Design of Non-uniform Circular Antenna Arrays Using Firefly Algorithm for Side Lobe Level Reduction

Gopi Ram, Durbadal Mandal, Rajib Kar, Sakti Prasad Ghoshal

Abstract—A design problem of non-uniform circular antenna arrays for maximum reduction of both the side lobe level (SLL) and first null beam width (FNBW) is dealt with. This problem is modeled as a simple optimization problem. The method of Firefly algorithm (FFA) is used to determine an optimal set of current excitation weights and antenna inter-element separations that provide radiation pattern with maximum SLL reduction and much improvement on FNBW as well. Circular array antenna laid on *x-y* plane is assumed. FFA is applied on circular arrays of 8-, 10-, and 12- elements. Various simulation results are presented and hence performances of side lobe and FNBW are analyzed. Experimental results show considerable reductions of both the SLL and FNBW with respect to those of the uniform case and some standard algorithms GA, PSO and SA applied to the same problem.

Keywords—Circular arrays, First null beam width, Side lobe level, FFA.

I. INTRODUCTION

A LOT of research works have been carried out in the past few decades on different antenna arrays in order to get improved radiation pattern. In many applications it is necessary to design antennas with very directive characteristics to meet the demand of long distance communication. An antenna array is formed by assembly of radiating elements in an electrical or geometrical configuration. Total field of the antenna array is found by vector addition of the fields radiated by all individual current excitation elements [1]. To provide a very directive pattern, it is necessary that the fields from the array elements must add constructively in some desired direction and add destructively and cancel each other in the remaining space. This is important to reduce interference from the side lobe of the antenna. There are several parameters by varying which the radiation pattern can be modified [1]-[10]. These parameters are geometrical configurations (e.g. linear, circular, planar, spherical etc.), inter-element spacing, individual excitations (amplitude and phase) and relative pattern of individual elements [1]. A circular array has all its elements placed along the perimeter of a circle. Circular arrays are arrays that have a configuration of very practical interest. Its applications span

radio detection finding, air and space navigation, underground propagation, radar, sonar and many other systems. In this work, the antenna array design problem consists of finding current excitation weights and antenna inter-element separations that provide a radiation pattern with maximum SLL reduction and FNBW reduction as well.

Different evolutionary optimization algorithms such as simulated annealing algorithms [11], genetic algorithm (GA) [2]-[12], particle swarm optimization (PSO) [13], [14] etc. have been widely used to the synthesis of design methods capable of satisfying constraints which would be unattainable. When considering global optimization methods for antenna array design, GA seems to be the promising one. Standard GA (herein referred to as Real Coded GA (RGA)) has a good performance for finding the promising regions of the search space, but it is prone to revisiting the same suboptimal solutions. Particle Swarm Optimization (PSO) is an evolutionary algorithm developed by Eberhart et al. [13]. PSO is simple to implement and its convergence may be controlled via few parameters. The limitations of the conventional PSO are that it may be influenced by premature convergence and stagnation problem. Recently, a new metaheuristic search algorithm, called Firefly algorithm (FFA), has been developed by Yang [15]-[18]. FFA mimics some characteristics of tropic firefly swarms and their flashing behavior. A firefly tends to be attracted towards other fireflies with higher flash intensity. This algorithm is thus different from PSO and can have two advantages: local attractions and automatic regrouping. As light intensity decreases with distance, the attraction among fireflies can be local or global, depending on the absorbing coefficient, and thus all local modes as well as global modes will be visited. In addition, fireflies can also subdivide and thus regroup into a few subgroups because neighboring attraction is stronger than long-distance attraction; thus it can be expected each subgroup will swarm around a local mode. The technique of FFA is used to solve electromagnetic problems due to their robustness, wide range of applications and readiness in their implementation. FFA is used in this paper to get the desired optimal pattern. The rest of the paper is arranged as follows: in Section II, the general design equations for the non-uniformly placed asymmetric circular antenna arrays are stated. Then, in Section III, FFA is introduced in brief. Numerical simulation results are presented in Section IV. Finally, the paper concludes with a summary of the work in Section V.

