International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:7, No:6, 2013

Design of the Propelling Nozzles for the Launchers and Satellites

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***Abstract*—**The aim of this work is to determine the supersonic nozzle profiles used in propulsion, for the launchers or embarked with the satellites. This design has as a role firstly, to give a important propulsion, i.e. with uniform and parallel flow at exit, secondly to find a short length profiles without modification of the flow in the nozzle. The first elaborate program is used to determine the profile of divergent by using the characteristics method for an axisymmetric flow. The second program is conceived by using the finite volume method to determine and test the profile found connected to a convergent.

***Keywords*—**Characteristic method, nozzles, supersonic flow, propellers.

I. Introduction

THE conduits constitute the essential body in the propulsion, in aeronautics as into aerospace. The relaxation of gases in the conduit since the combustion chamber where reigning the great pressure to the exit where reigning the low pressure, generally the atmospheric pressure or the vacuum, gives a force of propulsion which moves the vehicle. To achieve this objective, it should be taken care that the flow in the nozzle is without presence of shock wave and that it is uniform and parallel at the exit. The profile slope of the divergent starts from the throat with a zero value then it increases according to an arc of circle up to a maximum value which is function of the exit Mach number, and then it decreases gradually to the exit or it becomes zero, see Fig. 1.

Expansion waves are emitted by the first part, the arc of circle, and are propagated in the flow while entering in interaction with those which are emitted by the lower wall. In the second part of the divergent, which is the required part, the waves are eliminated since the slope of the wall decreases in such way that the flow becomes uniform and parallel at the exit. Two important questions arise, which must be the radius of the arc of circle R_{arc} and which is its maximum slope. The several time execution of the computational code will enable us to have the two necessary values to achieve the objective.

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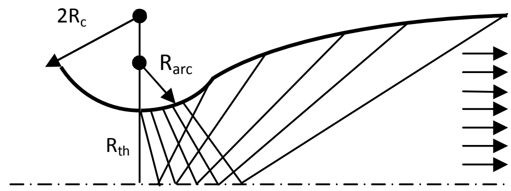

Fig. 1 Characteristics lines in the divergent

II. Mathematic Formulation

The equations of the method of the characteristics treating an axisymmetric compressible flow are given by two differential equations connecting the velocity V, the Mach number M and the angle θ, see Fig. 2 [1]. From each point two lines characteristic ξ and η are propagated forming an angle α with the vector speed V which itself formed an angle θ with the x axis.

$$u = V\cos\theta \quad ; v = V\sin\theta \quad ; \quad \sin\alpha = \frac{1}{M} \tag{1}$$

$$\frac{dy}{dx} = \tan(\theta \mp \alpha) \tag{2}$$

$$\frac{1}{V}\left(\frac{dv}{d\theta}\right) = \mp\tan\alpha + \frac{\sin\alpha.\tan\alpha.\sin\theta}{\sin(\theta\mp\alpha)}\frac{1}{r}\left(\frac{dr}{d\theta}\right) \tag{3}$$

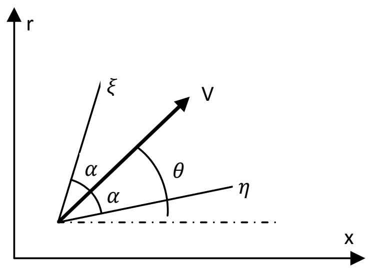

Fig. 2 Characteristics lines at a point

The procedure of calculation is to find the flow parameters at point 3, Fig. 3 represents the intersection of the two characteristics ξ and η resulting from two other points 1 and 2.

