

# Development of a Neural Network based Algorithm for Multi-Scale Roughness Parameters and Soil Moisture Retrieval

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**Abstract**—The overall objective of this paper is to retrieve soil surfaces parameters namely, roughness and soil moisture related to the dielectric constant by inverting the radar backscattered signal from natural soil surfaces.

Because the classical description of roughness using statistical parameters like the correlation length doesn't lead to satisfactory results to predict radar backscattering, we used a multi-scale roughness description using the wavelet transform and the Mallat algorithm. In this description, the surface is considered as a superposition of a finite number of one-dimensional Gaussian processes each having a spatial scale. A second step in this study consisted in adapting a direct model simulating radar backscattering namely the small perturbation model to this multi-scale surface description. We investigated the impact of this description on radar backscattering through a sensitivity analysis of backscattering coefficient to the multi-scale roughness parameters.

To perform the inversion of the small perturbation multi-scale scattering model (MLS SPM) we used a multi-layer neural network architecture trained by backpropagation learning rule. The inversion leads to satisfactory results with a relative uncertainty of 8%.

**Keywords**—Remote sensing, rough surfaces, inverse problems, SAR, radar scattering, Neural networks and Fractals.

## I. INTRODUCTION

THE retrieval of information related to physical surface parameters is a major objective of many studies in remote sensing investigations especially SAR(Synthetic Aperture Radar) applications. In that context, modeling radar backscattering through natural surfaces has become an important theme of research and active remote sensing has shown its utility for many applications in hydrology, geology and other fields.

Most traditional electromagnetic models [1][2] consider natural surfaces as single scale zero mean stationary Gaussian random processes. Roughness behavior is characterized by statistical parameters like the rms height  $s$  and the correlation length  $l$ .

Recent studies have shown that in natural conditions the agreement between experimental measurements and theoretical values is usually poor due to the large variability of the correlation function. As a consequence, backscattering models have often failed to predict correctly backscattering

[3,4]. Many mathematical works dealing with natural surfaces description have shown that they are better described as self-affine random processes than as stationary processes.

In the context of SAR applications, Mattia et al. [1][2][4] have described one dimensional rough surfaces as band limited fractal random processes and studied the impact of this multi-scale description on radar backscattering. However, natural surface roughness changes from one direction to another and one-dimensional profiles are then insufficient. Thus, bi-dimensional profiles are required to describe more adequately natural surfaces. In this paper, we propose a theoretical modeling approach using the small perturbation model to describe radar backscattering on multi-scale bi-dimensional surfaces.

Although significant progress has been made in the ability to acquire remotely sensed data, extracting soil moisture and roughness parameters of natural surfaces has been problematic for many reasons. In fact, many previous studies have dealt with model-based retrieval algorithm and have encountered many problems like the lack of information about the characteristics of natural surface roughness as well as the range of roughness parameters to use. In another hand, the uncertainties concerning the validity of the scattering models when applied to natural roughness conditions reduces the accuracy of the retrieval procedure. In addition, the relationship between the backscattering coefficient and soil parameters is non linear and the problem of retrieving parameters may be ill-posed and it may be not possible to separate the contributions from different mechanisms, making necessary the retrieval of several parameters simultaneously.

We propose in this paper a neural network based inversion procedure using a multi-layer neural network (NN) architecture trained by a backpropagation learning rule.

In the first section, we present the multi-scale surface description and study the impact of this multi-scale roughness description on radar backscattering using the small perturbation model by investigating the sensitivity of backscattering to the new surface parameters and to the dielectric constant.

In the second section, the neural network based inversion procedure is presented. The results and their accuracy are given in the last section.

II. CHARACTERISTICS OF NATURAL SURFACE ROUGHNESS AND THEIR EFFECTS ON RADAR BACKSCATTERING

The weakness of the classical description of natural surfaces is the large spatial variability which affects the correlation function and makes classical roughness parameters very variable [1][2]. In that context, many previous works suggested that natural surfaces are better described as self affine random processes (1/f processes) than as stationary processes. The statistical properties of 1/f processes are invariant for scale transformation so that phenomena described by 1/f random processes occur at every spatial scale. The self-affine property of 1/f processes is well represented by orthonormal wavelet decomposition [6] in the same way as stationary processes (single scale processes) are very well described by the Fourier decomposition.