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II. DESIGN EQUATIONS

Fig. 1 assumes the geometry of a circular array of N isotropic sources is laid on x-y plane having a radius 'a' and scanning at point P in the far field. The elements in the non-uniform circular antenna array are taken to be isotropic sources, so the radiation pattern of the array can be described by its array factor. In the x-y plane, the array factor for the circular array shown in Fig. 1 is given by [1]:

$$AF(\phi, I, d) = \sum_{n=1}^{N} I_n e^{j(ka\cos(\phi-\phi_n)+\alpha_n)} \qquad (1)$$

where

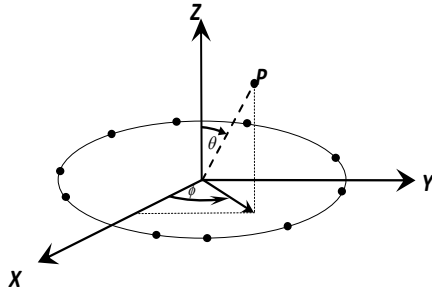$$ka = 2\pi a / \lambda = \sum_{i=1}^{N} d_i, \qquad (2)$$

Fig. 1 Geometry of non-uniform circular array laid on the *x-y* plane with N isotropic elements scanning at a point P in the far field

$$\phi_n = (2\pi / ka)\sum_{i=1}^{n} d_i ; \qquad (3)$$

$k = 2\pi / \lambda$, λ being the wavelength of operation;
$\theta =$ Elevation angle;
$\phi =$ Azimuth angle;
$\phi_n =$ Angular location of nth element along the x-y plane;

$$\alpha_n = -ka\cos(\phi_0 - \phi_n) \qquad (4)$$

In this case the array factor can be written

$$AF(\phi_n, I, d) = \sum_{n=1}^{N} I_n \exp\{jka[\cos(\phi-\phi_n) - \cos(\phi_0-\phi_n)]\} \qquad (5)$$

where $I = [I_1, I_2,........ I_N]$; I_n represent the excitation of the nth element of the array; $d = [d_1, d_2,...d_N]$; d_n represent the distance from element n to n+1. ϕ_0 is the angle where global maximum is attained in $\phi = [-\pi, \pi]$. In our design problem ϕ_0 is chosen as 0 degree; ϕ_0 is the maximum radiation angle. The design goal in the paper is to find the optimum set of values of I_n and d_n in order to get the optimal SLL reduction and

FNBW reduction as well in the radiation pattern in the desired direction ϕ. I_n and d_n are used to change the antenna pattern. After defining the array factor, the next step in the design process is to formulate the objective function which is to be minimized. The objective function J may be written as

$$J = W_1 \times |AF(\phi_{msl}, I_n)| / |AF(\phi_0, I_n)| + W_2 \times (FNBW_{computed} - FNBW(I_n = 1)) \qquad (6)$$

where $FNBW$ is an abbreviated form of first null beam width or in simple terms angular width between first nulls on either side of the main beam. Thus $FNBW_{computed}$ and $FNBW(I_n = 1)$ basically refer to the computed first null beam width in radian for the non-uniform excitation case and for uniform excitation case, respectively. The second term in (6) is computed only if $FNBW_{computed} < FNBW(I_n = 1)$ and the corresponding solution set of I_n and d_n is retained in the active population, otherwise discarded. Further W_1 and W_2 are the weighting factors. ϕ_{msl} is the angle where maximum side lobe $AF(\phi_{msl}, I_n)$ is attained on either side of the main beam. The weights W_1 and W_2 are chosen in such a way that optimization of SLL remains more dominant than optimization of $FNBW$ and J never becomes negative. Minimization of J means maximum reduction of SLL and lesser $FNBW_{computed}$ as compared to $FNBW(I_n = 1)$. The FFA technique employed for optimizing I_n and d_n resulting in the minimization of J and hence reduction in both the SLL and $FNBW$ are described in the next section.

