solution for dispersion of a solute in the peristaltic motion of a couple stress fluid through a porous medium with slip condition in the presence of both homogeneous and heterogeneous chemical reactions. The average effective dispersion coefficient has been found using Taylor's limiting condition and long wavelength approximation. The effects of various relevant parameters on the average coefficient of dispersion have been studied. The average effective dispersion coefficient tends to increase with permeability parameter but tends to decrease with homogeneous chemical reaction rate parameter, couple stress parameter, slip parameter and heterogeneous reaction rate parameter.

Keywords—Dispersion, Peristalsis, Couple stress fluid, Porous medium, Chemical reaction, Slip condition.

I. INTRODUCTION

DISPERSION (or diffusion) of a solute describes the spread of particles through random motion from regions of higher concentration to regions of lower concentration. The fluid mechanical aspects of dispersion of a solute received the attention of some investigators in the last few decades. The dispersion of a solute in a viscous liquid flowing in a circular pipe under laminar conditions was studied by Taylor [1]–[3] and Aris [4]. Studies reveal that in a wide variety of problems of chemical engineering, diffusion of a solute takes place with simultaneous chemical reaction in situations such as hydrolysis, gas absorption in an agitated tank, esterification, etc. (Padma and Ramana Rao [5]). Hence, Padma and Ramana Rao [5], Gupta and Gupta [6] and Ramana Rao and Padma [7]–[8] dealt with the effect of chemical reaction on dispersion in Newtonian fluids. Dutta et al. [9], Soundalgekar and Chaturani [10] and Shukla et al. [11] studied dispersion in non-Newtonian fluids by considering only homogeneous first-order chemical reaction in the bulk of the fluid. Further, a number of authors have studied the dispersion of a solute in a porous medium under different conditions (Dulal [12], Mehta and Tiwari [13]). Porosity has various physiological applications such as the flow of blood in the micro-vessels of the lungs which may be treated as a channel bounded by two thin porous layers (Misra and Ghosh [14]).

Peristalsis is a mechanism of fluid transport that occurs widely in several physiological situations such as flow through ureter, mixing of food and chyme movement in the intestine, movement of eggs in the fallopian tube, transport of the spermatozoa in the cervical canal, transport of bile in the bile

duct, transport of cilia, and circulation of blood in small blood vessels. There are also many other important applications of peristalsis such as the design of roller pumps and finger pumps which are used to transport blood or corrosive fluids. Peristaltic transport of Newtonian fluids has been studied by Fung and Yih [15], Shapiro et al. [16], and Misra and Pandey [17] under different conditions.

It is well known that most physiological fluids including blood behave as non-Newtonian fluids. Hence, the study of peristaltic transport of non-Newtonian fluids may help to get better understanding of the working biological systems. Motivated by this, several researchers studied peristaltic transport of non-Newtonian fluids (Misra and Ghosh [14], Radhakrishnamacharya [18], Misra and Pandey [19], Ramachandra Rao and Mishra [20]).

Couple stress fluids are fluids consisting of rigid, randomly oriented particles suspended in a viscous medium. Couple stress fluid is known to be a better model for bio-fluids, such as blood, lubricants containing small amount of high polymer additive, electro-rheological fluids and synthetic fluids. The main feature of couple stress fluids is that the stress tensor is anti-symmetric and their accurate flow behavior cannot be predicted by the classical Newtonian theory. Stokes [21] generalized the classical model to include the effect of the presence of the couple stresses and this model has been widely used because of its relative mathematical simplicity (Islam and Zhou [22]). For couple stress fluids, there have been a number of studies carried out due to its widespread industrial and scientific applications, such as the works of Stokes [21], Srivastava [23], Mekheimer and Abd elmaboud [24] and Sobh [25].

