

# Design and Analysis of a Novel 8-DOF Hybrid Manipulator

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**Abstract**—This paper presents kinematic and dynamic analysis of a novel 8-DOF hybrid robot manipulator. The hybrid robot manipulator under consideration consists of a parallel robot which is followed by a serial mechanism. The parallel mechanism has three translational DOF, and the serial mechanism has five DOF so that the overall degree of freedom is eight. The introduced manipulator has a wide workspace and a high capability to reduce the actuating energy. The inverse and forward kinematic solutions are described in closed form. The theoretical results are verified by a numerical example. Inverse dynamic analysis of the robot is presented by utilizing the Iterative Newton-Euler and Lagrange dynamic formulation methods. Finally, for performing a multi-step arc welding process, results have indicated that the introduced manipulator is highly capable of reducing the actuating energy.

**Keywords**—hybrid robot, closed form, inverse dynamic, actuating energy, arc welding

## I. INTRODUCTION

MOST manipulators designed are of the form of serial or parallel mechanisms. Serial manipulators, which are made of a series of links and joints that connect the base to the end-effectors, and are capable of achieving large workspace. Parallel manipulators, such as the Stewart Platform [1], are robotic mechanisms which are composed of a set of parallel limbs, connecting the base plate to the moving plate, and they are able to achieve high stiffness and high force-to-weight ratio. This property makes parallel mechanisms an excellent option for high loads applications. The most important disadvantage of parallel manipulators is their restricted workspace. Recently some researches have designed manipulators that present a compromise between the high rigidity of parallel manipulators and the extended workspace of serial manipulators. Many different types of hybrid robots have been investigated [2, 3, 4].

Applications of robot in wide workspace are interested in this paper. Many applications like arc welding, spot welding, gluing and polishing are examples of tasks often using end-effectors with at least one axis of symmetry. So 5-DOF robots are suitable for these applications. For such

applications, serial robots are excellent choices and are used extremely in industries due to many advantages like large workspace and the possibility of entering the tools into inner sections of work piece. But for larger workspace, the use of the bigger and heavier serial robot with excessive actuating energy is a unique mean. The conception of moving the robot's base to the desired position in 3D space can be efficient to decrease the actuating energy. For this purpose, hybrid serial-parallel robots whose parallel part can transfer serial mechanisms are appropriate. Huang et al. [5] studied the characteristics of 6 DOF parallel-serial hybrid manipulators which features a 3 DOF in series actuated module mounted on the moving plate of another 3 DOF in parallel actuated manipulator with prismatic actuators. Yang et al. [6] discussed the kinematics of hybrid type manipulation system with 6 DOF, which consist of a 3-DOF planar parallel platform and a 3-DOF serial robot arm. Huang et al. [7] studied a conceptual design and dimensional synthesis of a 3-DOF parallel mechanism module which forms the main body of a newly invented 5-DOF reconfigurable hybrid robot. LiangZhi et al. [8] studied a hybrid 5DOF manipulator based on the novel 3-RPS in-actuated parallel manipulator. In their design a 2DOF serial working table is placed over the mobile platform. Campos et al. [9] presented a new methodology to synthesize hybrid robots as a whole structure. Their method is based on Assure groups as the simplest basic blocks to build kinematic chains. The closed chains of some of them are known as parallel structures, one of them is a new hybrid structure that has too complex structure. However, little attention has been paid to the serial connections of serial and parallel manipulators that serial mechanism can do its task independently with appropriate DOF for such process. Regarding the previous works, the existing robots cannot be preferred option for interested applications. Some of the flaws existing in certain previous works include limited translational workspace or inability to make proper orientation to work piece, limited ability of entering tools in inner section of work piece, having no closed form for inverse or direct kinematic problems, improper DOF,

complex structure, planar translation for parallel robot, less attention to dynamic analysis and discuss only kinematic and the majority of them depending between moving the parallel and serial mechanisms.

In the sequence of designing the novel hybrid robots, this paper presents a hybrid robot that is appropriate for the stated applications by connecting two serial and parallel mechanisms. The hybrid-type robot has a wide workspace and serial mechanism that can perform the tasks exclusively. In this paper rigid body hypothesis and frictionless were assumed. The paper is organized as follows: In Section 2, description of the robot is discussed, robot structure and capability are described. In Section 3, the inverse and forward kinematic solutions are presented in the closed form. In Section 4, inverse dynamic analysis of the robot presented by of Iterative Newton-Euler and Lagrange dynamic formulation methods. Finally in Section 5, for performing a multi-step arc welding process, results have indicated that the introduced manipulator is highly capable of reducing the actuating energy.