From (2), one will have:

$$r_3 - r_1 = (x_3 - x_1)\tan(\theta - \alpha) \quad ; \quad \eta = const \tag{4}$$

$$r_3 - r_2 = (x_3 - x_2)\tan(\theta + \alpha) \quad ; \quad \xi = const \tag{5}$$

Multiplying the (3) by$d\theta.\cot\alpha$, we obtain the form

$$d\theta \pm \frac{\cot\alpha}{V}dV \mp \frac{\sin\alpha.\sin\theta}{\sin(\theta\mp\alpha)}\frac{dr}{r} = 0 \tag{6}$$

Thus

$$(\theta_3-\theta_1)+Q_1(V_3-V_1)-\frac{G_1}{r_1}(r_3-r_1)=0\ \ ;\ \eta=const\ (7)$$
$$(\theta_3-\theta_2)-Q_2(V_3-V_2)+\frac{G_2}{r_2}(r_3-r_2)=0\ \ ;\ \xi=const(8)$$

where:

$$Q=\frac{\cot\alpha}{V}$$
$$F=\frac{\sin\alpha.\sin\theta}{\sin(\theta+\alpha)}(\ \xi=const) \quad (9)$$
$$G=\frac{\sin\alpha\,.\sin\theta}{\sin(\theta-\alpha)}(\ \eta=const)$$

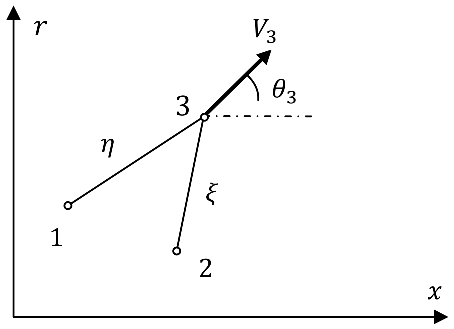

Fig. 3 Intersection of two characteristics

III. Calculation and Programming

In the first it is necessary to calculate the minimum radius of the arc of the circle. This arc must meet the Mach wave ξ coming on the opposite side. The angle of inclination of this wave is function of the throat Mach number M_{th} on the side of the wall such as:

$$\beta_{th}=f(M_{th})=\text{asin}\left(\frac{1}{M_{th}}\right) \quad (10)$$

The Mach number M_{th} is taken nearer to the unit. The problem consists here to find the radius of the arc R_{arc} in such way that the intersection would take place at the point where the angle of the wall equalθ_{max}. In this case, one supposed that the Mach wave does not become deformed and remains a straight line. It is shown easily that:

$$\frac{R_{arc}}{R_{th}}=\frac{2}{\sin\theta_{max}.\tan\beta_{th}-1+\cos\theta_{max}} \quad (11)$$

Fig. 4 shows the variation $\left(\frac{R_{arc}}{R_{th}}\right)_{min}$ according to the maximum angle of the wall. It is the minimum value to have a progressive distribution of the expansion waves resulting from the concave wall of the circle arc. More the ratio is lower than minimum more the expansion waves are concentrated at the beginning of the divergent follow-up of a longer conical wall.

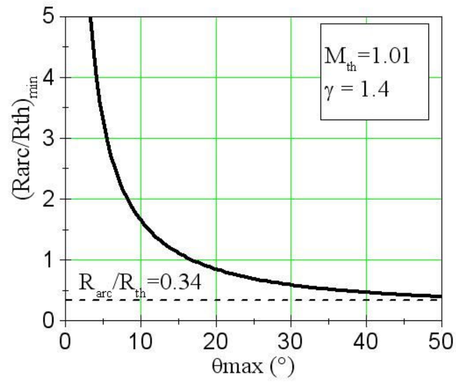

Fig. 4 Variation of minimum radius of the arc according to the angle

To avoid an exaggerated concentration of the relaxasion waves, it is necessary to increase the radius of the arc, with this intention, several executions of our programme were made until where the first wave eliminated on the convex part becomes closer to the last wave emitted by the concave part, in this case there will be a good racordemenet between the arc and the remainder of the divergent one. The same procedure must be done for different exit Mach number of the nozzle, i.e. for various angles θ_{max}. One thus obtains the graph of Fig. 5 for the plane and axisymmetric nozzles.

