International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:12, No:1, 2018

Design and Validation of an Aerodynamic Model of the Cessna Citation X Horizontal Stabilizer Using both OpenVSP and Digital Datcom

Marine Segui, Matthieu Mantilla, Ruxandra Mihaela Botez

***Abstract*—**This research is the part of a major project at the Research Laboratory in Active Controls, Avionics and Aeroservoelasticity (LARCASE) aiming to improve a Cessna Citation X aircraft cruise performance with an application of the morphing wing technology on its horizontal tail. However, the horizontal stabilizer of the Cessna Citation X turns around its span axis with an angle between -8 and 2 degrees. Within this range, the horizontal stabilizer generates certainly some unwanted drag. To cancel this drag, the LARCASE proposes to trim the aircraft with a horizontal stabilizer equipped by a morphing wing technology. This technology aims to optimize aerodynamic performances by changing the conventional horizontal tail shape during the flight. As a consequence, this technology will be able to generate enough lift on the horizontal tail to balance the aircraft without an unwanted drag generation. To conduct this project, an accurate aerodynamic model of the horizontal tail is firstly required. This aerodynamic model will finally allow precise comparison between a conventional horizontal tail and a morphed horizontal tail results. This paper presents how this aerodynamic model was designed. In this way, it shows how the 2D geometry of the horizontal tail was collected and how the unknown airfoil's shape of the horizontal tail has been recovered. Finally, the complete horizontal tail airfoil shape was found and a comparison between aerodynamic polar of the real horizontal tail and the horizontal tail found in this paper shows a maximum difference of 0.04 on the lift or the drag coefficient which is very good. Aerodynamic polar data of the aircraft horizontal tail are obtained from the CAE Inc. level D research aircraft flight simulator of the Cessna Citation X.

***Keywords*—**Aerodynamic, Cessna, Citation X, coefficient, Datcom, drag, lift, longitudinal, model, OpenVSP.

I. Introduction

THE "Morphing Wing" consists in replacing a conventional wing of an aircraft by a wing that will change during the flight. While the Wright brothers' talked about controlling their fixed wing with cables on the leading edge a century earlier [1], no one suspected that this idea would evolve, and nowadays, would become a promising idea to improve the aerodynamics of an aircraft and to reduce the amount of fuel needed for a given travel. Indeed, this technology named the "Morphing Wing" consists in replacing a conventional wing of an aircraft by a wing that will change during the flight. This wing can change its airfoil, its sweep angle, or its other parameters [2], [3]. Thus, by changing its shape during flight, the wing has the best aerodynamic configuration, with the minimum drag force induced for each flight condition. Researchers at the LARCASE have already studied the morphing technology on an Unmanned Aerial System (UAS), the UAS-S4 Ehécatl designed by the company Hydra Technology. As a matter of fact, the LARCASE has replaced the original wing of the UAS-S4 by a wing able to reduce the drag generated by changing its airfoil shape. Then experimental tests have been realized in a subsonic wind tunnel [4]-[8]. Results obtained by the LARCASE team have shown that the drag force induced by the wing can be reduced until 14% [9]. Another study conducted by Iannota [10] has shown that a reduction of 20% of drag can lead to a reduction of 18% in fuel consumption, in other words, there is a cause-and-effect relationship between the drag and the fuel consumption. Therefore, an aircraft equipped with a Morphing Wing technology will also improve its performance, and reduce its fuel required for the same mission.

Fig. 1 Cessna Citation X Research Aircraft Flight Simulator (RAFS)

In parallel, a global campaign aiming to reduce emissions of toxic gas in the atmosphere is conducted by International Civil Aviation Organization (ICAO) since 2010 [11]. As part of the general effort to reduce the aviation net carbon footprint, research is conducted in the LARCASE Laboratory to develop mathematical models and efficient algorithms optimization for

Marine Segui is an M.Sc.A Student with the Laboratory of Active Controls Avionics and AeroServoElasticity (LARCASE), at Ecole de Technology Supérieure (ETS), Montreal, QC H3C-1K3 CA.

Matthieu Mantilla is an Undergraduate Student with the LARCASE at ETS Montreal, QC H3C-1K3 CA.

Ruxandra Mihaela Botez is professor and Canada Research Chair Holder Level 1 in Aircraft Modelling and Simulation Technologies, LARCASE, ETS, Montreal, QC H3C-1K3 Canada (e-mail: ruxandra.botez@etsmtl.ca).

aircraft [12]-[26]. Models developed at the LARCASE are accurately validated using a level D Research Aircraft Flight Simulator (RAFS) of the Cessna Citation X (Fig. 1) designed by CAE Inc. The level D is given by the Federal Aviation Administration (FAA) to a flight simulator that reflects a flight dynamics model of the original aircraft with a maximal error of 5%. Based on these accurate models of performance, the LARCASE team is working on a new project aiming to predict and reduce fuel consumption for a business aircraft such as the Cessna Citation X equipped with the morphing wing technology on its horizontal stabilizer. Indeed, to balance the Cessna Citation X during a flight, its horizontal stabilizer can turn from -8 to +2 degrees around the y-axis (axis along the span of the horizontal tail) to balance the airplane. However, within this range of angles, the horizontal stabilizer generates certainly some unwanted drag. To cancel this drag, the LARCASE team proposes to balance the aircraft by a morphing horizontal stabilizer that can generate enough lift on the tail to balance the aircraft. In the same way, this study requires designing and validating an accurate aerodynamic model of the Cessna Citation X horizontal stabilizer.