Previous works [1][2] have described one-dimensional surfaces by means of wavelet transform. In this paper, we model natural roughness as a multi-scale process having an 1/f spectrum with a finite range of spatial scales going from a few millimeters b (b)/λ/10) to several meters (B/resolution cell) [1][2]. The surface is considered as a superposition of a finite number of one-dimensional gaussian processes each having a spatial scale [1].

Wornell has demonstrated that 1/f processes can be synthesized by exploiting a Karhunen-Loève expansion in terms of orthonormal wavelet functions [16].

One-dimensional natural surfaces description can be obtained by using an approximation of this expansion:

$$z_p(x) = \sum_{m=-p_1}^{p_2} \sum_{n=-\infty}^{+\infty} z_n^m \psi_n^m(x/L) \tag{1}$$

where  $z_n^m$  is a collection of gaussian random independent variables with variance  $\sigma^2$ ,  $x$  a normalized distance with respect to an arbitrary length  $L=2bb$  and  $\psi_n^m$  a collection of orthonormal wavelet. In this work, we have used 4th order Daubechies wavelets. In this model,  $z_p(x)$  is a superposition of a finite number of random gaussian processes, each characterized by an increasing spatial scale.

As natural roughness changes from one direction to another, one-dimensional profiles are insufficient. Thus, bi-dimensional profiles are required to describe more adequately natural surfaces. Wavelet theory can be extended from one-dimensional to two-dimensional case using the separable dyadic multi-resolution analysis introduced by Mallat [7] [8]. The bi-dimensional wavelet transform gives us respectively the vertical wavelet component (2), the horizontal wavelet (3) component and the diagonal wavelet component (4) of the height  $z$  considered as a 1/f process over a finite range of spatial scales going from an inner spatial scale  $b$  of a few millimeters to an outer spatial scale  $B$  of several meters.

$$z_p^V(x,y) = \sum_{m_x=0}^p \sum_{m_y=0}^p \sum_{n_x=-\infty}^{+\infty} \sum_{n_y=-\infty}^{+\infty} z_{n_x}^{m_x} z_{n_y}^{m_y} \psi\left(\frac{2^{m_x}}{B}x-n_x\right)\phi\left(\frac{2^{m_y}}{B}y-n_y\right) \tag{2}$$

$$z_p^H(x,y) = \sum_{m_x=0}^p \sum_{m_y=0}^p \sum_{n_x=-\infty}^{+\infty} \sum_{n_y=-\infty}^{+\infty} z_{n_x}^{m_x} z_{n_y}^{m_y} \phi\left(\frac{2^{m_x}}{B}x-n_x\right)\psi\left(\frac{2^{m_y}}{B}y-n_y\right) \tag{3}$$

$$z_p^D(x,y) = \sum_{m_x=0}^p \sum_{m_y=0}^p \sum_{n_x=-\infty}^{+\infty} \sum_{n_y=-\infty}^{+\infty} z_{n_x}^{m_x} z_{n_y}^{m_y} \psi\left(\frac{2^{m_x}}{B}x-n_x\right)\psi\left(\frac{2^{m_y}}{B}y-n_y\right) \tag{4}$$

where  $z_{n_x}^{m_x}$  and  $z_{n_y}^{m_y}$  are a collection of uncorrelated zero mean Gaussian random variables.

Their associated autocorrelation function (ACF) is given by the following equations:

$$\rho^i(x,y,x+\xi,y+\eta) = \langle z_p^i(x,y) z_p^i(x+\xi,y+\eta) \rangle \tag{5}$$

where  $i=V, H$  or  $D$ .

The stationary parts of  $\rho^H, \rho^V$  and  $\rho^D$  are respectively given by :

$$r_c^H(\xi,\eta) = \gamma_{0x}\gamma_{0y} \sum_{m_x=0}^p \sum_{m_y=0}^p 2^{-m_x(v_x-1)} 2^{-m_y(v_y-1)} R_{m_x}^S(\xi) R_{m_y}^M(\eta) \tag{6}$$

$$r_c^V(\xi,\eta) = \gamma_{0x}\gamma_{0y} \sum_{m_x=0}^p \sum_{m_y=0}^p 2^{-m_x(v_x-1)} 2^{-m_y(v_y-1)} R_{m_x}^M(\xi) R_{m_y}^S(\eta) \tag{7}$$