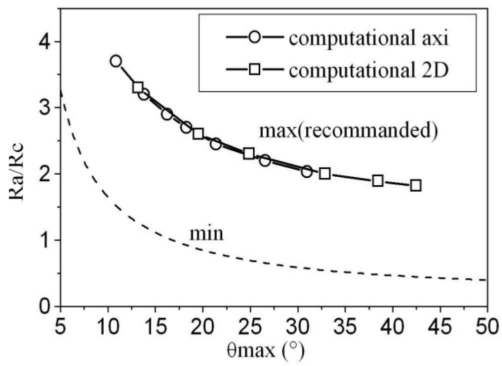

Fig. 5 Variation of maximum radius of thearc according to the angle

In 2D and axisymeric nozzles one proposes the following approximate relation:

$$\left(\frac{R_{arc}}{R_{th}}\right)max=1{,}75+1{,}75.exp\left(-\frac{\theta_{max}-12{,}03}{10{,}78}\right) \quad (12)$$

In this relation $\theta_{max}$is function of the exit Mach number of the nozzle which, in 2D, is given by the function of Prandtl Meyer $\nu(M)$ which is:

$$\nu(M)=\sqrt{\left(\frac{\gamma+1}{\gamma-1}\right)}artan\sqrt{\left(\frac{\gamma-1}{\gamma+1}\right)(M^2-1)}-artan\sqrt{(M^2-1)}(13)$$

The maximum angle of inclination is $\theta_{max}=\frac{\nu(M)}{2}$.

For the axisymmetric nozzle, θ_{max} can be obtained numerically, it is function of $\nu(M)$ and the effect of the axisymetry. The following value is proposed in Fig. 6.

$$\theta_{max} = \frac{\nu(M)}{4.\mathrm{k}} \tag{14}$$

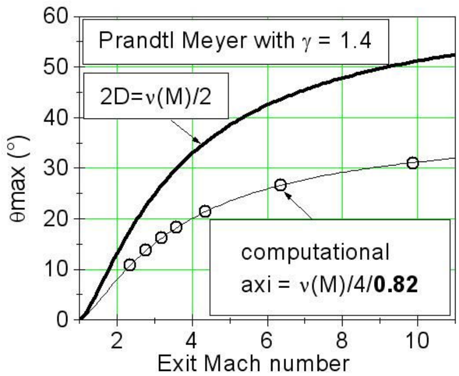

Fig. 6 Variation of the angle of inclination according to M

The coefficient K is function of nature of gas and exit Mach number. For air, $k = 0.82$ ($\gamma = 1.4$ and $M = 3$). For the steam, it is the case of the engines cryotechnic, k≈0.8, it varies little with the exit Mach number M_e of the nozzle, that is to say.

$$K = 0.81 - 4.167\ 10^{-4} * M_e + 2.5\ 10^{-4} * M_e^2 \tag{15}$$

Concerning the variation of the of section ratio of the nozzle according to the exit Mach number for the case 2D and axisymmetric is given in the Fig. 7, it is the same one as in unidimensional since the flow at the exit is uniform and parallel.

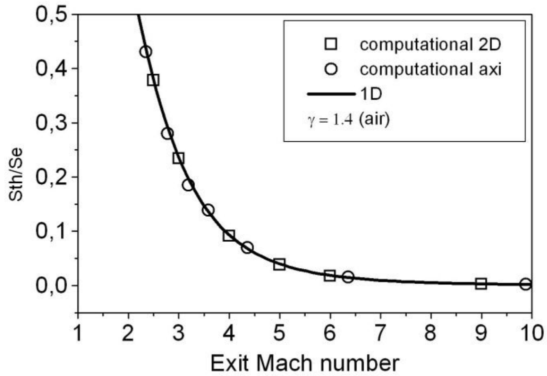

Fig. 7 Section ratio according to Me

IV. Results and Interpretation

A. Characteristic Method

The profile exactitude is function of the size grid i.e. the number of the Mach lines or division used n. In Fig. 8, we present several profiles according to the number of divisions, n=10, 20,40 and 60. We note that for n=60 the profile takes already its final form which does not change any more for n higher than 60. The same observation for an axisymmetric profile, see Fig. 9. The Fig. 10 shows the difference between 2D and axisymetric profiles giving at the exit the same Mach number with a uniforme and parallel flow. The section ratio is the same one for both. Let us notice that the plane conduit is large and longer than the axisymmetric nozzle.