## II. DESCRIPTION OF THE ROBOT

### A. Robot structure

This mechanism is made of two parts. The parallel robot is followed by a serial mechanism "Fig. 1". The first part is linked to the base by three legs in parallel. Its motion is restricted to only three translations by using the special linking of the platform to the base (fixed palate). This structure has simple structure and is made of only revolute joints [10]. The second part is designed with five revolute joints and has five DOFs. Therefore, the end-effector has eight DOFs with respect to the base.

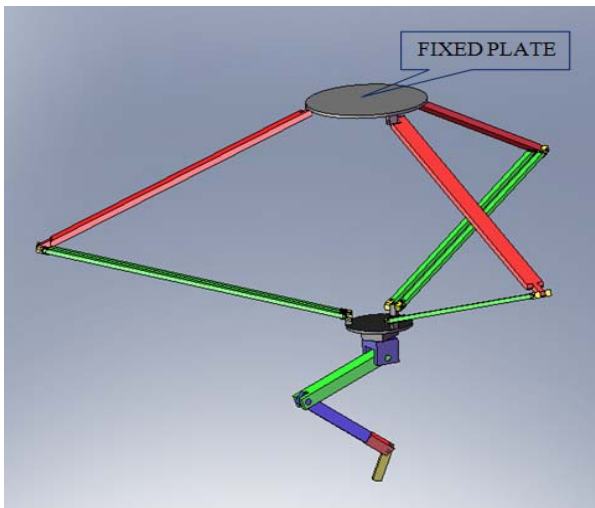


Fig. 1 Robot structure

### B. Robot capability

The design has several advantages: it presents the particularity ability to divide task in two sections for the

reduction of the actuating energy. For example, in applying a multi-step process, parallel robot transfers the serial mechanism to appropriate position then stops until serial part accomplishes its task, or moves it throughout the workspace. Moreover the continued workspace for the task can be built by moving both parts simultaneously. In addition this design presents a compromise between high rigidity of fully parallel manipulators and extended workspace of serial manipulators. Also, to avoid collision with work piece, the use of more DOF is proposed. Furthermore closed form of direct and inverse kinematic solutions were obtained for this type of mechanisms allowing the determination of all possible solutions of the forward and inverse analysis problem. This characteristic is a great help in investigating the capabilities of this type of robot and it is a valuable tool for the engineer to derive its controller. The parallel robot has high precision and high payload capability for transferring the serial mechanism and carries the wire feeder for arc welding applications. Parallel robot has almost wide workspace to carry the serial part and it merely utilizes the revolute joints, without singularity in its workspace. As a result it has a simple structure [10]. In addition special structure of serial part can create sufficient orientation tool and ability to enter the inner part without collision with work pieces.

## III. KINEMATICS PROBLEMS OF THE ROBOT

Mechanism kinematics deals with the study of the mechanism motion as constrained by the geometry of the links. Typically, the study of mechanism kinematics is divided into two parts: inverse kinematics and direct kinematics. The inverse kinematics problem involves mapping a known pose (position and orientation) of the tool of the mechanism to a set of input joint variables that will achieve that pose. The direct kinematics problem involves the mapping from a known set of input joint variables to a pose of the tool that results from those given inputs. In this section the inverse and forward kinematics problems of proposed mechanism are described in closed form.

### A. The forward kinematics

In forward kinematics the closed-form solution has been obtained for the parallel robot. Also it has been shown that two solutions exist [10]. With regard to pure translational movement of parallel robot, transformation matrices can be computed as follows. Ground base frame is named by  $\tilde{0}$  and center of parallel platform is named by  $P$ .

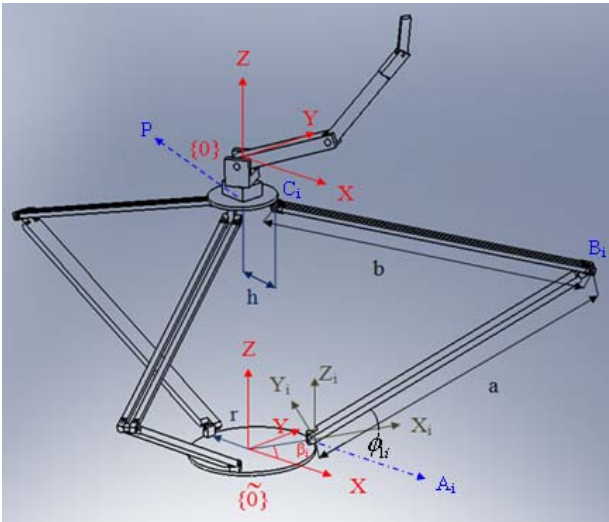


Fig. 2 Nomenclature of axis and length for parallel robot

$${}^0_P T = \begin{pmatrix} 1 & 0 & 0 & X_p \\ 0 & 1 & 0 & Y_p \\ 0 & 0 & 1 & Z_p \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (1)$$

In serial mechanism, from the value of the link parameter (Table I), the individual link transformation matrixes can be computed. Then the link transformation matrixes can be multiplied together to find the single transformation that relates frame (6) to frame (0).