$$r_c^D(\xi,\eta) = \gamma_{0x}\gamma_{0y} \sum_{m_x=0}^p \sum_{m_y=0}^p 2^{-m_x(v_x-1)} 2^{-m_y(v_y-1)} R_{m_x}^M(\xi) R_{m_y}^M(\eta) \tag{8}$$

where  $\xi$  and  $\eta$  are the spatial horizontal and vertical extension

$$R_{m_x}^S(\xi) = \int_{-\infty}^{+\infty} \phi(\alpha)\phi\left(\alpha + \frac{2^{m_x}}{B}\xi\right)d\alpha, R_{m_x}^M(\eta) = \int_{-\infty}^{+\infty} \psi(\alpha)\psi\left(\alpha + \frac{2^{m_x}}{B}\eta\right)d\alpha \tag{9}$$

The mean variance of  $z_p^i$  for  $i=D, H$  or  $V$  in (2), (3) and (4) is:

$$s^2 = r_c^H(0,0) = r_c^D(0,0) = r_c^V(0,0) \tag{10}$$

Fig. 1, Fig. 2, and Fig. 3 represent the height of three simulated multi-scale three-dimensional surfaces. We have studied the impact of the multi-scale surface parameters namely  $v$  (Fig. 1) and  $P$  (Fig. 2 and Fig. 3).

It can be seen in Fig. 1 that for  $v=1.1$  the surface is rougher with a maximum height of 3.7 cm whereas for  $v=2$  the surface is smoother with a maximum height of 2.3 cm.

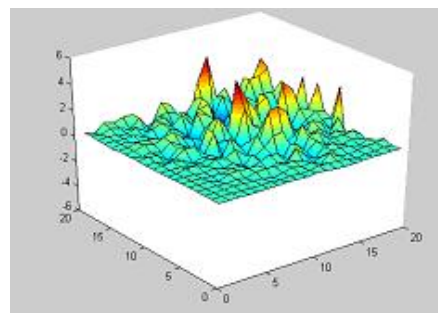


Fig. 1 3D representation of the height for a multi-scale two-dimensional surface with  $v_x=v_y=1.1$ ;  $\gamma_x=\gamma_1=0.2$  cm;  $\gamma_y=\gamma_2=0.3$  cm and  $Z_{max} = 3.7$  cm

In Figs. 2 and 3 we have varied the number of spatial scales respectively from  $P=5$  to  $P=10$ . We can notice that the surface with a larger number of spatial scales is more complex.

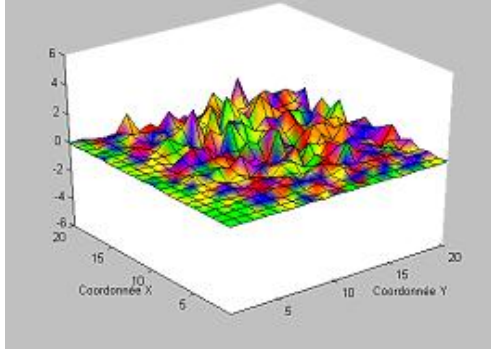


Fig. 2 3D representation of the height for a multi-scale two-dimensional surface with  $v_x=v_y=1.3$ ;  $\gamma_y=\gamma_x=0.3$  cm and  $P=5$

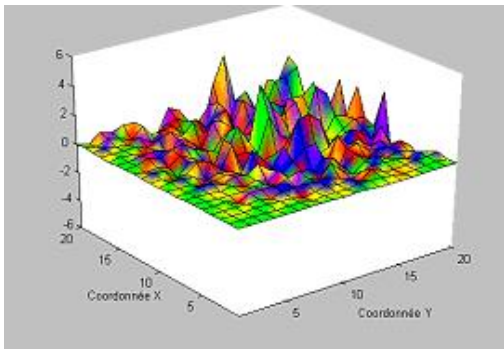


Fig. 3 3D representation of the height for a multi-scale two dimensional surface with  $v_x=v_y=1.3$ ;  $\gamma_y=\gamma_x=0.3$  cm and  $P=10$

### III. IMPACT OF MULTI-SCALE DESCRIPTION OF TWO-DIMENSIONAL NATURAL ROUGH SURFACES ON RADAR BACKSCATTERING

#### A. The Multi-Scale Small Perturbations Model (MLS SPM)

The main purpose of the present study is to develop an inversion model for soil moisture and multi-scale roughness parameters retrieval over bare soil surfaces using remotely sensed data.