Problems that involve slip boundary conditions would be useful models for flows through pipes in which chemical reactions occur at the walls, flows with laminar film condensation, certain two phase flows and flows in porous slider bearings (Terrill [26]). Curiously enough, there are two extremely different types of fluids which appear to slip. One class contains the rarefied gases (Kwang and Fang [27], Bhatt and Sacheti [28]) and the other fluids with much elastic character. In such fluids, some slippage occurs under a large tangential traction. It has been claimed that slippage can occur in non-Newtonian fluids, concentrated polymer solution and molten polymer. Further, in the flow of dilute suspensions of particles, a clear layer is sometimes observed next to the wall (Sobh [25]). Consequently, from a macroscopic viewpoint, there can be seen a slip velocity at the porous wall and the velocity

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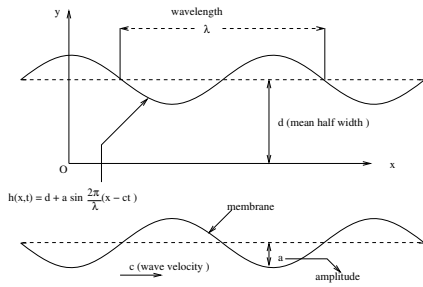


Fig. 1. Geometry of the problem.

gradient changes across the interface (Suga et al. [29]).

Dispersion of a solute in peristaltic motion of a couple stress fluid through a porous medium with slip condition has not received much attention. Peristalsis, diffusion and porosity are very important aspects in biological, chemical, environmental and bio-medical processes (Paul [30]). It is realized that peristalsis may have effect on dispersion of a solute in fluid flow. This, in turn, may help in better absorption of nutrients and drugs in physiological systems. Hence, the study of the interaction of peristalsis with diffusion under different conditions particularly under slip condition may lead to better understanding of the flow situation in physiological systems. In this paper, dispersion of a solute in peristaltic motion of a couple stress fluid through a porous medium with slip condition is investigated. Using long wavelength approximation and Taylor's approach, closed form solution has been obtained for the dispersion coefficient for both the cases of homogeneous first-order irreversible chemical reaction and combined first-order homogeneous and heterogeneous chemical reactions. The effects of various relevant parameters on the average effective dispersion coefficient are studied.

II. MATHEMATICAL FORMULATION

Consider the dispersion of a solute in peristaltic flow of couple stress fluid in an infinite uniform channel of width $2d$. It is assumed that the channel is filled with porous material. Further, the walls of the channel are supposed to be flexible on which traveling sinusoidal waves of long wavelength are imposed. Cartesian coordinate system (x, y) is chosen with the x -axis aligned with the center line of the channel. The traveling waves are represented by (Fig.1)

$$y = \pm h = \pm \left[d + a \sin \frac{2\pi}{\lambda} (x - ct) \right] \quad (1)$$

where a is the amplitude, c is the wave speed and λ is the wavelength of the peristaltic wave.

Under long wavelength approximation and neglecting body forces and body couples, the equations governing the peristaltic motion of incompressible couple stress fluid through a

porous medium for the present problem are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} - \eta' \frac{\partial^4 u}{\partial y^4} - \frac{\mu}{k_0} u = 0 \quad (3)$$

$$-\frac{\partial p}{\partial y} = 0 \quad (4)$$

where $u(x, y, t)$ and $v(x, y, t)$ are the velocity components in the x and y directions respectively, μ is the viscosity coefficient of classical fluid dynamics, η' is the couple stress viscosity and k_0 is permeability of the medium.

We assume that the walls are inextensible so that only lateral motion takes place and the horizontal displacement of the wall is zero. Thus, the relevant boundary conditions for the velocity are given by

$$u = -\frac{d\sqrt{Da}}{\alpha_1} \frac{\partial u}{\partial y} \quad \text{at } y = \pm h \quad (5)$$

$$\frac{\partial^2 u}{\partial y^2} = 0 \quad \text{at } y = \pm h \quad (6)$$

where (5) is the Saffman's slip boundary condition (Bhatt [28]), Da is the permeability parameter (or Darcy number) and α_1 is the slip parameter.

Solving (2)-(4) under the boundary conditions (5) and (6), the velocity is given as

$$u(y) = -\frac{k_0}{\mu} \frac{\partial p}{\partial x} [R_2 \cosh(m_1^* y) - R_1 \cosh(m_2^* y) + 1] \quad (7)$$

where

$$m_1^* = \sqrt{\frac{\mu}{2\eta'} \left(1 + \sqrt{1 - \frac{4\eta'}{\mu k_0}} \right)}, m_2^* = \sqrt{\frac{\mu}{2\eta'} \left(1 - \sqrt{1 - \frac{4\eta'}{\mu k_0}} \right)}$$