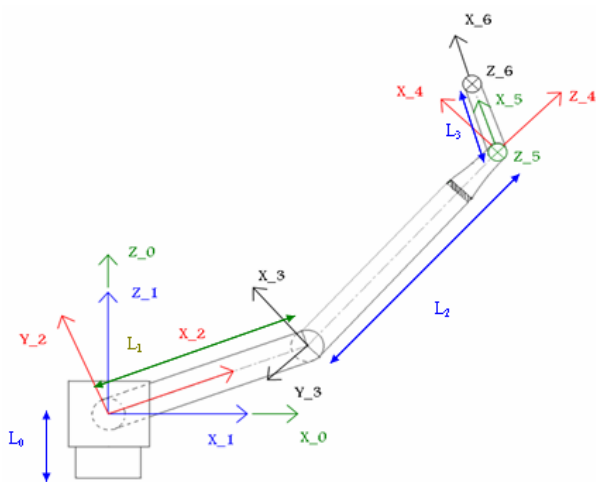


Fig. 3 Nomenclature of axis and length for serial robot

 TABLE I  
LINK DENAVIT-HARTENBERG PARAMETERS OF THE SERIAL MECHANISM

$i$	$\alpha_{i-1}$	$a_{i-1}$	$d_i$	$\theta_i$
1	$\alpha_0 = 0$	$a_0 = 0$	$d_1 = 0$	$\theta_1$
2	$\alpha_1 = +90$	$a_1 = 0$	$d_2 = 0$	$\theta_2$
3	$\alpha_2 = 0$	$a_2 = L_1$	$d_3 = 0$	$\theta_3$
4	$\alpha_3 = +90$	$a_3 = 0$	$d_4 = L_2$	$\theta_4$
5	$\alpha_4 = +90$	$a_4 = 0$	$d_5 = 0$	$\theta_5$
6	$\alpha_5 = 0$	$a_5 = L_3$	$d_6 = 0$	$\theta_6 = 0$

Transformation matrix,  $({}^0_6 T)$ , will be a function of all variables.

$${}^0_6 T = {}^0_P T {}^P_0 T {}^0_1 T {}^1_2 T {}^2_3 T {}^3_4 T {}^4_5 T {}^5_6 T \quad (2)$$

$${}^0_6 T = \begin{pmatrix} r_{11} & r_{12} & r_{13} & P_x + X_p \\ r_{12} & r_{22} & r_{23} & P_y + Y_p \\ r_{31} & r_{32} & r_{33} & P_z + L_0 + Z_p \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

### B. The inverse kinematics

In this section the position and orientation of end-effector or the matrix (3) is known and a set of input joint variables are desired. With defined value of parallel robot and serial mechanism, solutions can be derived in closed form.

Matrices  ${}^0_P T$  and  ${}^0_6 T$  are known. In parallel robot, two solutions exist in closed form for proper dimension [10]. For serial mechanism, Equation (4) must be solved for  $\theta_i$  ( $i = 1 \dots 6$ ) when  ${}^0_6 T$  is known as numeric values.

$${}^0_6 T = \begin{pmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}^0_1 T(\theta_1) {}^1_2 T(\theta_2) {}^2_3 T(\theta_3) {}^3_4 T(\theta_4) {}^4_5 T(\theta_5) {}^5_6 T(\theta_6) \quad (4)$$

Transformation matrix  ${}^0_5 T$  is pure translational so  ${}^0_5 T$  have used to find the input joint variables.

$${}^0_5 T = {}^0_6 T = \begin{pmatrix} r_{11} & r_{12} & r_{13} & -r_{11}L_3 + P_x \\ r_{21} & r_{22} & r_{23} & -r_{21}L_3 + P_y \\ r_{31} & r_{32} & r_{33} & -r_{31}L_3 + P_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

A restatement of (5) that puts the dependence to  $\theta_1$ , on the left hand side of the equation is

$$({}^0_5 T)^{-1} {}^0_6 T = {}^0_5 T \quad (6)$$

Inverting  ${}^0_5 T$ , equation (6) was written as

$$\begin{pmatrix} C1 & S1 & 0 & 0 \\ -S1 & C1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & \tilde{P}_x \\ r_{21} & r_{22} & r_{23} & \tilde{P}_y \\ r_{31} & r_{32} & r_{33} & \tilde{P}_z \\ 0 & 0 & 0 & 1 \end{pmatrix} = {}_5^4T \quad (7)$$