The retrieval of roughness and soil moisture soil surfaces parameters from backscattered data can be carried out by using analytical models like the Integral Equation Model [9][10], the Physical Optics or the Small perturbation model, empirical models like the Oh model or semi-empirical [11]-[12]. The complexity of analytical models makes the inversion procedure difficult. The theoretical model used in this paper is the small perturbations model.

Thus, before applying the inversion procedure, the sensitivity of the backscattering coefficient to surface and radar parameters is established by using the SPM model.

#### Validity of the Small Perturbations Model

The SPM model is used when the surface height standard deviation is much smaller than wavelength and the rms slope  $s$  is not high ( $ks \ll 0.3$ ).

#### Radar Backscattering Coefficient Expression:

The backscattering coefficient according to the SPM leads to:

$$\sigma_{pq}^0 = (8k^4 (\cos \theta)^4) |\alpha_{pq}|^2 W^{(1)}(2k_x) \quad (11)$$

where  $W^{(n)}$  is the Fourier transform of the  $n$ th power of the multi-scale ACF given by (2) :

$$W^{(n)}(-2k_x, 0) = \frac{2}{\pi} \int_0^\infty \int_0^\infty \left( \frac{r_c^i(\xi, \eta)}{v_c^i(0,0)} \right)^n \cos(2k_x \xi) d\xi d\eta \quad (12)$$

$k_x$  is the  $x$  component of the incident wave number and  $r_c^i$  is respectively the horizontal, the vertical and the diagonal autocorrelation function for  $i=H, V$  and  $D$ .

$$\text{and } \begin{cases} \alpha_{hh} = \frac{\cos \theta - \sqrt{\epsilon - \sin^2 \theta}}{\cos \theta + \sqrt{\epsilon - \sin^2 \theta}} \\ \alpha_{hv} = \alpha_{vh} = 0 \\ \alpha_{vv} = (\epsilon - 1) \frac{\sin^2 \theta - \epsilon(1 + \sin^2 \theta)}{[\epsilon \cos \theta + \sqrt{\epsilon - \sin^2 \theta}]^2} \end{cases} \quad (13)$$

To study the impact of the multi-scale roughness description, dielectric parameters and radar parameters on the backscattering coefficient we have varied each parameter of interest and analysed its impact on radar backscattering.

#### B. Final Stage Sensitivity to MLS Roughness Parameters

We have studied the sensitivity of the backscattering coefficient to MLS roughness parameters. We have represented the angular trends from 20 to 80 degrees. In the first step, we kept  $\gamma_0$  at 0.2 cm (Fig. 4 and Fig. 5) and varied the fractal parameter  $v_1$  de 1.1 à 2.1 in both HH and VV polarizations for ten spatial scales. Surfaces with  $v$  between 1.7 and 2.1 can be considered as smooth where as surfaces with  $v_1=1.1$  are quite rough. For all the simulations, the backscattering coefficient decreases with the incident angle. We notice that the backscattering coefficient decreases when  $V$  increases which corresponds to a decreasing fractal dimension  $D$  corresponding to a smoother surface whose diffusion properties are dominated by specular reflection.

In a following step, we kept  $v$  at 1.1 (Fig. 6 and Fig. 7) and at 2.1 and varied  $\gamma_0$  from 0.2 cm to 0.6 cm in both HH and VV polarization. In both cases, we notice that the backscattering coefficient increases with  $\gamma_0$ .

