Denosing ECG using Translation Invariant Multiwavelet

Jeong Yup Han, Su Kyung Lee, and Hong Bae Park

***Abstract*—**In this paper, we propose a method to reduce the various kinds of noise while gathering and recording the electrocardiogram (ECG) signal. Because of the defects of former method in the noise elimination of ECG signal, we use translation invariant (TI) multiwavelet denoising method to the noise elimination. The advantage of the proposed method is that it may not only remain the geometrical characteristics of the original ECG signal and keep the amplitudes of various ECG waveforms efficiently, but also suppress impulsive noise to some extent. The simulation results indicate that the proposed method are better than former removing noise method in aspects of remaining geometrical characteristics of ECG signal and the signal-to-noise ratio (SNR).

***Keywords*—**ECG, TI multiwavelet, denoise.

I. Introduction

ECG signal is one of the biomedical signals, which are widely studies and applied in clinic. A normal ECG waveform is usually composed of P wave, QRS complexes, and T wave, and the accurate detection of them is important to analyze ECG signal. However, because ECG signal is very faint, it is extremely easy to interfere by the different noises while gathering and recording. How to suppress noises effectively is always an important problem in the detection of ECG signal.

Recently, wavelet transform has been widely used in signal and image processing due to the time-frequency localization characteristics[1]. There are mainly two kinds of wavelet denoising methods used in denoising of ECG signal: one is wavelet transform modulus maxima method. This method can eliminate noises and remain the information of the original signal in maximum at the same time, but the amount of calculation is great, and the process of calculation may be unstable[2]. The other is wavelet thresholding denoising method. Wavelet thresholding denoising method deals with wavelet coefficients using a suitable threshold chosen in advance. The wavelet coefficients at different scales could be obtained by taking discrete wavelet transform (DWT) of the noisy signal[3].

Normally, those wavelet coefficients with smaller magnitudes than the preset threshold are caused by the noise and replaced by zero, and the others with larger magnitudes than the preset threshold are caused by original signal mainly and kept (hard-thresholding case) or shrunk (the soft-thresholding case)[4]. Then the denoised signal could be reconstructed from the resulting wavelet coefficients. This method is simple and easy to be used in denoising of ECG signal. But hard-thresholding denoising method may lead to the oscillation of the reconstructed ECG signal. The soft-thresholding denoising method may reduce the amplitudes of ECG waveforms, and especially reduce the amplitudes of the R waves. To overcome these disadvantages mentioned above, an improved thresholding denoising method is proposed firstly. It is a compromising method between the hard- and soft-thresholding. Secondly, a new translation-invariant (TI) wavelet denoising method with improved thresholding is presented to eliminate the noise of ECG signal.

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II. Denoising methods

A. Wavelet Transform

The wavelet transform is a convolution of the wavelet function $\psi(t)$ with the signal $x(t)$. Orthonormal dyadic discrete wavelets are associated with scaling functions $\phi(t)$. The scaling function can be convolved with the signal to produce approximation coefficients S. The DWT can be written by [1]

$$T_{m,n} = \int_{-\infty}^{\infty} x(t)\psi_{m,n}(t)\,dt. \tag{1}$$

By choosing an orthonormal wavelet basis, $\psi_{m,n}(t)$, one can reconstruct the original. The approximation coefficients of the signal at scale m and n can be presented by

$$S_{m,n} = \int_{-\infty}^{\infty} x(t)\phi_{m,n}(t)\,dt. \tag{2}$$

A discrete input signal, $S_{0,n}$ is of finite length N, which is an integer power of 2: $N = 2^M$. Thus the range of scales that can be investigated is $0 < m < M$. A discrete approximation of the signal can be shown as be shown as