Equating the (2, 4) elements from both sides of (7)

$$-S1\tilde{P}_x + C1\tilde{P}_y = 0 \quad (8)$$

Therefore, the two solutions for  $\theta_1$  was written as

$$\theta_1 = A \tan 2(\tilde{P}_y, \tilde{P}_x) \quad \theta_1 = A \tan 2(-\tilde{P}_y, -\tilde{P}_x) \quad (9), (10)$$

Equating both the (1, 4) elements and the (3, 4) elements from two sides of (7), obtained

$$C1\tilde{P}_x + S1\tilde{P}_y = C2S3L_2 + C2L_1 + S2C3L_2 \quad (11)$$

$$\tilde{P}_z = S2S3L_2 + S2L_1 - C2C3L_2 \quad (12)$$

If square equation (11), (12), (8) and add the resulting equations, obtained

$$S3 = \frac{\tilde{P}_x^2 + \tilde{P}_y^2 + \tilde{P}_z^2 - L_1^2 - L_2^2}{2L_1L_2} = K \quad (13)$$

Therefore, two solution for  $\theta_3$  was written as

$$\theta_3 = A \tan 2(K, \sqrt{1-K^2}) \quad \theta_3 = A \tan 2(K, -\sqrt{1-K^2}) \quad (14), (15)$$

Rewrite (5) that all the left hand side are function of only determined joint variables and  $\theta_2 + \theta_3$

$$({}_3^0T)^{-1} {}_5^0T = {}_3^2T \quad (16)$$

$$\begin{bmatrix} c1c(2+3) & s1c(2+3) & s(2+3) & -c3L_1 \\ -c1s(2+3) & -s1s(2+3) & c(2+3) & s3L_1 \\ s1 & -c1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & \tilde{P}_x \\ r_{21} & r_{22} & r_{23} & \tilde{P}_y \\ r_{31} & r_{32} & r_{33} & \tilde{P}_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c4c5 & -c4s5 & s4 & 0 \\ -s5 & -c5 & 0 & -L_2 \\ s4c5 & -s4s5 & -c4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

Equating the (1, 4), (2, 4) elements from both sides of (17), there are two unknown and two equation of  $\theta(2+3)$

$$(C1\tilde{P}_x + S1\tilde{P}_y)C(2+3) + (\tilde{P}_z)S(2+3) = C3L_1 \quad (18)$$

$$(\tilde{P}_z)C(2+3) + (-C1\tilde{P}_x - S1\tilde{P}_y)S(2+3) = -L_2 - S3L_1 \quad (19)$$

That solved as follow and founding  $\theta(2+3)$

$$\theta(2+3) = A \tan 2\left(\frac{s(2+3)}{c(2+3)}\right) \quad (20)$$

$$\theta_2 = \theta(2+3) - \theta_3 \quad (21)$$

Therefore, there are four solutions for  $\theta_2$

Equating the (1, 3), (3, 3) elements from both sides of (17)

$$r_{33}S(2+3) + r_{23}S1C(2+3) + r_{13}C1C(2+3) = S4 \quad (22)$$

$$-S1r_{13} + r_{23}C1 = C4 \quad (23)$$

There are four solutions for  $\theta_4$

$$\theta_4 = A \tan 2\left(\frac{s4}{c4}\right) \quad (24)$$

Rewrite (5) that all the left hand side is a function of only determined joint variables and  $\theta_5$

$$({}_4^0T)^{-1} {}_5^0T = {}_4^4T \quad (25)$$

$$\begin{bmatrix} C4C1C(2+3) + S1S4 & C4S1C(2+3) - C1S4 & S(2+3)C4 & -C4C3L_1 \\ -S4C1C(2+3) + S1C4 & -S4S1C(2+3) - C1C4 & -S(2+3)S4 & S4C3L_1 \\ C1S(2+3) & S1S(2+3) & -C(2+3) & -L_2 - S3L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} & \tilde{P}_x \\ r_{21} & r_{22} & r_{23} & \tilde{P}_y \\ r_{31} & r_{32} & r_{33} & \tilde{P}_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} C5 & -S5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S5 & C5 & 0 & 0 \end{bmatrix} \quad (26)$$

Equating the (1, 4), (3, 4) elements from both sides of (26),

$$C5 = r_{11}(C4C1C(2+3) + S1S4) + r_{21}(C4S1C(2+3) - C1S4) + r_{31}(C4S(2+3)) \quad (27)$$

$$S5 = (r_{11}C1 + r_{21}S1)S(2+3) - r_{31}C(2+3) \quad (28)$$

So, there are four solutions for  $\theta_5$

$$\theta_5 = A \tan 2\left(\frac{S5}{C5}\right) \quad (29)$$

Finally, whereas four Solutions have been found for serial mechanism, consequently total solution for inverse kinematic of the hybrid robot is eight.