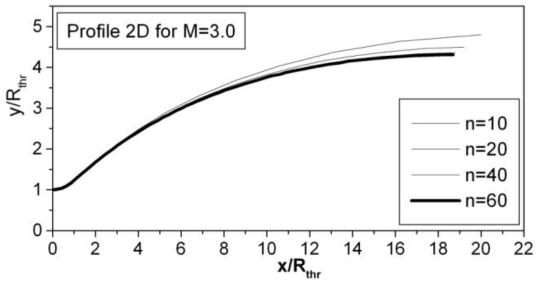

Fig. 8 2D Profile according to the size of the grid

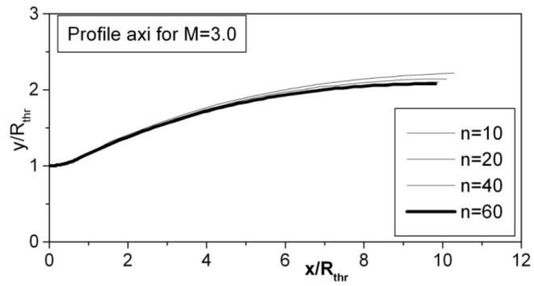

Fig. 9 Axisymetric profile according to the size profile

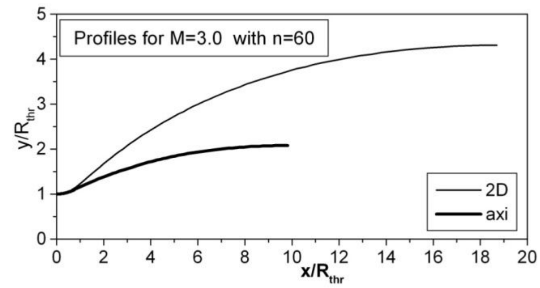

Fig. 10 Axisymmetric and 2D profiles for M=3.0

To be sure that calculation is well made, it is necessary to draw the characteristic lines in the calculation domain. Here, one presents only the axisymmetric profiles. Fig. 11 shows the characteristic lines propagating since the wall in arc then enter in interaction with those which come from the lower wall, all the lines are eliminated by the convex wall before reaching the exit of the nozzle.

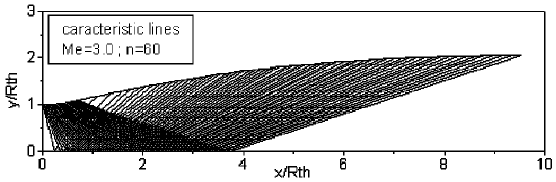

Fig. 11 Characteristics lines in the diverging of the nozzle

We can also draw the Mach number contours in the divergent one to see the axisymetry of the flow, see Fig. 12.

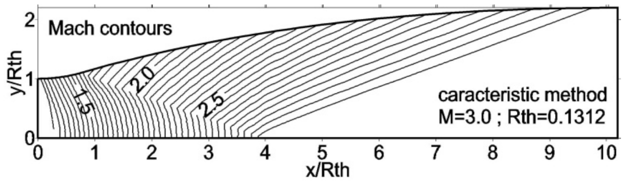

Fig. 12 Mach-contours in the diverging of the nozzle

Our objective is to determine the exact profile for a cryotechnic engine whose characteristics are given in Table I.

TABLE I
CHARACTERISTICS OF CRYOTECHNIC ENGINE

Gaz	γ	r (J/Kg.K)	De(m)	L(m)
H2O	1.329	462	**1.76**	**3.0**
Qm(Kg/s)	Ve(m/s)	S/Sth	P0(bars)	T0(K)
270	4000	**45**	110	4000

This type of nozzle is used in the cryotechnic launchers, called Vulcan engine. Considering the ideal nozzle obtained with the characteristics method are long, one uses truncated ideal nozzle with 1/3 of the divergent throat since the large variation of kinetic energy is done in the beginning of the divergent. The advantage of the truncated nozzles is to gain much over the length therefore the weight without much loss of the thrust.

One tests several exit Mach number in such way that to 1/3 of the throat to finds the diameter De=1.76m. One obtains the results appearing in Table II.

The ideal nozzle is presented on Fig. 13. The divergent profile of the truncated ideal nozzle is represented on Fig. 14.