In order to explain how this aerodynamic model was designed, the methodology of this paper is organized following three main sub-sections. The first sub-section presents the Cessna Citation X business jet and its flight domain, the second sub-section presents how the geometrical data of the Cessna Citation X horizontal tail were collected, and, the third sub-section presents how the airfoil of the Cessna Citation X horizontal stabilizer was found and validate.

II. Conception of an Aerodynamic Model of the Cessna Citation X Horizontal Stabilizer

This section presents the methodology used to design the Cessna Citation X horizontal stabilizer aerodynamic model. Thus, a global presentation of the Cessna Citation X business aircraft is given in the first subsection. Then, second subsection presents how 2D geometrical data of the horizontal stabilizer has been recovered from airplane drawings. Finally, the third subsection is focused on finding the horizontal stabilizer airfoil.

Fig. 2 Cessna Citation X Business Aircraft

A. Presentation of the Cessna Citation X Business Aircraft

The Cessna Citation X (Fig. 2) is a transcontinental business jet that can fly at a speed up to Mach number equal to 0.935. Powered by two Rolls-Royce AE 3007C-1 turbofan engines, this aircraft is one of the fastest business aircraft on the market. This airplane has remarkable performances, notably in the cruise regime, where it is certified to fly up to a maximum certified altitude of 15,545 meters and a maximum Mach number certified of 0.92 (Table I). For the takeoff regime it is restricted by a maximum weight of 35,000 pounds, nine passengers and two pilots.

TABLE I
General Properties of the Cessna Citation X

Designation	Value	Unit
Aircraft		
Height Overall	5.87	m
Length Overall	22.05	m
Mach Max Certified	0.92	-
Max Certified Altitude	15,545	m
Wing		
Wingspan	19.38	m
Area	48.96	m^2

The Cessna Citation X has a low position wing in a and a T-tail configuration with a vertical tail and a horizontal tail. Engines are installed at the rear of the fuselage and have the aim to not disturb the fluid perceived by the horizontal tail.

The horizontal tail is composed of two parts: the horizontal stabilizer and the elevators. The horizontal stabilizer is a wing that turns to balance the aircraft during its flight. The elevators are two parts situated on the trailing edge of the horizontal tail allowing a change of direction. Because of the fact that the following study is done in a part of a project focused on the cruise regime, the horizontal tail can be modeled without elevators. In this way, following terms "horizontal tail" or "horizontal stabilizer" represents the same thing.

B. Geometrical Data Acquisition by Digitization of Cessna Citation X Drawings

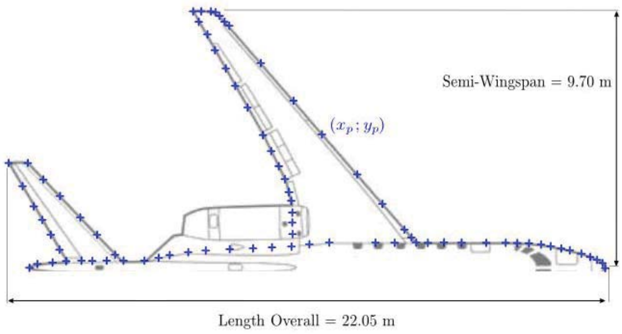

Fig. 3 Cessna Citation X 3D drawing with points recovered by the digitalization software

The shape of the Cessna Citation X horizontal tail was acquired from 3D drawings of the airplane. Indeed, a method of digitization was used. This method consists in scan drawing through a software called "Eugauge Digitizer" and marks out borders of the horizontal tail with their coordinates (x_p, y_p) shown on Fig. 3. In this way, each recovered point has a coordinate x_p along the longitudinal axis of the airplane, and a coordinate y_p along the wing span axis. As a matter of fact, coordinates (x_p, y_p) shown on the wing are given through the unknown scale of its 3D drawing. To restore the airplane scale, some reference lengths are required on the drawing. Thus, the length overall value of 22.05 m (Table I) scales x_p coordinates, and, the height overall value of 5.87 m (Table I) scales y_p coordinates (Fig. 3).

TABLE II
GEOMETRICAL PROPERTIES OF THE CESSNA CITATION X HORIZONTAL TAIL

Designation	Value	Unit
Wingspan	7.95	m
Mean Aerodynamic Chord	1.62	m
Root Chord	2.35	m
Tip Chord	0.90	m
Sweep angle (2 % Chord)	42	deg
Area	11.15	m^2

Through the digitization process, only the 2D geometry of the horizontal tail borders and the main dimensions of the aircraft (Table II) have been recovered. Indeed, the airfoil with which is equipped the horizontal tail could not be accurately defined through this methodology. However, to design a reliable aerodynamic model of a wing, the airfoil shape is also required. As a result, in order to obtain the airfoil shape, another methodology detailed in the following subsection was used.

C. Horizontal Tail Airfoil Research

Once 2D geometrical data of the horizontal tail have been collected, it is thus necessary to find the shape of the horizontal tail airfoil to build its complete aerodynamic model. For this purpose, lift and drag coefficients of the Cessna Citation X horizontal tail $C_{L_{ht}}$ and $C_{D_{ht}}$are accurately given by CAE Inc. data through the RAFS (Fig. 1) according to the angle of attack of the horizontal tail α_{tail} and the Mach number of the aircraft *Mach*. It is important to specify that the lift coefficient $C_{L_{ht}}$ given by the RAFS for zero angle of attack is zero; this property is the main feature of a symmetrical airfoil. More generally, this property informs that the Cessna Citation X horizontal tail airfoil is a symmetrical airfoil.