#### IV. NUMERICAL EXAMPLE

In this section a numerical example is solved for direct and inverse kinematics. First direct kinematics is solved by these data: ( $\phi_{11} = 70$ ,  $\phi_{12} = 85$ ,  $\phi_{13} = 65$ ,  $\theta_1 = 45$ ,  $\theta_2 = 50$ ,  $\theta_3 = 160$ ,  $\theta_4 = 60$ ,  $\theta_5 = 35$ ). Then determine the

transformation matrix  ${}^0_6T$ . After that by the introduced analytical method all eight solutions are derived. That shows that they include direct data. In the case of an actual manipulator, there are well-defined limits on these variables, which have to be satisfied at the task. Here, units (m, kg, s, degree) have been used for all quantities. Assumption length are as follow: ( $L_0 = 0.05$ ,  $L_1 = 0.310$ ,  $L_2 = 0.370$ ,  $L_3 = 0.140$ ,  $a = 1$ ,  $b = 1.04$ ,  $r = 0.19$ ,  $h = 0.1$ ).

TABLE II  
THE EIGHT SOLUTIONS OF THE INVERSE KINEMATIC PROBLEM

i	$\phi_{11}$ (deg)	$\phi_{12}$ (deg)	$\phi_{13}$ (deg)	$\theta_1$ (deg)	$\theta_2$ (deg)	$\theta_3$ (deg)	$\theta_4$ (deg)	$\theta_5$ (deg)
1	70.0000	85.0000	65.0000	45.0000	101.6560	20.0000	51.9678	334.1937
2	70.0000	85.0000	65.0000	45.0000	50.00000	160.0000	60.0000	35.0000
3	70.0000	85.0000	65.0000	225.0000	52.0475	160.0000	157.6007	256.4691
4	70.0000	85.0000	65.0000	225.0000	463.7035	20.0000	174.8721	292.9193
5	106.7820	126.4843	103.8992	45.0000	101.6560	20.0000	51.9678	334.1937
6	106.7820	126.4843	103.8992	45.0000	50.00000	160.0000	60.0000	35.0000
7	106.7820	126.4843	103.8992	225.0000	52.0475	160.0000	157.6007	256.4691
8	106.7820	126.4843	103.8992	225.0000	463.7035	20.0000	174.8721	292.9193

Regard to the (Table II) all eight solutions are derived and solution number 2 is the direct data exactly.

#### V. DYNAMICS ANALYSIS

In this section inverse dynamic equations have been derived and validated with the MSC software package. Analysis of the robot was presented by methods of Iterative Newton-Euler and Lagrange dynamic formulation. The external force and moment of tool have been taken as zero.

However, the program includes external forces and moments and if the force data are available, they are automatically handled by the program.

Firstly, for serial part, inverse dynamic analysis is derived by Iterative Newton-Euler dynamic formulation. This analysis has been done by including the position, velocity, and acceleration of serial's base caused by parallel robot.

$${}^{i+1}F_{i+1} = m_{i+1} {}^{i+1}\dot{V}_{C_{i+1}} \quad (30)$$

$${}^{i+1}N_{i+1} = C_{i+1} I_{i+1} {}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times C_{i+1} I_{i+1} {}^{i+1}\omega_{i+1} \quad (31)$$

$${}^i f_i = {}^{i+1}R {}^{i+1}f_{i+1} + {}^i F_i \quad (32)$$

$${}^i n_i = {}^i N + {}^i R_{i+1} {}^{i+1}n_{i+1} + {}^i P_{C_i} \times {}^i F_i + {}^i P_{i+1} \times {}^i R_{i+1} {}^{i+1}f_{i+1} \quad (33)$$

$$\tau_i = {}^i n_i^T {}^i \hat{Z}_i \quad (34)$$

With defining a trajectory, actuator torques are calculated. Clearly, these torques of serial mechanism are functions of the parallel movement. Results of actuator torques are demonstrated as follows.