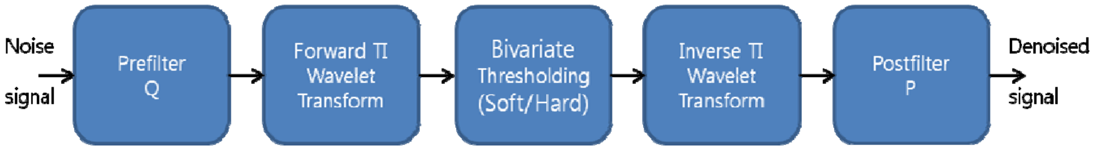


Fig. 1 Block diagram of the TI multiwavelet denosing

$$x_0(t) = x_m(t) + \sum_{m=1}^{M} d_m(t) \tag{3}$$

Where, the mean signal approximation at scale M is

$$x_m(t) = S_{M,n}\phi_{M,n}(t) \tag{4}$$

And the detail signal approximation corresponding to scale m is defined for a finite length signal as

$$d_m(t) = \sum_{n=0}^{2^{M-m}-1} T_{m,n}\psi_{m,n}(t) \tag{5}$$

Adding the approximation of the signal at scale index M to the sum of all detail signal components across scales gives the approximation of the original signal at scale index 0. The signal approximation at a specific scale was a combination of the approximation and detail at the next lower scale.

$$x_m(t) = x_{m-1}(t) - d_m(t) \tag{6}$$

If scale $m = 3$ was chosen, it can be shown that the signal approximation is given by

$$x_3(t) = x_0(t) - d_1(t) - d_2(t) - d_3(t) \tag{7}$$

Corresponding to the successive stripping of high frequency information (contained within the $d_m(t)$) from the original signal at each step[8]. This is referred to as multi-resolution analysis of a signal using wavelet transform, and is the basic of our procedure.

B. *Translation-Invariant Multiwavelet Denoising*

TI denoising suppresses noise by averaging over thresholded signals of all circular shifts. The TI table is a fast way of implementation, rather than having to do a transform on the original signal n times. We can realize the TI multiwavelet denoising algorithm in the following steps.

1) Shifting the noisy ECG signal within range of cycle spinning to get a new shifted ECG signal, which has some phase-shift compared with the noisy ECG signal.
2) Transform the new shifted ECG signal by DWT and apply the improved thresholding to get the estimated wavelet coefficients, which are used to reconstruct denoised ECG signal by IDWT.
3) Inverse-shifting the denoised ECG signal to get the associated denoised ECG signal whose phase is the same as the noisy ECG signal.
4) Repeat the procedure 1)-3) for the next shift constantly to get a series of denoised ECG signals.
5) Calculating the average for all the obtained denoised ECG signals to get the final denoised ECG signal.

Fig. 1 shows the block diagram of the TI multiwavelet denoising scheme. The TI table in [7] is for the discrete single wavelet transform. Below, we describe the TI table for the discrete multiwavelet transform: The TI table is an n by WM matrix, where W is the number of decomposition levels, and $0 \le W \le \log_2 n$. For convenience, we partition the TI table into W column groups with columns $(j-1)M+1$ through jM into column group j for $1 \le j \le W$. Similarly, the j th column group is partitioned into 2^j "boxes," where each box is a $n/2^j$ by the M matrix. These boxes contain all the multiwavelet coefficients at scale j for different shifts. We also need an $n \times M$ matrix, which is denoted by S, to store the lowpass coefficients. This matrix is dynamically filled during each resolution scale. The fill-in of the TI table and S are fulfilled by the series of decimation and filtering operations. Let G and H stand for the usual downsampling highpass and lowpass operation of the wavelet theory. In addition, let R_h stand for circular shift by h. Let $S_{0,0}$ be the multiple streams by applying a prefilter to f, and initialize

$$\begin{aligned} D_{1,0} &= GR_0S_{0,0}, \quad D_{1,1} = GR_1S_{0,0} \\ S_{1,0} &= HR_0S_{0,0}, \quad S_{1,1} = HR_1S_{0,0} \end{aligned} \tag{8}$$