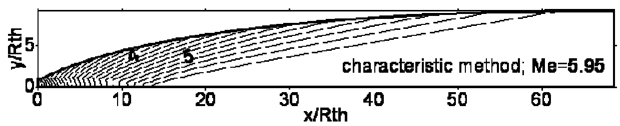

Fig. 13 Mach-contours in the ideal nozzle with characteristic methods

Truncated nozzle profile starts with an arc whose radius equal 0.2719m until a maximum slope of 27.068° connected to a convex part whose approximate form is exponential see Fig. 14.

We propose the following form such as:

$$\frac{y}{R_{th}} = y1 + A1.\exp\left[-\frac{\left(\frac{x}{Rth}-x1\right)}{t1}\right] \qquad (16)$$

$$0.943 \leq \frac{x}{Rth} \leq 22.8 \quad ; \quad Rth = 0.13m$$

with:

$$y1 = 8.77\,,\ A1 = -7.90$$
$$x1 = 0,\ t1 = 17.158$$

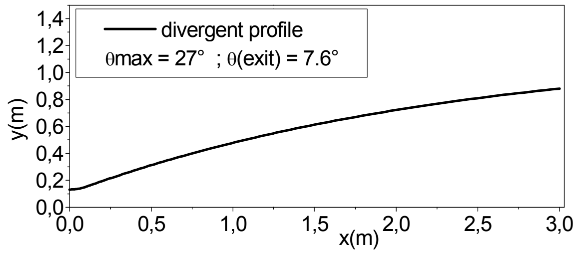

Fig. 14 Ideal truncated nozzle profile

TABLE II
CHARACTERISTICS OF IDEAL AND TRUNCATED NOZZLES

$R_{th} = .1312m$; $Rarc = 0.2719$, $\gamma = 1.329$, $T_0 = 4000K$, $\theta_{max} = 27.068°$			
ideal nozzle (characteristic method)			
M	Velocity (m/s)	L(m)	De(m)
5.95	3570	9	2.426
Fp (KN)	Qm (Kg/s)	S/S*	θ_{exit}(°)
1024.19	285.37	88	0
truncated ideal nozzle (CFD)			
M	Velocity (m/s)	L(m)	De(m)
Axis = 5.62 Wall = 4,78	Axis = 3542 Wall = 3440	**3.0**	**1.76**
Fp (KN)	Qm (Kg/s)	S/S*	θ_{exit}(°)
1019.74	285.12	**45**	7.63

Fig. 15 shows that the curve of approximation passes by all points constituting the profile of the truncated nozzle. The profile given by the preceding expression is reserved only for this nozzle, Table II, it corresponds well to the engine Vulcan 1 of the launcher ARIANE 5. It is the first family of profiles.

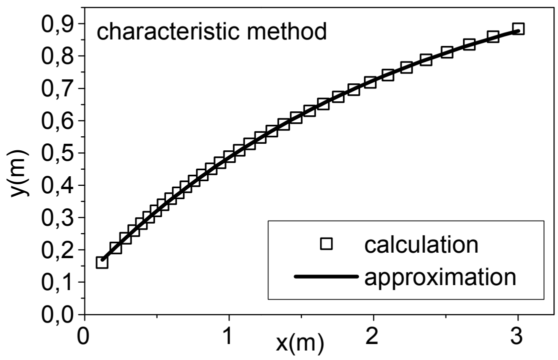

Fig. 15 Truncated nozzle profile approximation

B. CFD Application

We will test the profile found by the characteristics method with our computer code by using the finite volume method [2], [3]-[4] in order to determine the flow parameters in the nozzle which starts by a convergent of 45° of conicity followed by an arc of circle to the throat. The convergent is connected to the divergent profile found by the characteristics method. The grid used is represented on Fig. 16.

The results are function of grid size, number of iterations and the CFL [5]. The interest to study of the flow by using the finite volume method is firstly, to compare with the characteristics method and to see how the expansion waves are propagated in the nozzle and the inexistence of shock wave. Secondly, to be able to calculate the parameters at the exit of the truncated nozzle especially flow rate and the thrust. Fig. 17 shows the Mach contours in all truncated nozzle, they almost forms inclined lines in the right part of the divergent. It should be noted that the flow is modified a little compared to the characteristic method.