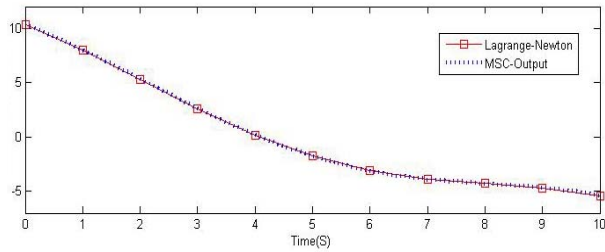


Fig. 4 Actuator torque for  $\tau_1$  (N.M)

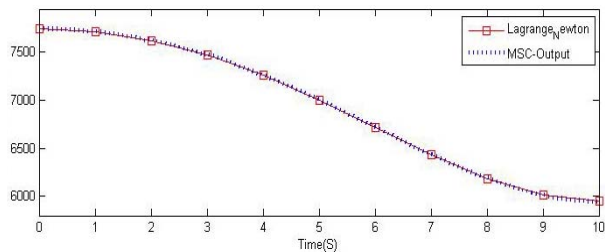


Fig. 5 Actuator torque for  $\tau_2$  (N.M)

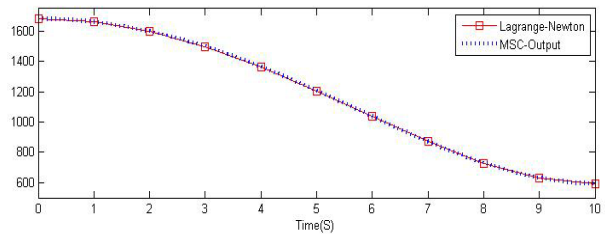


Fig. 6 Actuator torque for  $\tau_3$  (N.M)

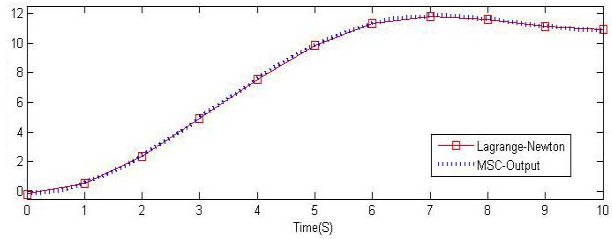


Fig. 7 Actuator torque for  $\tau_4$  (N.M)

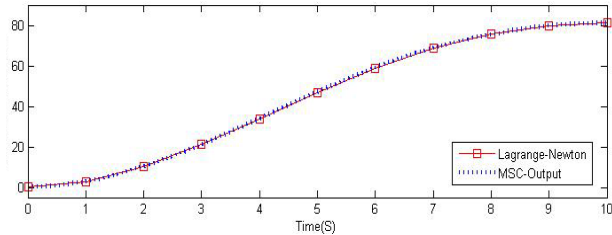


Fig. 8 Actuator torque for  $\tau_5$  (N.M)

In parallel robot, inverse dynamic analysis is derived by Lagrange multiplayer dynamic formulation.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = Q_j + \sum_{i=1}^3 \lambda_i \frac{\partial \Gamma_i}{\partial q_j} \quad j = 1 \dots 6 \quad (35)$$

Where  $\Gamma_i$  denotes the  $i$ th constraint function,  $\lambda_i$  is the Lagrangian multiplier.  $Q_j$  represents the generalized force contributed by an externally applied force. The numbers of generalized coordinates are six as follow.

$$q_1 = \phi_{11} \quad q_2 = \phi_{12} \quad q_3 = \phi_{13} \quad q_4 = X_p \quad q_5 = Y_p \quad q_6 = Z_p \quad (36)$$

Equation (35) represents a system of six equations in six variables. The six variables are  $\lambda_i$  ( $i = 1 \dots 3$ ), and the three actuator torques,  $Q_j$  ( $j = 4, 5, 6$ ).

Generalized forces,  $Q_j$  ( $j = 1 \dots 3$ ), represented x, y, and z components of an external force exerted at the center of moving platform. These forces are caused by weight and movement of serial mechanism in the introduced manipulator. The constrain equations are obtained from the fact that distance between joints B and C is always equal to the length of the connecting rod of the upper arm (b).

$$\Gamma_i = \left| \overline{B_i C_i} \right|^2 - b^2 = (X_p + hc \beta_i - rc \beta_i - ac \beta_i c \phi_i)^2 \quad (37)$$

$$+ (Y_p + hs \beta_i - rs \beta_i - as \beta_i c \phi_i)^2 + (Z_p - as \phi_i)^2 - (b)^2 = 0$$

To simplify the analysis, we assume that the mass of each connection rod,  $m_b$ , in the upper arm assembly is divided and concentrated at the two endpoints  $B_i$  and  $C_i$ . Then the Lagrange function L has been driven as follows.

$$K = K_p + \sum_{i=1}^3 (K_{ai} + K_{bi}) \quad (38)$$

Where  $K_p$  is the kinetic energy of the moving platform,  $K_{ai}$  is the kinetic energy of the input link on limb  $i$ , and  $K_{bi}$  is the kinetic energy of the two connecting rods of limb  $i$ .

$$K_p = 0.5m_p(\dot{X}_p^2 + \dot{Y}_p^2 + \dot{Z}_p^2) \quad (39)$$

$$K_{ai} = 0.5\left(\frac{1}{3}m_a a^2\right)\dot{\phi}_i^2 \quad (40)$$

$$K_{bi} = 0.5m_p(\dot{X}_p^2 + \dot{Y}_p^2 + \dot{Z}_p^2) + 0.5m_b a^2 \dot{\phi}_i^2 \quad (41)$$

Where  $m_p$  is the mass of the moving platform,  $m_a$  is the mass of the input link,  $m_b$  is the mass of one from the two connecting rods.