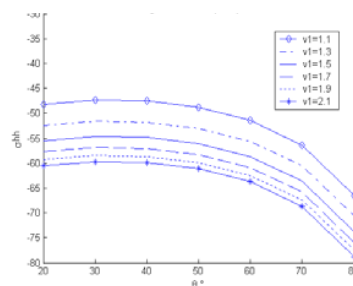


Fig. 4 Backscattering angular trends in HH polarization for  $\gamma_0$  kept at 0.2 cm and  $v$  varied from 1.1 to 2.1 and  $f=5.3$  Ghz

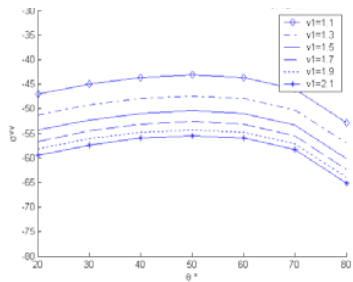


Fig. 5 Backscattering angular trends in VV polarization for  $\gamma_0$  kept at 0.2 cm and  $\nu$  varied from 1.1 to 2.1;  $f=5.3$  GHz and  $P=10$

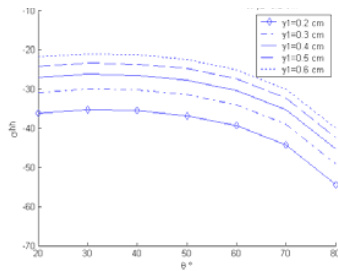


Fig. 6 Backscattering angular trends in HH polarization for  $\nu$  kept at 1.1 and  $\gamma_0$  varied from 0.2cm to 0.6cm;  $f=5.3$ GHz and  $P=10$

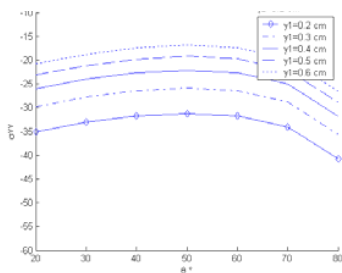


Fig. 7 Backscattering angular trends in VV polarization for  $\nu$  kept at 1.1 and  $\gamma_0$  varied from 0.2cm to 0.6cm;  $f=5.3$  GHz and  $P=10$

C. Sensitivity to the Number of Spatial Scales

We have studied the impact of the number of spatial scales  $P$  on radar backscattering for the same surfaces with the same multi-scales roughness parameters  $\nu$  and  $\gamma_0$  for HH and VV polarization. (Fig. 8 and Fig. 9).

These curves show that surfaces with the same roughness characteristics having a different number of spatial scales produce different backscattering coefficients. It can be seen that when  $P$  increases the backscattering coefficient decreases. In fact, surfaces with a higher number of spatial scales are more sensitive to roughness variations and specular reflection become more important.

D. Document Modification Sensitivity to Soil Moisture

Soil moisture is related to the complex dielectric constant  $\epsilon$ . We have studied the impact of the real part of the dielectric constant  $\epsilon_1$  and the imaginary part  $\epsilon_2$  separately. Figs 10 and 11 show the behavior of the backscattering coefficient when  $\epsilon_1$

is varied from 1 to 5 and  $\epsilon_2$  kept at 1. We notice that the backscattering coefficient  $\sigma_0$  increases when  $\epsilon_1$  increases but the variation is not very important.

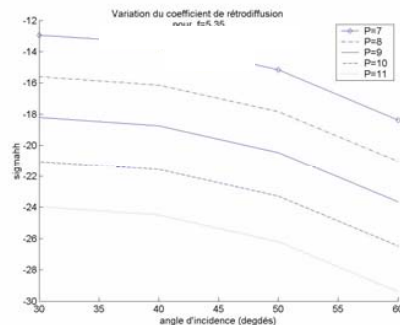


Fig. 8 Backscattering angular trends in HH polarization for  $P$  varied from 7 to 11 and  $f=5.35$  GHz

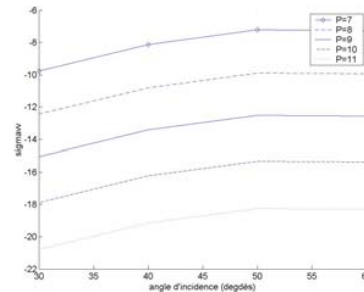


Fig. 9 Backscattering angular trends in VV polarization for  $P$  varied from 7 to 11 and  $f=5.35$  GHz

E. Sensitivity to Radar Polarization

We have studied the Impact of radar polarization on radar backscattering (Fig. 12). We notice that the signal in VV polarization is higher than in HH polarization.

F. Sensitivity to Radar Frequency

We have studied the impact of radar frequency on the backscattering coefficient.