$$R_1 = \frac{(m_1^*)^2 \cosh(m_1^* h)}{a_1^* - a_2^*}, R_2 = \frac{(m_2^*)^2 \cosh(m_2^* h)}{a_1^* - a_2^*},$$

$$a_1^* = (m_1^*)^2 \cosh(m_1^* h) \left[\cosh(m_2^* h) + \frac{d\sqrt{Da}}{\alpha_1} m_2^* \sinh(m_2^* h) \right],$$

$$a_2^* = (m_2^*)^2 \cosh(m_2^* h) \left[\cosh(m_1^* h) + \frac{d\sqrt{Da}}{\alpha_1} m_1^* \sinh(m_1^* h) \right].$$

Further, the mean velocity is defined as

$$\bar{u} = \frac{1}{2h} \int_{-h}^{+h} u(y) dy. \quad (8)$$

Substituting (7) in (8) we get,

$$\bar{u} = -\frac{k_0}{\mu} \frac{\partial p}{\partial x} \left[\frac{R_2}{m_1^* h} \sinh(m_1^* h) - \frac{R_1}{m_2^* h} \sinh(m_2^* h) + 1 \right]. \quad (9)$$

If we now consider convection across a plane moving with the mean speed of the flow, then relative to this plane, the fluid velocity is given by

$$\begin{aligned}
u_x &= u - \bar{u} \\
&= -\frac{k_0}{\mu} \frac{\partial p}{\partial x} [R_2 \cosh(m_1^* y) - R_1 \cosh(m_2^* y) \\
&\quad - \frac{R_2}{m_1^* h} \sinh(m_1^* h) + \frac{R_1}{m_2^* h} \sinh(m_2^* h)] \quad (10)
\end{aligned}$$

A. Diffusion with a Homogeneous First-order Chemical Reaction

It is assumed that a solute diffuses and simultaneously undergoes a first order irreversible chemical reaction in peristaltic transport of a couple stress fluid in a channel filled with porous material. Assuming isothermal conditions and $\frac{\partial^2 C}{\partial x^2} \ll \frac{\partial^2 C}{\partial y^2}$, the equation for the concentration C of the solute for the present problem satisfies the diffusion equation

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - k_1 C \quad (11)$$

where D is the molecular diffusion coefficient and k_1 is the first order reaction rate constant.

For typical values of physiologically relevant parameters of this problem, it is realized that $\bar{u} \approx c$. Using this condition and following Taylor [1]–[3], we assume partial equilibrium is maintained and then making use of the following dimensionless quantities

$$\theta = \frac{t}{\bar{t}}, \quad \bar{t} = \frac{\lambda}{\bar{u}}, \quad \eta = \frac{y}{d}, \quad \xi = \frac{x - \bar{u}t}{\lambda}, \quad H = \frac{h}{d}, \quad (12)$$

equation (10) reduces to

$$\begin{aligned}
u_x &= \frac{k_0}{\mu} \frac{\partial p}{\partial x} [R_2 \cosh(m_1 \eta) - R_1 \cosh(m_2 \eta) \\
&\quad - \frac{R_2}{m_1 H} \sinh(m_1 H) + \frac{R_1}{m_2 H} \sinh(m_2 H)] \quad (13)
\end{aligned}$$

where

$$\begin{aligned}
m_1 &= m_1^* d = \sqrt{\frac{m^2}{2} \left(1 + \sqrt{1 - \frac{4}{m^2 Da}} \right)}, \quad a_1 = a_1^* d^2, \\
m_2 &= m_2^* d = \sqrt{\frac{m^2}{2} \left(1 - \sqrt{1 - \frac{4}{m^2 Da}} \right)}, \quad a_2 = a_2^* d^2, \\
m &= d(\mu/\eta')^{1/2}, \quad Da = \frac{k_0}{d^2},
\end{aligned}$$

and m is the couple stress parameter.

Further, (11) becomes

$$\frac{\partial^2 C}{\partial \eta^2} - \frac{k_1 d^2}{D} C = \frac{d^2}{\lambda D} u_x \frac{\partial C}{\partial \xi} \quad (14)$$

Assuming that there is no absorption at the walls, the boundary conditions for the concentration C are

$$\frac{\partial C}{\partial \eta} = 0 \quad \text{for} \quad \eta = \pm H = \pm [1 + \epsilon \sin(2\pi\xi)] \quad (15)$$

where $\epsilon = a/d$ is the amplitude ratio.