Because of the fact that the concept is to find an unknown symmetrical airfoil, a retro-engineering method can be used (Fig. 4). Thereby, this method consists in scanning a large range of airfoils by a parameterized curve and computes their aerodynamic coefficients $\widehat{C_L}$ and $\widehat{C_D}$ when they are the same as the ones on a wing that has the same dimension of the horizontal stabilizer of the Cessna Citation X (Table II). The effectiveness of an airfoil is measured by a small gap between reference aerodynamic coefficients (C_{Lht} and C_{Dht}) given by the RAFS and those computed for a given airfoil shape using an aerodynamic solver ($\widehat{C_{L_{ht}}}$ and $\widehat{C_{D_{ht}}}$). All these steps have been described in the following sub-sections. The first sub-section detailed the method for setting a symmetrical airfoil using the Bezier-Parsec parameterization curve. The second and the third sub-sections are focused on the design of the horizontal stabilizer using OpenVSP and Digital Datcom software. Finally, the fourth sub-section presents how Digital Datcom and OpenVSP software are related to find the horizontal stabilizer airfoil shape of the Cessna Citation X.

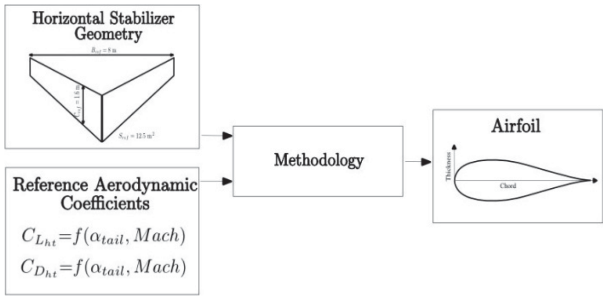

Fig. 4 Retro-engineering method to find an unknown airfoil of a given wing

1. Bezier-Parsec Parameterization Curve

The Bezier-Parsec parameterization [27], [28] is a methodology which allows to design a large collection of airfoils (NACA, EPPLER, FX or supercritical). To design a symmetrical airfoil, the upper curve is parameterized following a "thickness curve", and, the inner curve is the symmetry of the upper curve with respect to the chord.

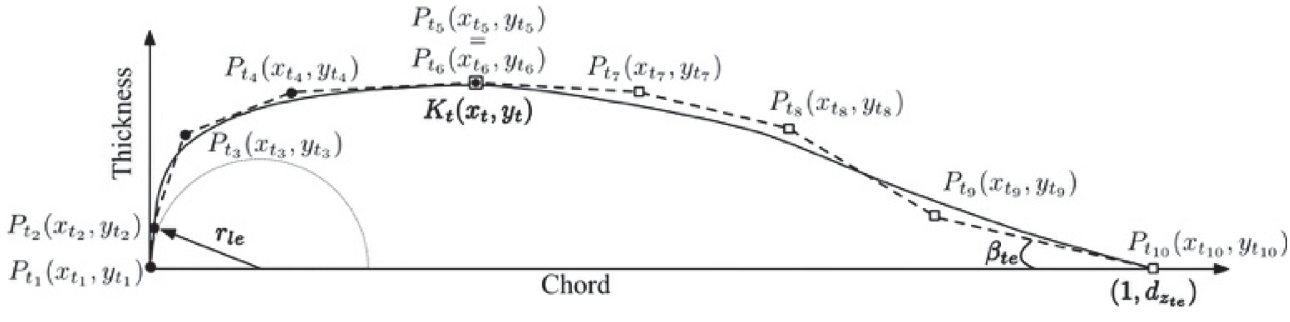

Fig. 5 Design parameters for the airfoil curve

The "thickness curve" is defined following two sub-curves, one describes the leading edge (*LE*) and the other one designs the trailing edge (*TE*). Using such a separation allows avoiding some disadvantages of a classical unique Bezier curve that generally leads to some unviable airfoils. Thus, sub-curves *LE* and *TE* are both based on the Bezier-Bernstein polynomial given by (1). Thus, the thickness curve is defined according to m Bezier control points using coordinates (x_{tm}, y_{tm}). The abscissa x_{tm} defined the position of a control point along the airfoil chord, and the ordinate y_{tm} defined the position of the control point along the thickness of the airfoil. Control points correspond to fictive points that aim to structure the shapes of the curves [27], [28].

$$B(u) = \sum_{j=0}^{m} P_j \frac{m!}{j!(m-j)!} u^j (1-u)^{m-j} \quad (1)$$

where *B(u)* is the Bezier curve, P_j are the Bezier control points, *u* is a linear vector which is distorted by Bezier control points, and finally, *m* is the order of the method.

Because of the fact that a 4-order Bezier curve allows to design a larger range of airfoils, specifically airfoils that have a flattened leading edge or a pronounced inflexion in the trailing edge, a 4-order Bezier curve was chosen to model the leading edge and for the trailing edge. As a consequence, this method can be named *BP44*.