III. EVOLUTIONARY TECHNIQUE EMPLOYED

FFA, developed by Yang [15]-[18], is inspired by the flash pattern and characteristics of fireflies. The basic rules for FFA are:
1) All fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex;
2) Attractiveness is proportional to their brightness, thus for any two flashing fireflies, the less bright one will move towards the brighter one, and the brightness decreases as their distance increases. If there is no brighter one than a particular firefly, it will move randomly;
3) The brightness of a firefly is affected or determined by the landscape of the cost function. For a minimization problem, the brightness can simply be inversely proportional to the value of the cost function. In this work, the cost function is *J*.

In the simplest case for minimization optimization problems, the brightness *B* of a firefly at a particular location *x* can be chosen as $B(x) = 1 / f(x)$, where *f(x)* is *J* in this work. However, the attractiveness β is relative; it should be seen in the eyes of the beholder or judged by the other fireflies. Thus, it will vary with the distance r_{ij} between firefly *i* and firefly *j*.

For a given medium with a fixed light absorption coefficient, γ, the light intensity varies with the distance r. That is

$$B = B_0 e^{-\gamma r} \qquad (7)$$

where B_0 is the original light intensity; r is the Euclidian distance between the fireflies. As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, the attractiveness / repulsiveness β of a firefly can be defined by

$$\beta = \beta_0 e^{-\gamma r^2}, \qquad (8)$$

where β_0 is the attractiveness (positive sign)/repulsiveness (negative sign) at $r=0$.

The distance between any two fireflies i and j at x_i and x_j, respectively, is the Euclidian distance.

$$r_{ij} = \| x_i - x_j \| = \sqrt{\sum_{k=1}^{D} \left(x_{i,k} - x_{j,k} \right)^2}, \qquad (9)$$

where $x_{i,k}$ is the kth component of the special coordinate x_i of the ith firefly; D is the dimension of each x_i and x_j. The movement of a firefly i is attracted by another more attractive (brighter) firefly j *or* repelled by more repulsive (less bright) firefly j is determined by

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2} (x_j - x_i) + \alpha(rand - \frac{1}{2}) \qquad (10)$$

where the second term is due to the attraction or repulsion. The third term is randomized with a control parameter α, which makes the exploration of search space more efficient. Usually, $\beta_0 = 1$, $\alpha \in [0,1]$ for most applications. By adjusting the parameters γ, α and β_0, the performance of the algorithm can be improved.

Steps of FFA are as follows:

Step 1. Generate initial firefly vectors $x_i = (x_{i1},..., x_{iD})$, $(i = 1,...,120)$, where $D = 2N$ (N current element excitation weights $I \in [0,1]$ plus N number of inter-element spacing $d \in [\lambda/2, \lambda]$). Set the maximum allowed number of iterations to 100. $\beta_0 = 0.6$, $\gamma = 0.2$, and $\alpha = 0.01$ (these values, and the population size, 120, were determined as optimal in a series of 30 preliminary trials).

Step 2. Computation of initial J of the total population.

Step 3. Computation of the population based best solution (gbest) vector corresponding to the historical population best and least value, J

Step 4. Update firefly positions:

1) Compute the Euclidian distance r_{ij} between the first particle vector (i^{th}) and the second particle vector (j^{th}) as per (9);
2) Compute β with the help of β_0 as per (8);
3) If the cost function J of the second particle is < cost
4) Function J of the first particle, then, update the first particle as per (10) with $+ \beta_0$ (case of attraction), otherwise with $- \beta_0$, (case of repulsion); Steps 3-4 are repeated till the maximum iteration cycles. Determine the optimal N number of current excitation weight coefficients and N number of inter-element separations from the final solution of optimal (gbest) vector.

IV. NUMERICAL RESULTS

The FFA described in the previous section has been implemented to study the behavior of the radiation pattern for non-uniform circular antenna arrays. In this case radiation pattern of the circular array with main lobe steered to $\phi_0 = 0$ degree is considered. This section gives simulation results of both the SLL and FNBW in radiation pattern. Three circular antenna arrays having 8-, 10-, and 12- elements are assumed. For every array FFA has been executed for 100 iterations. The population size is fixed at 60. FFA algorithm is initialized using random values. Table I shows the control parameters for FFA. Figs. 2-4 show comparisons between the radiation patterns for a uniform circular antenna array ($d = \lambda/2$) and the corresponding non-uniform circular antenna array optimized with the use of FFA algorithm. In the case of uniform circular array, inter-element separation d is the arc distance between consecutive elements arranged in a circle of radius $a = N.\lambda/(4\pi)$.