Assuming that $\partial C/\partial \xi$ is independent of η at any cross-section and solving (14) under the boundary conditions (15), the solution for the concentration of the solute C is given as

$$\begin{aligned}
C(\eta) &= A \cosh(\alpha \eta) - \frac{d^2}{\lambda D} \frac{\partial C}{\partial \xi} \frac{k_0}{\mu} \frac{\partial p}{\partial x} \left\{ \frac{R_2}{m_1^2 - \alpha^2} \cosh(m_1 \eta) \right. \\
&\quad - \frac{R_1}{m_2^2 - \alpha^2} \cosh(m_2 \eta) + \frac{R_2}{\alpha^2 m_1 H} \sinh(m_1 H) \\
&\quad \left. - \frac{R_1}{\alpha^2 m_2 H} \sinh(m_2 H) \right\} \quad (16)
\end{aligned}$$

where

$$A = \frac{d^2}{\lambda D} \frac{\partial C}{\partial \xi} \frac{k_0}{\mu} \frac{\partial p}{\partial x} \frac{1}{L} \left[\frac{m_1 R_2}{m_1^2 - \alpha^2} \sinh(m_1 H) - \frac{m_2 R_1}{m_2^2 - \alpha^2} \sinh(m_2 H) \right], \quad (17)$$

$$L = \alpha \sinh(\alpha H) \text{ and } \alpha = d(k_1/D)^{1/2}.$$

The volumetric rate Q at which the solute is transported across a section of the channel of unit breadth is defined by

$$Q = \int_{-H}^{+H} C u_x d\eta. \quad (18)$$

Substituting (16) and (13) in (18), we get the volumetric rate Q as

$$Q = -\frac{2d^6}{\lambda \mu^2 D} \frac{\partial C}{\partial \xi} \left(\frac{\partial p}{\partial x} \right)^2 F(\xi, \epsilon, \alpha, Da, m, \alpha_1) \quad (19)$$

where

$$\begin{aligned}
F(\xi, \epsilon, \alpha, Da, m, \alpha_1) &= (Da)^2 \times \\
&\left\{ \frac{R_2}{L(m_1^2 - \alpha^2)} \left(\frac{m_1 R_2 \sinh(m_1 H)}{m_1^2 - \alpha^2} - \frac{m_2 R_1 \sinh(m_2 H)}{m_2^2 - \alpha^2} \right) B_1 \right. \\
&+ \frac{R_1}{L(m_2^2 - \alpha^2)} \left(\frac{m_2 R_1 \sinh(m_2 H)}{m_2^2 - \alpha^2} - \frac{m_1 R_2 \sinh(m_1 H)}{m_1^2 - \alpha^2} \right) B_2 \\
&+ \frac{R_1 R_2}{\alpha^2 H} \sinh(m_1 H) \sinh(m_2 H) \left(\frac{m_1}{m_2(m_1^2 - \alpha^2)} + \frac{m_2}{m_1(m_2^2 - \alpha^2)} \right) \\
&- \frac{R_2^2 \sinh^2(m_1 H)}{\alpha^2 H(m_1^2 - \alpha^2)} - \frac{R_1^2 \sinh^2(m_2 H)}{\alpha^2 H(m_2^2 - \alpha^2)} - \frac{R_2^2}{m_1^2 - \alpha^2} \left(H + \frac{\sinh(2m_1 H)}{2m_1} \right) \\
&+ \frac{R_2^2 \sinh^2(m_1 H)}{m_1^2 H(m_1^2 - \alpha^2)} + \frac{R_1 R_2}{m_1^2 - \alpha^2} \left(\frac{1}{m_1^2 - \alpha^2} + \frac{1}{m_2^2 - \alpha^2} \right) B_3 \\
&- \frac{R_1 R_2}{m_1 m_2 H} \sinh(m_1 H) \sinh(m_2 H) \left(\frac{1}{m_1^2 - \alpha^2} + \frac{1}{m_2^2 - \alpha^2} \right) \\
&\left. - \frac{R_1^2}{m_2^2 - \alpha^2} \left(H + \frac{\sinh(2m_2 H)}{2m_2} \right) + \frac{R_1^2 \sinh^2(m_2 H)}{m_2^2 H(m_2^2 - \alpha^2)} \right\}, \quad (20)
\end{aligned}$$