Assuming that the acceleration of gravity points is in the  $z$  direction, the total potential energy of the manipulator is

$$U = U_p + \sum_{i=1}^3 (U_{ai} + U_{bi}) \quad (42)$$

Whereas  $U_p$  is the potential energy of the moving platform,  $U_{ai}$  is the potential energy of the two connecting rods of the  $i$ th limb.

$$U_p = m_p g Z_p \quad (43)$$

$$U_{ai} = 0.5m_a g a \sin \phi_i \quad (44)$$

$$U_{bi} = m_b g (Z_p + a \sin \phi_i) \quad (45)$$

Therefore, the Lagrange function is

$$L = 0.5(m_p + 3m_b)(\dot{X}_p^2 + \dot{Y}_p^2 + \dot{Z}_p^2) + 0.5\left(\frac{1}{3}m_a a^2 + m_b a^2\right)(\dot{\phi}_{11} + \dot{\phi}_{12} + \dot{\phi}_{13}) - (m_p + 3m_b)gZ_p - (0.5m_a + m_b)ga(S\phi_{11} + S\phi_{12} + S\phi_{13}) \quad (46)$$

Taking the derivatives of Lagrange function and  $\Gamma_i$  with respect to the six generalized coordinates and application of equation (35), yields a system of dynamical equation

$$(m_p + 3m_b)\ddot{X}_p = F_{PX} + 2\sum_{i=1}^3 \lambda_i (X_p + hC\beta_i - rC\beta_i - aC\beta_i C\phi_i) \quad (47)$$

$$(m_p + 3m_b)\ddot{Y}_p = F_{PY} + 2\sum_{i=1}^3 \lambda_i (Y_p + hS\beta_i - rS\beta_i - aS\beta_i C\phi_i) \quad (48)$$

$$(m_p + 3m_b)\ddot{Z}_p + (m_p + 3m_b)g = F_{PZ} + 2\sum_{i=1}^3 \lambda_i (Z_p - aS\phi_i) \quad (49)$$

$$\left(\frac{1}{3}m_a a^2 + m_b a^2\right)\ddot{\phi}_{11} + \left(\frac{1}{2}m_a + m_b\right)gaC\phi_{11} \quad (50)$$

$$= Q_4 + 2a\lambda_1 [(X_p C\beta_1 + Y_p S\beta_1 + h - r)S\phi_{11} - Z_p C\phi_{11}]$$

$$\left(\frac{1}{3}m_a a^2 + m_b a^2\right)\ddot{\phi}_{12} + \left(\frac{1}{2}m_a + m_b\right)gaC\phi_{12} = \quad (51)$$

$$Q_5 + 2a\lambda_2 [(X_p C\beta_2 + Y_p S\beta_2 + h - r)S\phi_{12} - Z_p C\phi_{12}]$$

$$\left(\frac{1}{3}m_a a^2 + m_b a^2\right)\ddot{\phi}_{13} + \left(\frac{1}{2}m_a + m_b\right)gaC\phi_{13} \quad (52)$$

$$= Q_6 + 2a\lambda_3 [(X_p C\beta_3 + Y_p S\beta_3 + h - r)S\phi_{13} - Z_p C\phi_{13}]$$

Where  $Q_i$  ( $i = 4 \dots 6$ ), are actuator torques and  $F_{PX}$ ,  $F_{PY}$  and  $F_{PZ}$  are external forces that are exerted at the

center of moving platform in  $x$ ,  $y$ , and  $z$  directions respectively. These forces are caused by weight and movement of serial mechanism in the introduced manipulator. Clearly, these forces are functions of parallel robot movement. Torques of parallel robot are function of base of serial mechanism force or on the other hand are function of parallel movement. Actuator torques calculated. Result of Actuator torques is demonstrated as follow.