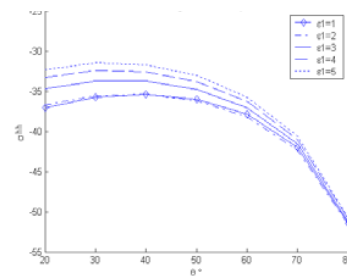


Fig. 10 Backscattering coefficient angular trends for  $\epsilon_2=1$  and  $\epsilon_1$  from 1 to 5 for HH polarization

Fig. 13 and Fig. 14 show the SPM MLS behavior when the frequency is varied from 3 to 6.2 GHz for different incident angles. We notice an increasing trend of the backscattering

coefficient with the radar frequency.

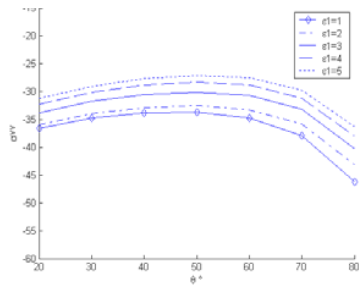


Fig. 11 Backscattering coefficient angular trends for  $\epsilon_2=1$   $\epsilon_1$  varied from 1 to 5 for VV polarization

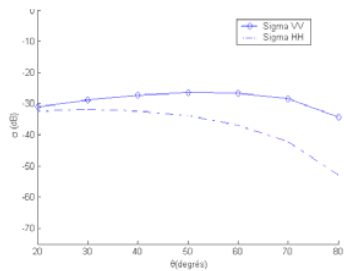


Fig. 12 Backscattering angular trends for HH and VV polarizations for the same multi-scale surface

#### IV. METHODOLOGY OF THE RETRIEVAL PROCEDURE

##### A. Inversion Procedure

In this section, an algorithm to retrieve multi-scale roughness parameters and soil moisture parameter is illustrated. The method consists of inverting the SPM direct model using a multi-layer perceptron architecture [10] and [17] (Fig. 27).

The inversion consists in retrieving roughness and soil moisture parameters  $\gamma_1, \gamma_2, v_1, v_2, \epsilon_1$  and  $\epsilon_2$  by using as input parameters the radar backscattering coefficients  $\sigma_{HH}, \sigma_{VV}$  and varying the incident angle  $\theta$  from 30 to 60 degrees. The NN is trained by learning rules using the backpropagation method.

Simulated data sets based on the SPM surface scattering model are used to train the neural network.

Before the training of the neural network for the parameters retrieval some considerations concerning the information content of the training data need to be made. If the training data are not sensitive to some of the parameters of interest the inversion for these parameters would be ineffective [10].

In this study, the direct problem is represented by the SPM model. Thus, a sensitivity analysis of the SPM model has been performed and presented in the second section to examine the dependence of the output of the scattering model to the inputs parameters. When the outputs of the scattering model became saturated or insensitive to a parameter, the parameter inversion range was narrowed. The range of parameters used in this inversion method is given in Table I.

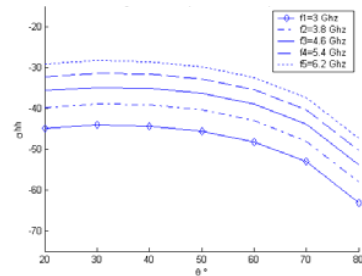


Fig. 13 Backscattering coefficient angular trends in HH polarization for 5 different frequencies from 3 to 6.2 KHz

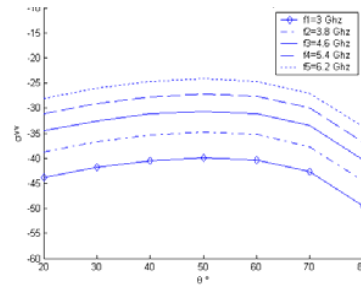


Fig. 14 Backscattering coefficient angular in VV polarization for 5 different radar frequencies from 3 to 6.2 GHz

TABLE I  
RANGE OF PARAMETERS

Range	Lower bound	Upper bound
Vertical roughness $\gamma_0$	0.2 cm	0.8 cm
horizontal roughness $v$	1.1	2.1
Complex dielectric constant: $\epsilon_1 - j \epsilon_2$	1-j	8-5 j

##### B. Neural Network Training

The first step in the inversion procedure is the generation of a set of training patterns. In this study, a total of 320 training patterns were generated by using each of the signal models  $\sigma$  ( $\Xi$ ) of the SPM backscattering coefficient. The parameters of interest  $\Xi$  used to generate the training patterns were randomly selected from within the range of parameters given by the sensitivity analysis.