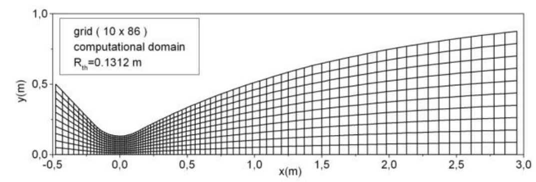

Fig. 16 Grid in the nozzle

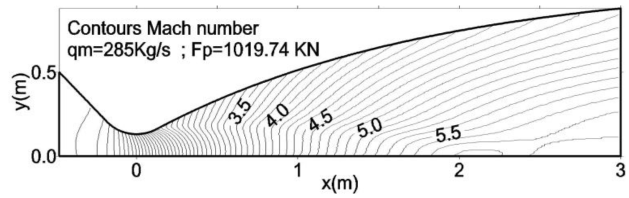

Fig. 17 Mach contours in the nozzle

The variation of the other parameters is represented on Figs. 18, 19, and 20. It is noticed that the axis Mach number is higher than that on the level of the wall, at the exit of the truncated nozzle is 5.62 and 4,78 respectively and the temperature is 647K and 844K respectively. For the pressure it is of 0.055 bars and 0.192 bars respectively. Let us notice that the pressure at the exit is lower than the atmospheric pressure at the time of launcher takeoff, what causes a compression of gas downstream from the nozzle which will disappear after 15 km of altitude.

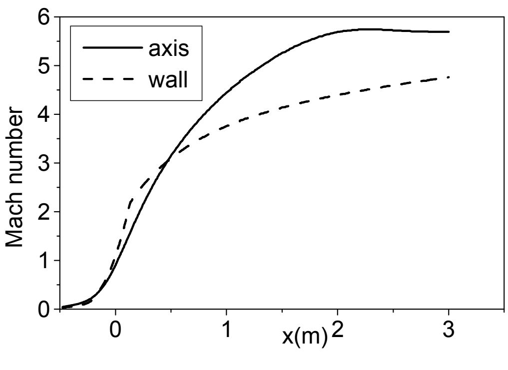

Fig. 18 Mach number evolution in the nozzle

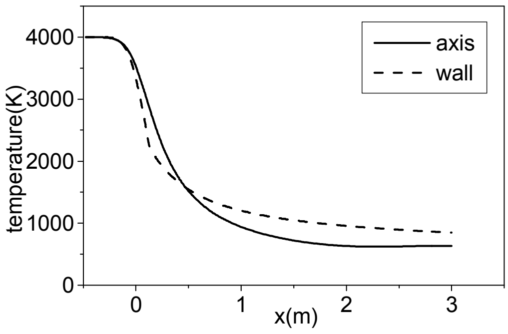

Fig. 19 Temperature evolution in the nozzle

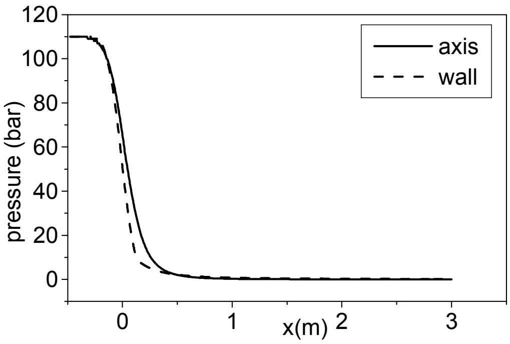

Fig. 20 Pressure evolution in the nozzle

V. Conclusion

In conclusion, we can confirm that the profiles of the propelling nozzles are traced by using, firstly, the method of characteristics in such way that the flow at exit is uniform and parallel; in this case one obtains the ideal nozzle form which is longer. Secondly, it is necessary to use the truncated nozzle deduced from the ideal nozzle which must be tested by a computer code which takes account of the flow at inlet of convergent to the exit of the nozzle. Let us note of course that the results found in Table II are identical to those of the engine Vulcan 1 of the launcher ARIANE 5.

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