$$B_1 = m_1 \cosh(\alpha H) \sinh(m_1 H) - \alpha \cosh(m_1 H) \sinh(\alpha H),$$

$$B_2 = m_2 \cosh(\alpha H) \sinh(m_2 H) - \alpha \cosh(m_2 H) \sinh(\alpha H),$$

and

$$B_3 = m_1 \cosh(m_2 H) \sinh(m_1 H) - m_2 \cosh(m_1 H) \sinh(m_2 H).$$

Comparing (19) with Fick's law of diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^* = 2 \frac{d^6}{\mu^2 D} \left(\frac{\partial p}{\partial x} \right)^2 F(\xi, \epsilon, \alpha, Da, m, \alpha_1). \quad (21)$$

Let the average of F be \bar{F} and is defined by

$$\bar{F} = \int_0^1 F(\xi, \epsilon, \alpha, Da, m, \alpha_1) d\xi. \quad (22)$$

B. Diffusion with Combined Homogeneous and Heterogeneous Chemical Reactions

We now discuss the problem of diffusion with a first-order irreversible chemical reaction taking place both in the bulk of the medium (homogeneous) as well as at the walls (heterogeneous) of the channel which are assumed to be catalytic to chemical reaction. The diffusion equation is same as given by (11), i.e.,

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) - k_1 C.$$

The differential material balance at the walls gives the boundary conditions

$$\frac{\partial C}{\partial y} + fC = 0 \quad \text{at} \quad y = h = \left[d + a \sin \frac{2\pi}{\lambda} (x - \bar{u}t) \right], \quad (23)$$

$$\frac{\partial C}{\partial y} - fC = 0 \quad \text{at} \quad y = -h = - \left[d + a \sin \frac{2\pi}{\lambda} (x - \bar{u}t) \right]. \quad (24)$$

If we introduce the dimensionless variables (12) and assume the limiting condition of Taylor [1]–[3], the diffusion equation remains as (14) but subject to the boundary conditions

$$\frac{\partial C}{\partial \eta} + \beta C = 0 \quad \text{for} \quad \eta = H = [1 + \epsilon \sin(2\pi\xi)], \quad (25)$$

$$\frac{\partial C}{\partial \eta} - \beta C = 0 \quad \text{for} \quad \eta = -H = -[1 + \epsilon \sin(2\pi\xi)], \quad (26)$$

where $\beta = fd$ is the heterogeneous reaction rate parameter corresponding to catalytic reaction at the walls.

The solution of (14) satisfying the boundary conditions (25) and (26) is

$$\begin{aligned} C(\eta) = & A' \cosh(\alpha\eta) \\ & + \frac{d^2}{\lambda D} \frac{\partial C}{\partial \xi} \frac{k_0}{\mu} \frac{\partial p}{\partial x} \left[\frac{R_1}{m_1^2 - \alpha^2} \cosh(m_1\eta) \right. \\ & - \frac{R_2}{m_2^2 - \alpha^2} \cosh(m_2\eta) + \frac{R_1}{\alpha^2 m_1 H} \sinh(m_1 H) \\ & \left. - \frac{R_2}{\alpha^2 m_2 H} \sinh(m_2 H) \right] \end{aligned} \quad (27)$$

where

$$\begin{aligned} A' = & \frac{d^2}{\lambda D} \frac{\partial C}{\partial \xi} \frac{k_0}{\mu} \frac{\partial p}{\partial x} \frac{1}{L'} \left[\frac{m_1 R_1}{m_1^2 - \alpha^2} \sinh(m_1 H) \right. \\ & - \frac{m_2 R_2}{m_2^2 - \alpha^2} \sinh(m_2 H) + \frac{\beta R_1}{m_1^2 - \alpha^2} \cosh(m_1 H) \\ & - \frac{\beta R_2}{m_2^2 - \alpha^2} \cosh(m_2 H) + \frac{\beta R_1}{\alpha^2 m_1 H} \sinh(m_1 H) \\ & \left. - \frac{\beta R_2}{\alpha^2 m_2 H} \sinh(m_2 H) \right] \end{aligned} \quad (28)$$

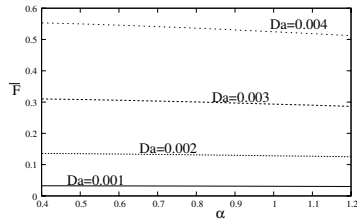
and $L' = \alpha \sinh(\alpha H) + \beta \cosh(\alpha H)$.