By considering an index i going from 0 to *n*, the leading edge is defined for $i \in [0; n/2]$ and the trailing edge is defined for $i \in [n/2+1; n]$. As the leading edge and the trailing edge airfoils are modeled by two different curves, it is important to ensure the continuity between the upper and lower surfaces shapes. For this reason, at *i* = *n*/2, $X_{tLE} = X_{tTE}$ and $Y_{tLE} = Y_{tTE}$. Finally, the *BP44* method defines an airfoil using *n* successive points with coordinates $(X_{tLE}(i), Y_{tLE}(i))$ for the leading edge curve (2), and coordinates $(X_{tTE}(i), Y_{tTE}(i))$ for the trailing edge curve (3).

$$X_{tLE}(i) = [1-u(i)]^4.x_{t1} + 4.[1-u(i)]^3.u(i).x_{t2} + 6.[1-u(i)]^2.u(i)^2.x_{t3} + 4.[1-u(i)].u(i)^3.x_{t4} + u(i)^4.x_{t5}$$
$$Y_{tLE}(i) = [1-u(i)]^4.y_{t1} + 4.[1-u(i)]^3.u(i).y_{t2} + 6.[1-u(i)]^2.u(i)^2.y_{t3} + 4.[1-u(i)].u(i)^3.y_{t4} + u(i)^4.y_{t5} \quad (2)$$

where, $x_{t1}, x_{t2}, x_{t3}, x_{t4}, x_{t5}$ and $y_{t1}, y_{t2}, y_{t3}, y_{t4}, y_{t5}$ are coordinates of Bezier-Parsec control points for the leading edge airfoil.

$$X_{tTE}(i) = [1-u(i)]^4.x_{t6} + 4.[1-u(i)]^3.u(i).x_{t7} + 6.[1-u(i)]^2.u(i)^2.x_{t8} + 4.[1-u(i)].u(i)^3.x_{t9} + u(i)^4.x_{t10}$$
$$Y_{tTE}(i) = [1-u(i)]^4.y_{t6} + 4.[1-u(i)]^3.u(i).y_{t7} + 6.[1-u(i)]^2.u(i)^2.y_{t8} + 4.[1-u(i)].u(i)^3.y_{t9} + u(i)^4.y_{t10} \quad (3)$$

where $x_{t6}, x_{t7}, x_{t8}, x_{t9}, x_{t10}$, and, $y_{t6}, y_{t7}, y_{t8}, y_{t9}, y_{t10}$ are coordinates of Bezier-Parsec control points for the trailing edge airfoil.

$$Q_t(x) = \left(\frac{27}{4}.K_t^2\right).x^4 + \left(-27.K_t^2.y_t + \frac{81}{2}.K_t^2.x_t^2\right).x^2 + (2.r_{le} - 18.K_t.x_t.y_t - 27.K_t^2.x_t^3).x + \left(3.y_t^2 + 9.K_t.x_t^2.y_t + \frac{27}{4}.K_t^2.x_t^4\right) = 0 \quad (4)$$

where r_{le} is the radius of curvature of the airfoil's leading edge. Bezier-Parsec control points coordinates (x_{tm}, y_{tm}) are defined in (5) following several parameters [27], [28] (Fig. 5) which are: K_t the curvature at the thickness crest, coordinates (x_t, y_t) corresponding to the position of the thickness crest, the wedge angle β_{te}, the half thickness of the trailing edge $d_{z_{te}}$, abscissa value of the fourth point x_{t4}, full coordinates of the eight point (x_{t8}, y_{t8}) and r_t the smallest root of Q_t (4).

$$\begin{aligned}
x_{t1} &= 0 & y_{t1} &= 0 \\
x_{t2} &= 0 & y_{t2} &= y_t + \frac{3}{2}.K_t.(x_t - r_t)^2 \\
x_{t3} &= r_t & y_{t3} &= y_t \\
x_{t4} &= x_{t4} & y_{t4} &= y_t \\
x_{t5} &= x_t & y_{t5} &= y_t \\
x_{t6} &= x_t & y_{t6} &= y_t \\
x_{t7} &= 2.x_t - r_t & y_{t7} &= y_t \\
x_{t8} &= x_{t8} & y_{t8} &= y_{t8} \\
x_{t9} &= 1 + \cot(\beta_{te}).\left[(d_{z_{te}} - \left(\frac{3}{2}.K_t.(x_t - r_t)^2 + y_t\right)\right] & y_{t9} &= \frac{3}{2}.K_t.(x_t - r_t)^2 + y_t \\
x_{t10} &= 1 & y_{t10} &= 0
\end{aligned} \quad (5)$$

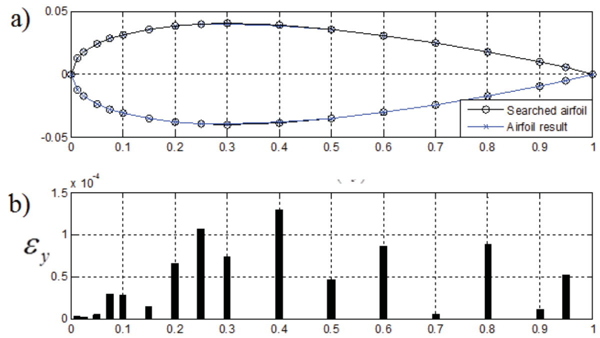

Fig. 6 Detailed error deviation for each point between the symmetrical airfoil NACA0008 and the airfoil find by BP44