Then, we have the recursive equations

$$\begin{aligned} D_{j+1,2k} &= GR_0S_{j,k}, \quad D_{j+1,2k+1} = GR_1S_{j,k} \\ S_{j+1,2k} &= HR_0S_{j,k}, \quad S_{j+1,2k+1} = HR_1S_{j,k} \end{aligned} \tag{9}$$

for $j = J-1, \cdots, W-1$ and $k = 0, 1, \cdots, 2^{j-1}-1$. It should be mentioned that the scale W is normally chosen to be smaller than $\log_2 n$. We should place the vector $D_{j,k}$ in box k of column group j. Additionally, we need to place the $S_{j,k}$ in box k of the average matrix S. After thresholding on the TI table, we can reverse the steps during the fill-in process. Let G^* and H^* stand for the usual upsampling highpass and lowpass operations. At scale j, for each k in the range $0 \le k \le 2^j - 1$, compute

$$\gamma_k = \left(R_0 G^* S_{j,k} + R_{-1} G^* S_{j,2k+1}\right) / 2$$
$$\delta_k = \left(R_0 H^* D_{j,k} + R_{-1} H^* D_{j,2k+1}\right) / 2 \qquad (10)$$

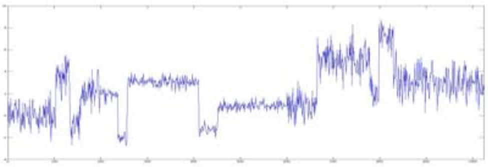
and

$$S_{j-1,k} = \gamma_k + \delta_k \qquad (11)$$

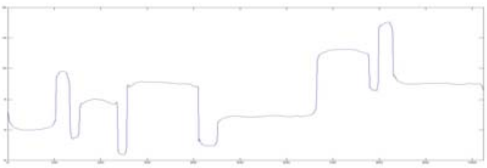
for $j = W, \cdots, 1$ and $k = 0, 1, \cdots, 2^{j-1} - 1$. The denoised multiple stream multiwavelet coefficients are then given by $S_{0,0}$. A postfilter has to be applied in order to get the denoised signal $\tilde{f}_i$.

III. Simulations

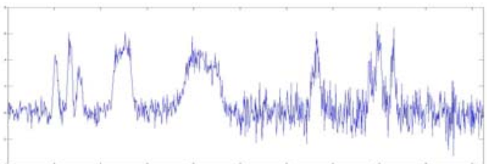
In our simulation, we examine whether the proposed method is good or bad by that signal: *Blocks, Bumps, HeaviSine, and Doppler*. Gaussian white noise is added to the signals.



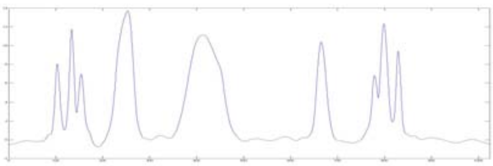
(a) The noise Blocks signal.



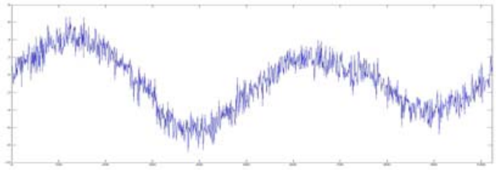
(b) The denoise Blocks signal.



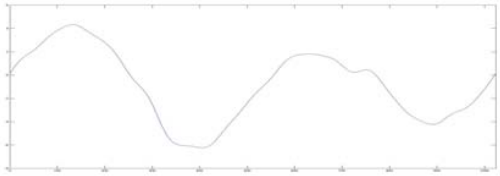
(c) The noise Bumps signal.



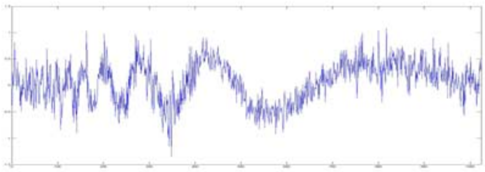
(d) The denoise Bumps signal.



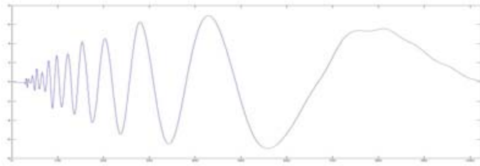
(e) The noise HeaviSine signal.



(f) The denoise HeaviSine signal.



(g) The noise Doppler signal.