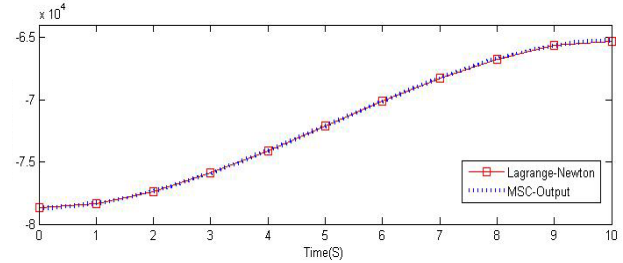


Fig. 9 Actuator torque for  $Q_4$  (N.M)

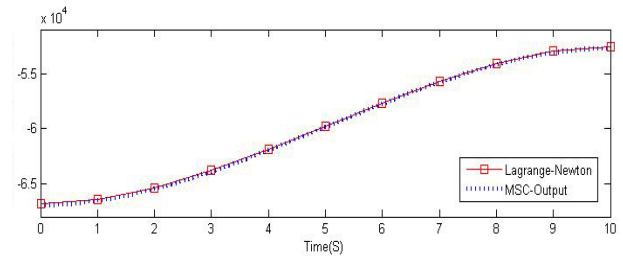


Fig. 10 Actuator torque for  $Q_5$  (N.M)

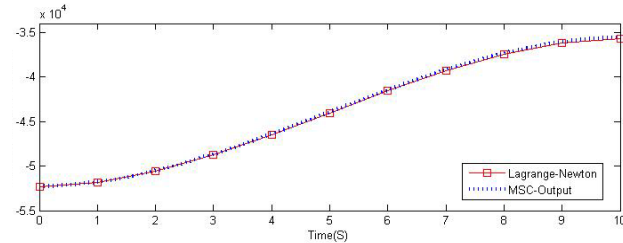


Fig. 11 Actuator torque for  $Q_6$  (N.M)

## VI. ENERGY INVESTIGATION

In "Fig. 12" a multi-step process is demonstrated for compromise between a traditional serial robot and introduced manipulator to represent its ability to reduce the consumed energy. By hybrid manipulator, parallel robot transfers the serial mechanism to the desired position then parallel robot stops and serial mechanism accomplish its task alone. This capacity can save energy a lot.

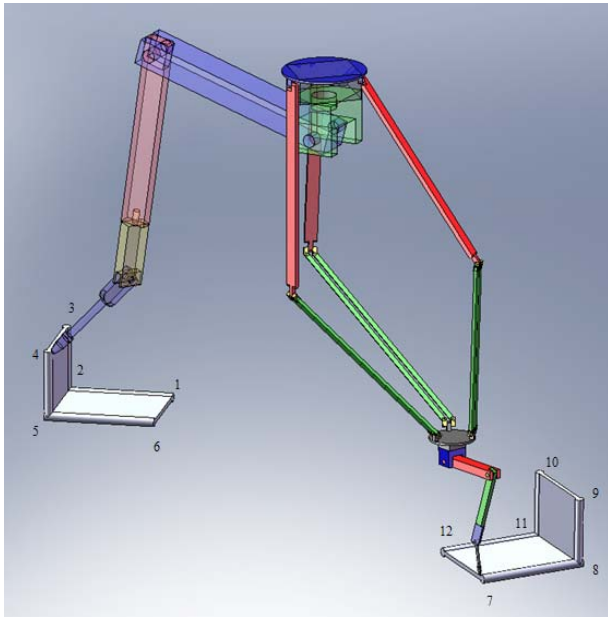


Fig. 12 Multi-step process

Result of described process was demonstrated in (Table. III).

TABLE III  
THE CONSUME ENERGY OF THE PROPOSED HYBRID ROBOT AND  
TRADITIONAL SERIAL ROBOT (J).

Step	1-2	2-3	3-4	4-5	5-6	6-7
Serial	11.1403	24.1564	4.9468	36.0494	6.2893	77.2069
Hybrid	0.6181	1.3263	0.2686	1.9857	0.3482	114.3226
Step	7-8	8-9	9-10	10-11	11-12	TOTAL
Serial	9.7774	25.6590	4.7528	19.2781	6.6487	225.9051
Hybrid	0.5368	1.4009	0.2663	1.0520	0.3618	122.4873

Regarding the (Table. III), it is obvious that the designed robot has great ability to reduce the actuating energy. Results establish that for the defined process the introduced mechanism can decrease the actuating energy about 46 %.

## VII. CONCLUSION

This paper presents kinematic and dynamic analysis of a novel 8-DOF hybrid robot manipulator. The introduced mechanism has a wide range of workspace and high capability to reduce the actuating energy. The inverse and forward kinematic solutions are described in closed form and the theoretical results have been verified numerically. Inverse dynamic analysis of the robot is presented by utilizing Iterative Newton-Euler and Lagrange dynamic formulation methods and the results have been verified by MSC software. Finally, for performing a multi-step arc welding process, results have indicated that the introduced manipulator is highly capable of reducing the actuating energy.

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