In order to test the *BP44* method, 31 symmetrical airfoils coming from different well known families such as NACA, supercritical, Eppler and Wortmann-FX have been selected. To be tested, the *BP44* has to recognize the shape of a known airfoil. For example, results obtained for the research of the NACA0008 are presented in Figs. 6 (a) and (b). Because of the fact that airfoils are displayed using series of *n* points, the error ε_y between the real desired airfoil and the designed airfoil by the *BP44* methodology is expressed with (6):

$$\varepsilon_y = \sum [y_{Bezier}(upper) - y_{airfoil}(upper)]^2 \quad (6)$$

where $y_{Bezier}(upper)$ corresponds to the y-coordinates of the airfoil designed by *BP44* methodology, and $y_{airfoil}(upper)$ represents the y-coordinates of the real airfoil tested.

Finally, Fig. 7 (b) shows the average of error ε_y obtained for each family of airfoils (Fig. 7 (a)). Globally the *BP44* method seems to better predict NACA airfoil families with an average error of 2.10^7 than 3.10^7 of difference for other families. In spite of this small difference, all airfoils are very well found.

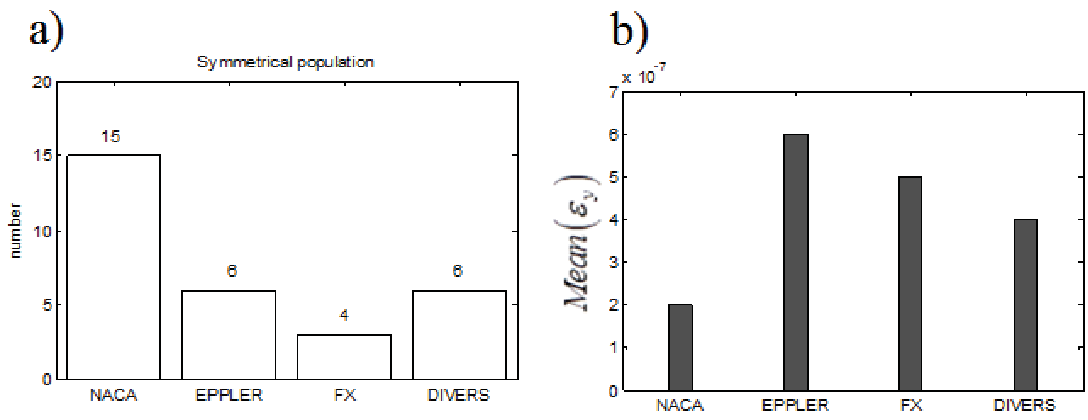

Fig. 7 Mean error ε_y observed between a research airfoil by the *BP44* method and the real airfoil for different families of airfoils

After designing an airfoil using this *BP44* methodology, it is necessary to quantify how close it is to the original horizontal tail's airfoil. For this purpose, each airfoil is going to be compared with original airfoil through their aerodynamic coefficients. To define aerodynamic coefficients $\widehat{C_{L_{ht}}}$ and $\widehat{C_{D_{ht}}}$ of a designed airfoil, it is required to equip a wing that has the same dimensions as the horizontal stabilizer of the Cessna Citation X (Table II) with the designed airfoil, and, finally, to make an aerodynamic computation. Aerodynamic computation can be made using different software such as OpenVSP and Datcom, depending on the appropriate solving method required in each case.

2. Model Conception Using OpenVSP Software

In order to design an accurate model of the horizontal tail, OpenVSP was especially chosen for its user-friendly environment. OpenVSP interface allows designing a wing while having a global vision on the exact geometry taking into account by the software. OpenVSP (Vehicle Sketch Pad) is an aircraft modeling software developed by the National Aeronautics and Space Administration (NASA) [29], [30]. It is available for research and industry under NASA Open Source Agreement (NOSA). This software is based on two computation methods: The Panel Method (PN) and the Vortex Lattice Method (VLM). The PN is used to solve incompressible flow over three dimensions geometries. Contrarily, the VLM reduces the three dimensions of the object to two cross planes, and takes into account vortex horseshoes induced by the incoming wind onto the geometries. VLM method computes aerodynamic coefficients C_L and C_D, whereas, the PN method computes the lift coefficient C_L and the induced drag C_{Di}. It is important to notice that the total drag coefficient C_D is a sum of the induced drag C_{Di} and the zero-lift drag C_{D0}.

Because of the fact that the researched airfoil of the horizontal tail is symmetrical, the projection of the thickness on two cross planes by the VLM method will not be a good approximation for it. For this reason, OpenVSP was used only with the PN, so, without the C_{D0} computation. Concerning the design of the horizontal tail model, a wing component of OpenVSP has been set to dimensions stocked in Table II, and the airfoil was designed with the *BP44* method. Fig. 8 shows the horizontal tail of the Cessna Citation X in OpenVSP interface. Finally, computation was done with Mach numbers equal to 0.6 to 0.9, and angles of attack α_{tail} from -8 degrees to +2 degrees.