Substituting (27) and (13) in (18), we get

$$Q = -2 \frac{d^6}{\lambda \mu^2 D} \frac{\partial C}{\partial \xi} \left(\frac{\partial p}{\partial x} \right)^2 G(\xi, \epsilon, \alpha, \beta, Da, m, \alpha_1). \quad (29)$$

where

$$\begin{aligned} G(\xi, \epsilon, \alpha, \beta, Da, m, \alpha_1) = & (Da)^2 \times \\ & \left\{ \frac{R_2}{L'(m_1^2 - \alpha^2)} \left(\frac{m_1 R_2 \sinh(m_1 H)}{m_1^2 - \alpha^2} - \frac{m_2 R_1 \sinh(m_2 H)}{m_2^2 - \alpha^2} \right) B_1 \right. \\ & + \frac{R_1}{L'(m_2^2 - \alpha^2)} \left(\frac{m_2 R_1 \sinh(m_2 H)}{m_2^2 - \alpha^2} - \frac{m_1 R_2 \sinh(m_1 H)}{m_1^2 - \alpha^2} \right) B_2 \\ & - \frac{R_2^2 \sinh^2(m_1 H) \sinh(\alpha H)}{\alpha H L' (m_1^2 - \alpha^2)} - \frac{R_1^2 \sinh^2(m_2 H) \sinh(\alpha H)}{\alpha H L' (m_2^2 - \alpha^2)} \\ & + \frac{R_1 R_2}{\alpha H L'} \left(\frac{m_1}{m_2 (m_1^2 - \alpha^2)} + \frac{m_2}{m_1 (m_2^2 - \alpha^2)} \right) \times \\ & \quad \sinh(m_1 H) \sinh(m_2 H) \sinh(\alpha H) \\ & + \frac{\beta R_2}{L' (m_1^2 - \alpha^2)} \left(\frac{R_2 \cosh(m_1 H)}{m_1^2 - \alpha^2} - \frac{R_1 \cosh(m_2 H)}{m_2^2 - \alpha^2} \right) B_1 \\ & + \frac{\beta R_1}{L' (m_2^2 - \alpha^2)} \left(\frac{R_1 \cosh(m_2 H)}{m_2^2 - \alpha^2} - \frac{R_2 \cosh(m_1 H)}{m_1^2 - \alpha^2} \right) B_2 \\ & + \frac{\beta R_2 \cosh(m_1 H) \sinh(\alpha H)}{\alpha H L' (m_1^2 - \alpha^2)} \left(\frac{R_1 \sinh(m_2 H)}{m_2} - \frac{R_2 \sinh(m_1 H)}{m_1} \right) \\ & + \frac{\beta R_1 \cosh(m_2 H) \sinh(\alpha H)}{\alpha H L' (m_2^2 - \alpha^2)} \left(\frac{R_2 \sinh(m_1 H)}{m_1} - \frac{R_1 \sinh(m_2 H)}{m_2} \right) \\ & + \frac{\beta R_2}{\alpha^2 H L' (m_1^2 - \alpha^2)} \left(\frac{R_2 \sinh(m_1 H)}{m_1} - \frac{R_1 \sinh(m_2 H)}{m_2} \right) B_1 \\ & + \frac{\beta R_1}{\alpha^2 H L' (m_2^2 - \alpha^2)} \left(\frac{R_1 \sinh(m_2 H)}{m_2} - \frac{R_2 \sinh(m_1 H)}{m_1} \right) B_2 \\ & + \frac{\beta R_2 \sinh(m_1 H) \sinh(\alpha H)}{\alpha^3 m_1 H^2 L'} \left(\frac{R_1 \sinh(m_2 H)}{m_2} - \frac{R_2 \sinh(m_1 H)}{m_1} \right) \\ & + \frac{\beta R_1 \sinh(m_2 H) \sinh(\alpha H)}{\alpha^3 m_2 H^2 L'} \left(\frac{R_2 \sinh(m_1 H)}{m_1} - \frac{R_1 \sinh(m_2 H)}{m_2} \right) \\ & - \frac{R_2^2}{m_1^2 - \alpha^2} \left(H + \frac{\sinh(2m_1 H)}{2m_1} \right) + \frac{R_2^2 \sinh^2(m_1 H)}{m_1^2 H (m_1^2 - \alpha^2)} \\ & + \frac{R_1 R_2}{m_1^2 - \alpha^2} \left(\frac{1}{m_1^2 - \alpha^2} + \frac{1}{m_2^2 - \alpha^2} \right) B_3 \\ & - \frac{R_1 R_2}{m_1 m_2 H} \sinh(m_1 H) \sinh(m_2 H) \left(\frac{1}{m_1^2 - \alpha^2} + \frac{1}{m_2^2 - \alpha^2} \right) \\ & \left. - \frac{R_1^2}{m_2^2 - \alpha^2} \left(H + \frac{\sinh(2m_2 H)}{2m_2} \right) + \frac{R_1^2 \sinh^2(m_2 H)}{m_2^2 H (m_2^2 - \alpha^2)} \right\}. \end{aligned} \quad (30)$$