Fig. 2 indicates that all side lobes are suppressed to a level less than -15.83 dB as a result of the FFA based optimization. FNBW is reduced to 81.36 degrees. Using the results in Table II for N=8, Fig. 2 shows the array pattern, which is much better as compared to those obtained in [11], [12], [14].

Fig. 3 illustrates the case for N=10. For this value of N, FFA method generates a set of I_n and d_n that provides a radiation pattern with -14.60 dB SLL, i.e. the maximum SLL reduction. Fig. 3 shows the array factor obtained using the results in Table II for N=10, which is much better as compared to those obtained in [11], [12], [14]. In this case also, FNBW is reduced to 46.08 degrees, which is very much improved as those obtained in [11], [12], [14].

Lastly, Fig. 4 illustrates the case for N=12. For this value of N, FFA method generates a similar set of I_n and d_n that provides a radiation pattern with -15.79 dB SLL, and 46.08 degrees FNBW i.e. reduction of both the SLL and FNBW. In this case also the array pattern obtained for N=12 is much better as compared to those obtained in [11], [12], [14]. In the uniform case, the current excitations for the array elements $I_1, I_2 .... I_N$ are normalized using $\max(I_i) = 1$.

Table II illustrates that as the number of antenna elements N

increases, both SLL reduction and FNBW reduction occur for non-uniform circular antenna arrays. It should be noted that the size of the circular array obtained is slightly larger than that obtained in [11], [12], [14] because in this work inter-element separation is maintaining the minimum distance within $(0 < d < 2\lambda)$ as adopted in [11], [12], [14] to avoid the mutual coupling effect.
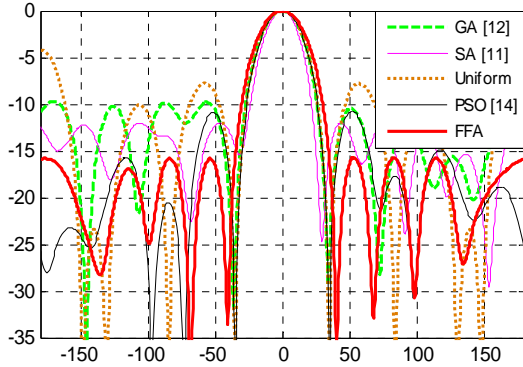

Fig. 2 Best array pattern found by FFA for the 8-element non-uniform circular array
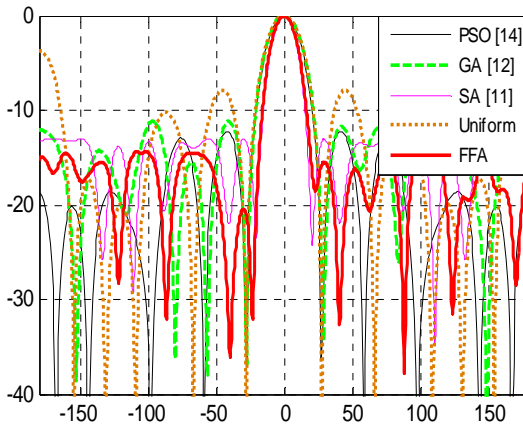

Fig. 3 Best array pattern found by FFA for the 10-element non-uniform circular array
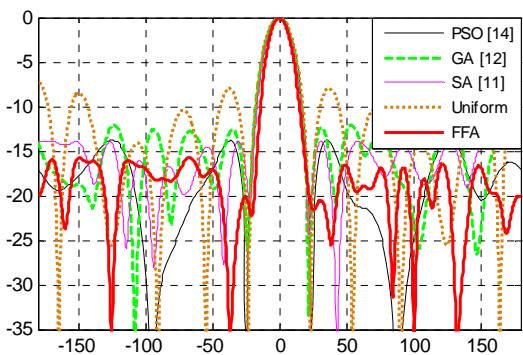