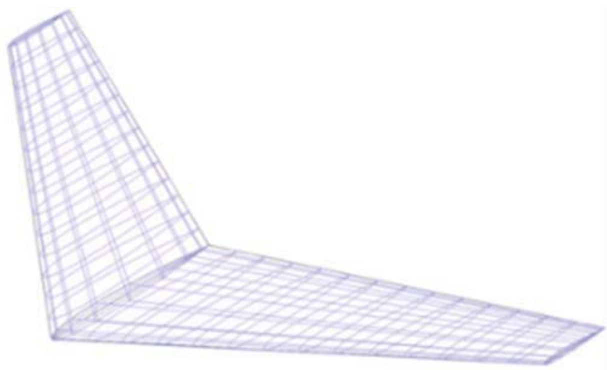

Fig. 8 Cessna Citation X horizontal tail model in OpenVSP interface

3. Model Design Using Digital Datcom Software

To finalize a complete aerodynamic study, lift and drag coefficients C_L and C_D are required. Because of the fact that OpenVSP gives only C_{Di} that is a part of the required C_D, it is necessary to compute the zero-lift drag coefficient of a wing C_{D0}. Digital Datcom computes accurately the C_{D0}, thus it is used to define this coefficient in this study.

Digital Datcom software is using computation methods contained in the United States Air Force (USAF) Stability and Control Datcom (Data Compendium) [31]. This software computes static stability characteristics, so, lift and drag coefficients C_L and C_D of a geometrical configuration (wing, wing-fuselage or wing-fuselage-tail) for a given altitude, Mach number and an angle of attack. Moreover, this software is particularly interesting for its accurate zero-lift drag C_{D0} computation. Finally, it is important to notice that Digital Datcom uses in most cases a semi-empirical method. Thus, an aerodynamic computation for a wing configuration is done in less than 10 seconds.

Using Digital Datcom, a model is constructed from an input text-file which contains information regarding the flight condition in *FLTCON*, the geometrical data of the wing in *OPTINS* and in *WGPLNF*, and finally the airfoil data in *WGSCHR*. The set of parameters stocked into *FLTCON*, *OPTINS*, *WGPLNF*, and *WGSCHR,* used to design the Cessna Citation X horizontal tail model, are presented in Table III.

TABLE III
SET OF PARAMETERS FOR THE CESSNA CITATION X HORIZONTAL TAIL MODEL USING DIGITAL DATCOM SOFTWARE

Designation	Value	Unit
	FLTCON	
Mach number	[0.6, ... , 0.9]	-
Altitude	30 000	ft
Angle of Attack	[-8.0, ... , 0.0]	deg
	OPTINS	
Reference Span	25.60	ft
Reference Area	120.00	ft^2
Mean Aerodynamic Chord	5.05	ft
	WGPLNF	
Tip Chord	2.43	ft
Root Chord	6.95	ft
Sweep at Inboard Panel	42.0	deg
Semi Span	12.76	m
	WGSCHR	
Number of Points (n)	28	-
X coordinates	X_t	-
Y Upper coordinates	Y_t	-
Y Lower coordinates	$-Y_t$	-

4. Airfoil Shape Research Algorithm

Models of the horizontal tail using OpenVSP and Digital Datcom allow obtaining complete aerodynamic coefficients: The lift coefficient $\widehat{C_{L_{ht}}}$ obtained by OpenVSP and the drag coefficient $\widehat{C_{D_{ht}}}$ obtained by the addition of the induced drag coefficient C_{D_i} computed by OpenVSP and the zero-lift drag coefficient C_{D_0}computed by Digital Datcom.

The final sub-section consists in putting in relationship: The airfoil designed with the *BP44* method with models of the horizontal tail using OpenVSP and Digital Datcom. Moreover, because there are a many combinations of design parameters for the *BP44*, it is required to integrate an optimization algorithm in this research. More precisely, a fitness function in relation with a Genetic Algorithm was build (Fig. 9). Firstly, the fitness function has for role to receive 8 random parameters of *BP44*: $x_t, y_t, \beta_{te}, r_{le}, K_t, x_v, x_{t8}$, and y_{t8}. Then, with these parameters, the fitness function can design the corresponding airfoil. Then, using this airfoil, the fitness function builds a model using OpenVSP, and another model using Digital Datcom. Finally, each software computes aerodynamic coefficients $\widehat{C_{L_{ht}}}, \widehat{C_{D_i}}, \widehat{C_{D_0}}$ of the horizontal tail and gives final coefficients $\widehat{C_{L_{ht}}}$and $\widehat{C_{D_{ht}}}$. In order to measure how the airfoil design by *BP44* is close to the airfoil researched the error *err* is computed (7) in which $C_{L_{ht}}$ and $C_{D_{ht}}$are the aerodynamic coefficients obtained from the RAFS (Fig. 1), and coefficients $\widehat{C_{L_{ht}}}$and $\widehat{C_{D_{ht}}}$are obtained using the horizontal tail model designed in sub-sections 2 and 3. Thus, for each selected Mach number, the error *err* given by the fitness function is minimized by a Genetic Algorithm. Finally, 5 optimized airfoils are obtained; one airfoil is obtained for each Mach number.

$$err = \left[C_{L_{ht}} - \widehat{C_{L_{ht}}}\right]^2 + 100.\left[C_{D_{ht}} - \widehat{C_{D_{ht}}}\right]^2 \quad (7)$$

To obtain only one airfoil for the global aerodynamic envelope, a geometrical average was done between these 5 optimized airfoils.