Fig. 15 represents the inversion process configuration. We have a total of 8 inputs corresponding to the backscattering coefficients  $\sigma_{HH}, \sigma_{VV}$  for 4 incident angles and 6 outputs. We have used 2 hidden layers containing 40 neurons after several tests.

##### C. Inversion Algorithm Results

To illustrate the inversion techniques described in the previous section we apply them to data simulated by the SPM.

Before using the NN for the inversion we have to calculate the mean rms error of the network. We found that it converges well to a value smaller than 0.05 after 18000 iterations so that



the NN is ready for the inversion procedure.

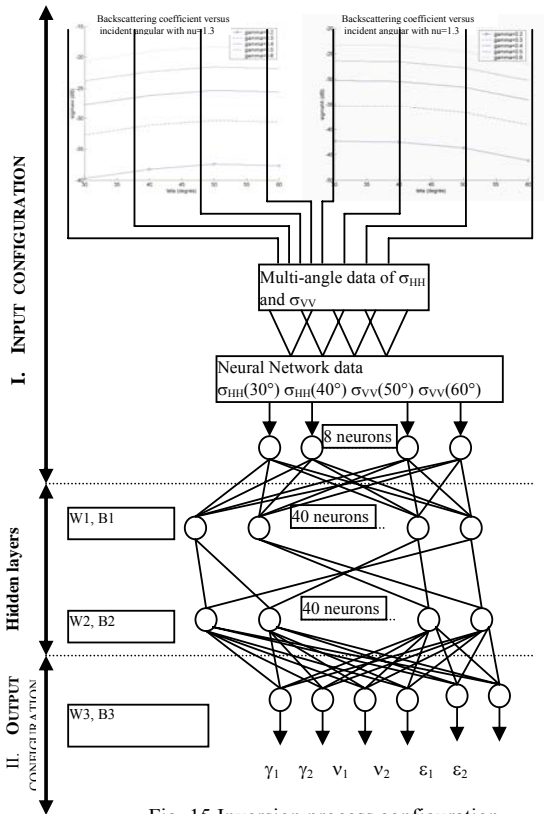


Fig. 15 Inversion process configuration

Table II and Table III present respectively the original and the retrieved data for three sets of data,  $S_1$ ,  $S_2$  and  $S_3$ .

TABLE II  
ORIGINAL VALUES

	$S_1$	$S_2$	$S_3$
$\epsilon_1$	2.0000	4.0000	6.0000
$\epsilon_2$	2.0000	1.0000	3.0000
$\nu_1$	1.3000	1.3000	1.8000
$\nu_2$	1.3000	2.1000	1.1000
$\gamma_1$ (cm)	0.2000	0.3000	0.1000
$\gamma_2$ (cm)	0.2000	0.3000	0.5000

TABLE III  
RETRIEVED VALUES AFTER THE INVERSION BY THE NN

	$S_1$	$S_2$	$S_3$
$\epsilon_1$	2.3611	3.9045	5.9694
$\epsilon_2$	1.9111	0.7259	3.0217
$\nu_1$	1.2800	1.3152	1.7843
$\nu_2$	1.3875	1.9710	1.1493
$\gamma_1$ (cm)	0.2130	0.2771	0.1093
$\gamma_2$ (cm)	0.2079	0.2861	0.4997

We can notice that the inversion has given quite satisfactory results as the original values were retrieved with an error of 8%.

## V. CONCLUSION

In this study, we have used a multi-scale roughness description using the wavelet transform and the Mallat algorithm to describe natural surface roughness and investigated the impact of this description on radar backscattering through a sensitivity analysis of backscattering coefficient to the multi-scale roughness parameters. This sensitivity study allowed us to determine the range of parameters to use in the training of the network.

To perform the inversion of the small perturbation multi-scale scattering model (MLS SPM) we used a multi-layer neural network trained by a backpropagation learning rule.

The inversion procedure has given quite satisfactory results with a mean error of 8 %.

Future work will be dedicated to the inversion of real data.

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