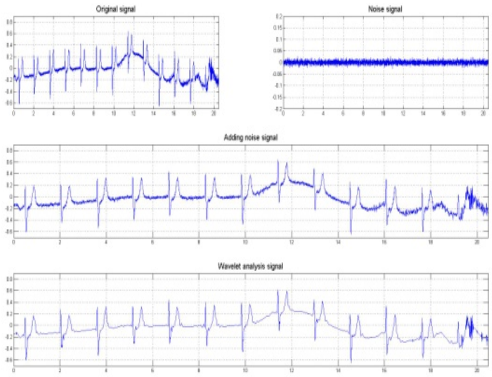
(h) The denoised Doppler signal.
Fig. 2 Blocks, Bumps, HeaviSine, and Doppler signals.

And we compute the mean square error (MSE). The MSE is shown in the Table 1.

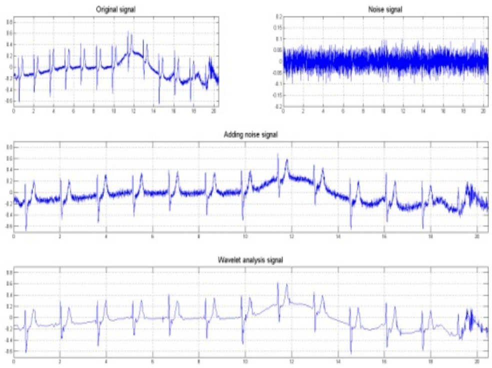
TABLE I
Mean Square Error (MSE) for Multiwavelet Denoising.

	Blocks	Bumps	Heavisine	Doppler
TI D4	34.175	35.173	11.838	25.383
TI GHM (Univariate)	27.108	24.965	**11.157**	14.869
TI GHM (Bivariate)	**23.869**	**20.428**	11.387	**13.586**
GHM (Univariate)	29.763	32.647	12.380	19.937
GHM (Bivariate)	28.246	30.582	13.001	21.297

In this section, American MIT-BIH database is used to validate the superiority of the proposed methods in this paper. At first, a “clean” ECG signal in MIT-BIH database is intercepted to be the original ECG signal. The length of the original ECG signal (i.e., the number of the sample points) is N = 2048. The original ECG signal and the ECG signal added to White Gaussian noise by other amplitude are shown in Fig. 2.

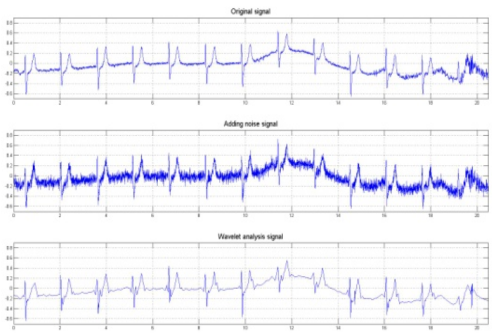


(a) The noise amplitude is 0.05.

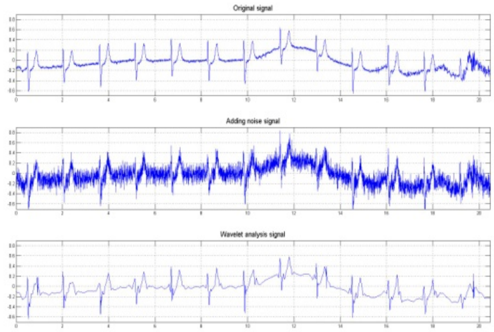


(b) The noise amplitude is 0.1.
Fig. 3 Denoised ECG signal by the noise amplitude.

In Fig. 3., we presented the original ECG signal and the ECG signal added to Gaussian white noise by other SNR.



(a) The noise SNR is 10.



(b) The noise SNR is 5.
Fig. 4 Denoised ECG signal by the noise SNR.

IV. Conclusions

In this paper, we proposed ECG signal denoising by using the TI wavelet transform. The number of wavelet coefficients is selected according to the noise level estimated in the ECG in order to avoid spending data for noise. Simulation results showed a clean ECG signal that is possible to use detecting and analyzing.

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