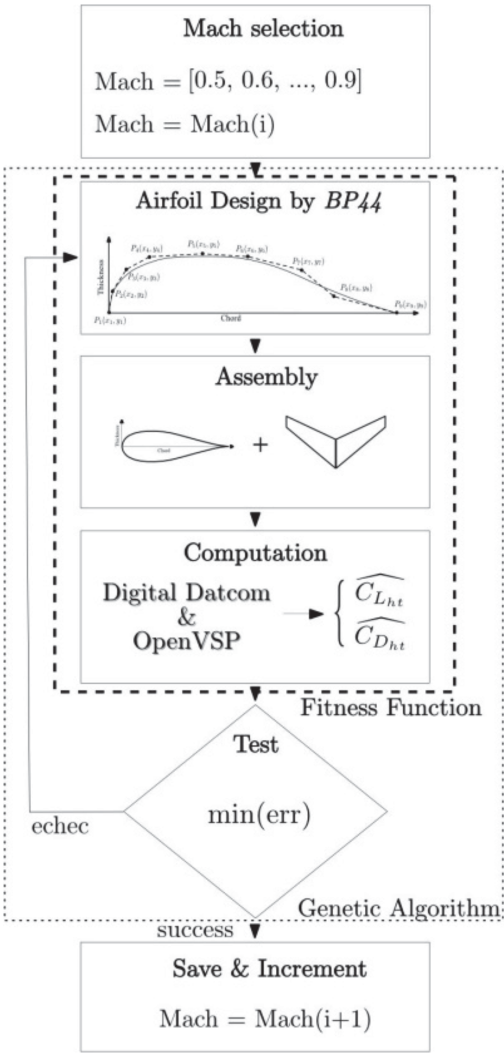

Fig. 9 Representation of the fitness function coupled with a Genetic Algorithm to find the Cessna Citation X Horizontal Tail airfoil

III. RESULTS

This section is going to present results that have been obtained at each step of the last part of the methodology described previously. Firstly, Fig. 10 shows the 5 airfoils that give the smallest error *err* for each Mach number. Then, Fig. 11 (a) presents a superposition of each airfoil (that has been previously presented in Fig. 10). Fig. 11 (b) presents the average airfoil obtained from the 5 airfoils fund for each Mach number.

Finally, the Cessna Citation X horizontal tail airfoil is here modeled by the average airfoil shown in Fig. 11 (b), and the 2D geometry of the horizontal tail given in Table II. Both OpenVSP and Digital Datcom are used to compute aerodynamic coefficients for the geometry found for the horizontal tail. In order to measure how this model represents the real Cessna Citation X horizontal tail airfoil model, an aerodynamic computation is made on this airfoil for Mach numbers from 0.5 to 0.9, and for angles of attack from 0 to -8 degrees. Then, these results are compared to aerodynamic polar obtained by the Research Aircraft Flight Simulator, which constitute an accurate reference. Fig. 12 (a) shows the aerodynamic polar comparisons for each Mach number, between the model (in the blue line with cross) and the RAFS (in black line with round). Fig. 12 (b) shows, for each Mach number, the absolute difference obtained between lift coefficients C_L given by the model and those given by the RAFS. Fig. 12 (c) shows the absolute differences obtained between drag coefficients C_D obtained by the RAFS and those given by the model for each Mach number. According to Fig. 12, all the aerodynamic coefficients C_L and C_D can be predicted by the model with a maximum difference of 0.04. More precisely, in the cruise regime, the Cessna Citation X flies most of the time at Mach number of 0.7 or 0.8 and at angles of attack of the horizontal tail from 0 to 4 degrees. As a consequence, if only these conditions are selected, a maximum difference of 0.031 is obtained for the lift coefficient, and, a maximum difference of 0.004 is obtained for the drag coefficient. These very good results allowed validation of this Cessna Citation X horizontal tail model in the cruise regime.

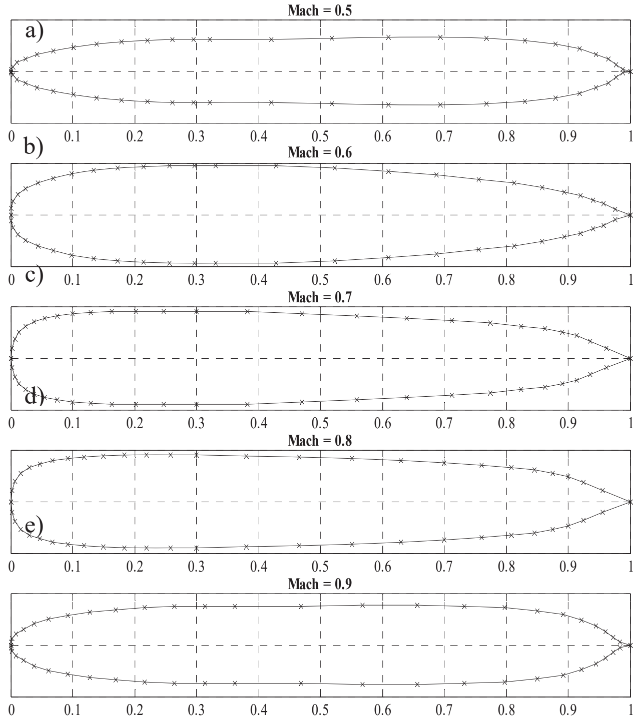

Fig. 10 Preview of airfoils founded for each Mach number by the research algorithm