Fig. 2. Effect of Da on \bar{F} for $m = 2.0$, $\alpha_1 = 0.02$ and $\epsilon = 0.2$.

Comparing (29) with Fick's Law of Diffusion, we find that the solute is dispersed relative to a plane moving with the mean speed of the flow with an effective dispersion coefficient D^* given by

$$D^* = 2 \frac{d^6}{\mu^2 D} \left(\frac{\partial p}{\partial x} \right)^2 G(\xi, \epsilon, \alpha, \beta, Da, m, \alpha_1). \quad (31)$$

The average of G denoted by \bar{G} is defined as

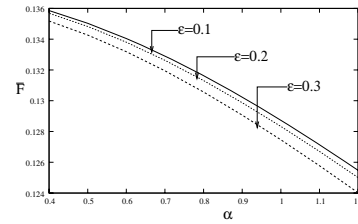
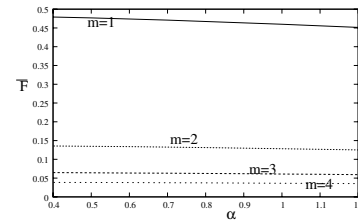
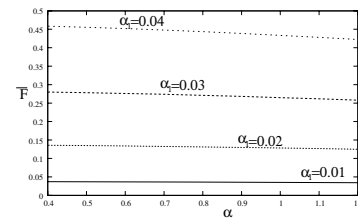
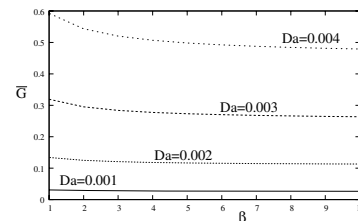
$$\bar{G} = \int_0^1 G(\xi, \epsilon, \alpha, \beta, Da, m, \alpha_1) d\xi. \quad (32)$$

III. RESULTS AND DISCUSSION

The effects of various parameters on the average effective dispersion coefficient can be observed through the functions $\bar{F}(\xi, \epsilon, \alpha, Da, m, \alpha_1)$ (for homogeneous case) and $\bar{G}(\xi, \epsilon, \alpha, \beta, Da, m, \alpha_1)$ (for combined homogeneous and heterogeneous case) given by equations (22) and (32), respectively. \bar{F} and \bar{G} have been obtained by numerical integration using MATHEMATICA software for different values of relevant parameters and presented graphically. The important parameters involved in the expressions are: the amplitude ratio ϵ , the homogeneous reaction parameter α , the heterogeneous reaction rate parameter β , the permeability parameter (or Darcy number) Da , the couple stress parameter m , and the slip parameter α_1 .

A. Homogeneous Chemical Reaction

Figs.2-5 show that average effective dispersion coefficient \bar{F} decreases with homogeneous reaction rate parameter α . This implies that homogeneous chemical reaction tends to decrease the dispersion of the solute. This result is expected since increase in α leads to increasing number of moles of solute undergoing chemical reaction, which results in the decrease of dispersion. The result that dispersion decreases with α agrees with previous results obtained by Gupta and Gupta [6], Dutta et al. [9], Ramana Rao and Padma [7]- [8], Padma and Ramana Rao [5], Shukla et al. [11]. Further, average dispersion increases with permeability parameter (or Darcy number) Da (Fig.2) but decreases with amplitude ratio ϵ (Fig.3), couple stress parameter m (Fig.4) and slip parameter α_1 (Fig.5). The result that dispersion decreases with couple stress parameter m agrees with the result obtained by Soundalgekar and Chaturani [10].