Fig. 4 Best array pattern found by FFA for the 12-element non-uniform circular array

V. CONVERGENCE PROFILES OF FFA

The minimum J values are recorded against number of iteration cycles to get the convergence profile for each array design. Fig. 5 portrays the convergence profiles of J for circular arrays having 10- and 12-elements. The programming was written in MATLAB language using MATLAB 7.5 on dual core(TM) processor, 2.88 GHz with 1 GB RAM.
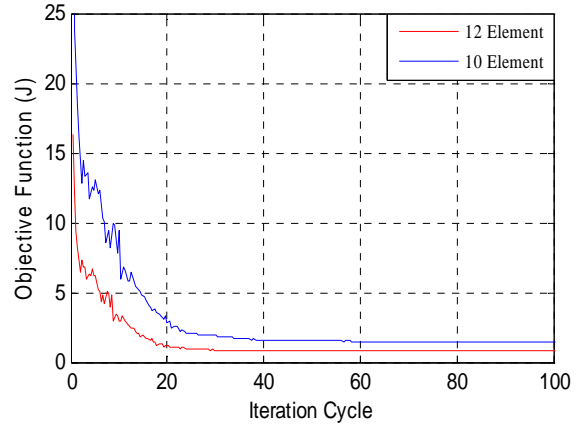

Fig. 5 Convergence profiles for FFA in case of 10- and 12-element non-uniform circular arrays.

TABLE I
CONTROL PARAMETERS OF FFA

Parameters	FFA
Population Size	120
Iteration cycle	100
α, γ, β_0	0.01, 0.2, 0.6

TABLE II
ELEMENT DISTRIBUTIONS AND THE RESULTING EXCITATION DISTRIBUTION FOR NON-UNIFORM CIRCULAR ANTENNA ARRAYS OBTAINED BY FFA FOR DIFFERENT NUMBERS OF ANTENNA ELEMENTS

No. of Ele.	SLL (dB)	SLL (dB) in [12], [14], [11]	FNBW (deg)	FNBW (deg) in [12], [14], [11]	$[I_1,I_2,I_3,I_4.......I_N]$, $[d_1,d_2,d_3,d_4,..........d_N]$ in $\lambda's$				
8	-15.83	-9.811, -10.799, -12.00	81.36	70.27, 70.27, NR	0.6810 0.0026 0.3392 0.3283	0.3621 0.7100 0.7676 0.4615	0.9428 0.2548, 0.8498 0.3263	0.9770 0.6080	0.3364 0.8459
10	-14.60	-9.811, -12.307, - 13.00	46.08	55.85, 55.85, NR	0.1304 0.1405 0.5772 0.5733	0.1051 0.0606 1.0382 1.1843	0 0.1167 1.0242 1.4350	0.0309 0.0862 1.0967 0.9970	0.1421 0.1488, 0.5705 0.6201
12	-15.79	-11.83, -13.670, -13.910	46.08	46.26, 46.26, NR	0.2641 0.4789 0.2885 0.6096 0.5558 0.9843	0.3068 0.1453 0.4628, 1.0830 1.1447 0.5071	0 0.0429 0.6975 0.5370	0.1845 0.2102 1.2350 0.8699	0.4672 0.0943 1.5782 0.8299

NR= Not Reported.

VI. CONCLUSION

This paper illustrates how to model the design of non-uniform circular antenna arrays for maximum reduction of side lobe level and first null beam width as well. Firefly algorithm (FFA) is proposed for the solution of this design. FFA efficiently computes optimal set of current excitation weights and antenna inter-element separations to generate a radiation pattern with maximum side lobe level reduction and improved first null beam width. Experimental results reveal that FFA provides the considerable side lobe reduction and improved first null beam width with respect to the uniform case and also those presented in [11], [12], [14].

Future research will be aimed at dealing with other geometries and constraints. Many different areas of antenna design and analysis require a feasible and versatile procedure, being able to perform array synthesis by tuning antenna characteristics and parameters. Because of the versatility of the method of FFA it seems a good candidate to face this problem.

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