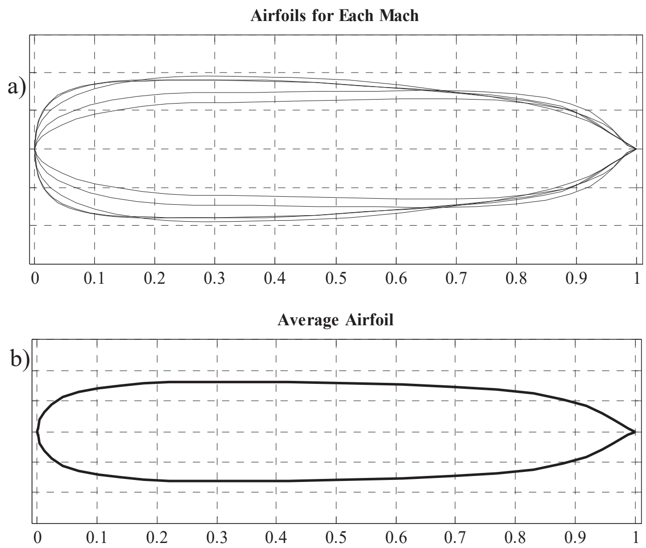

Fig. 11 Superposition of airfoils founded for each Mach numbers at the top and the average airfoil at the bottom

IV. Conclusion

This article shows the methodology used to design a geometrical aerodynamic model of the horizontal tail that the airfoil is unknown. A digitization for 2D dimension of the wing and an optimization algorithm coupled to a Bezier-Parsec parameterization curve were used. Finally, an airfoil was found, and aerodynamic coefficients of the wing obtained for this and aerodynamic coefficients given by the Research Aircraft Flight Simulator were compared, and a very small error of 0.04 was obtained. As a consequence, this airfoil was chosen for the horizontal tail model as it was validated.

Acknowledgment

This work was performed within the Canada Research Aircraft Chair in Modeling and Simulation Technologies, at the LARCASE. The Research Aircraft Flight Simulator was obtained by Dr Ruxandra Botez, Full Professor, thanks to the grants that were approved by the Canadian Foundation of Innovation (CFI), the Ministère du Développement Économique, de l'Innovation et de l'Exportation (MDEIE) and the contribution of CAE Inc. The authors thank to CAE Inc. team led by Mr Ken Dustin, and to Mr. Oscar Carranza Moyao for their support in the development of the Research Aircraft Flight Simulator at the LARCASE. The authors also thank to Mrs. Odette Lacasse at ETS for her support.

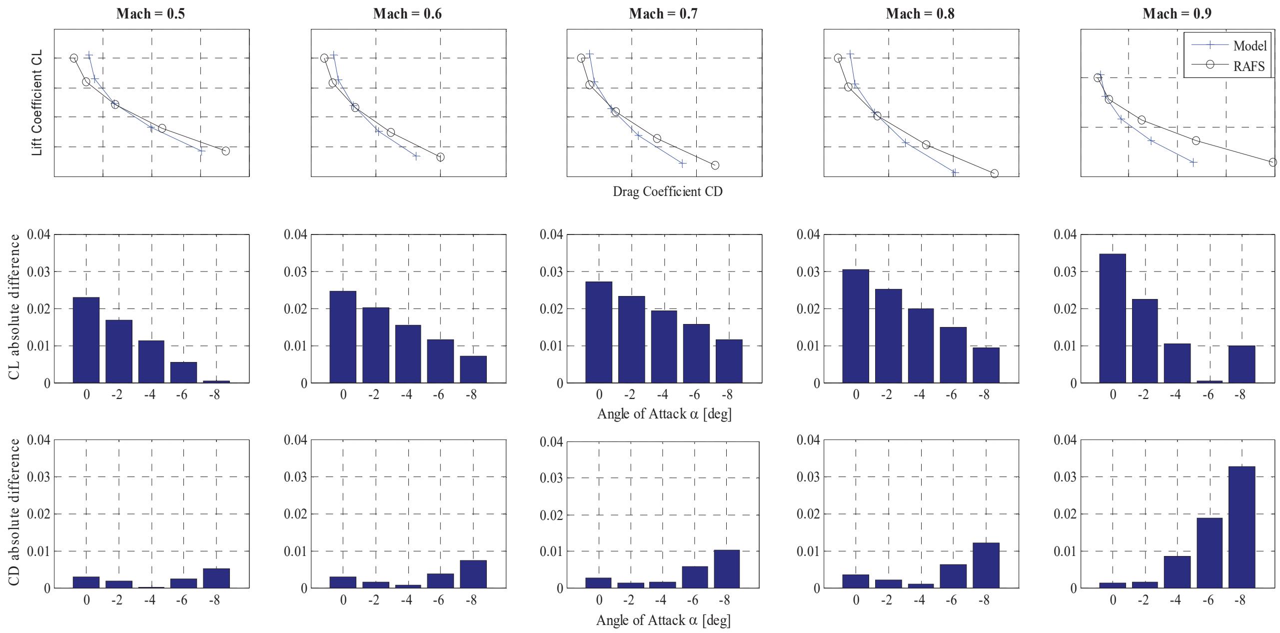

Fig. 12 Horizontal tail aerodynamic polar comparison between results obtained by the modeled wing equipped with the average airfoil (Model) and experimental reference data obtained from the Research Aircraft Flight Simulator (RAFS)

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