Fig. 3. Effect of ϵ on \bar{F} for $m = 2.0$, $\alpha_1 = 0.02$ and $Da = 0.002$.Fig. 4. Effect of m on \bar{F} for $\epsilon = 0.2$, $\alpha_1 = 0.02$ and $Da = 0.002$.Fig. 5. Effect of α_1 on \bar{F} for $\epsilon = 0.2$, $m = 2$ and $Da = 0.002$.Fig. 6. Effect of Da on \bar{G} for $\alpha = 1.0$, $m = 2.0$, $\alpha_1 = 0.02$ and $\epsilon = 0.2$.

B. Combined Homogeneous and Heterogeneous Chemical Reactions

Figs.6-10 show the effects of various parameters on the average dispersion coefficient \bar{G} for the case of combined first order chemical reactions both in the bulk and at the walls. Average dispersion coefficient \bar{G} increases with permeability parameter Da (Fig.6). But \bar{G} decreases with amplitude ratio ϵ (Fig.7), couple stress parameter m (Fig.8), slip parameter α_1 (Fig.9) and homogeneous chemical reaction parameter α (Fig.10). Further, Figs.6-10 reveal that dispersion decreases with heterogeneous reaction parameter β . The result that dispersion decreases with heterogeneous reaction rate parameter β agrees with previous results obtained by Gupta and Gupta [6], Ramana Rao and Padma [7]- [8], Padma and Ramana Rao [5]. This implies that heterogeneous chemical reaction tends to decrease the dispersion of the solute.

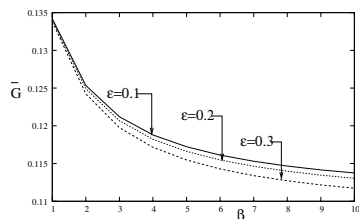


Fig. 7. Effect of ϵ on \bar{G} for $m = 2.0$, $Da = 0.002$, $\alpha_1 = 0.02$ and $\alpha = 1.0$.

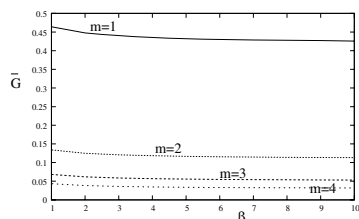


Fig. 8. Effect of m on \bar{G} for $\alpha = 1.0$, $Da = 0.002$, $\alpha_1 = 0.02$ and $\epsilon = 0.2$.

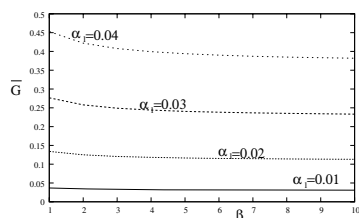


Fig. 9. Effect of α_1 on \bar{G} for $m = 2.0$, $Da = 0.002$, $\alpha = 1.0$ and $\epsilon = 0.2$.

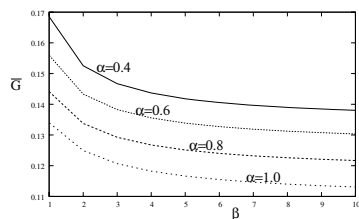


Fig. 10. Effect of α on \bar{G} for $m = 2.0$, $Da = 0.002$, $\alpha_1 = 0.02$ and $\epsilon = 0.2$.

IV. CONCLUSION

The dispersion of a solute in peristaltic motion of a couple stress fluid through a porous medium with slip condition in the presence of both homogeneous and heterogeneous chemical reactions has been studied under long wavelength approximation and Taylor's limiting condition. It is observed that average effective coefficient of dispersion tends to increase with permeability parameter Da . But it tends to decrease with homogeneous chemical reaction rate parameter α , couple stress parameter m , amplitude ratio ϵ , slip parameter α_1 and heterogeneous chemical reaction rate parameter β .

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