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Delay-range-dependent exponential synchronization of Lur'e systems with Markovian switching

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Abstract—The problem of delay-range-dependent exponential synchronization is investigated for Lur'e master-slave systems with delay feedback control and Markovian switching. Using Lyapunov-Krasovskii functional and nonsingular M-matrix method, novel delay-range-dependent exponential synchronization in mean square criterions are established. The systems discussed in this paper is advanced system, and takes all the features of interval systems, Itô equations, Markovian switching, time-varying delay, as well as the environmental noise, into account. Finally, an example is given to show the validity of the main result.

Keywords—Synchronization, Delay-range-dependent, Markov chain, Generalized Itô's formula, Brownian motion, M-matrix.

I. INTRODUCTION

CHAOS is very interesting nonlinear phenomenon and has been intensively studied in the last three decades. It is found to be useful or has great potential in many disciplines. Since Pecora and Carroll [1] addressed the synchronization problem of chaotic systems using a drive-response conception, the subject of chaotic synchronization has received considerable attentions [2-11]. Synchronization has been widely explored in a variety of fields, such as physical, chemical and ecological systems, human heartbeat regulation, secure communications, and so on.

Recently, the effect of delay on synchronization between two chaotic systems has been reported in many literatures due to the unavoidable signal propagation delay. In [12], Yalcin ME, Suykens JAK, and Vandewalle studied the master-slave synchronization of Lur'e systems with time-delay of the form

$$\mathfrak{M}:\begin{cases}\dot{x}(t)=Ax(t)+Bf(Cx(t))\\ p(t)=Hx(t)\end{cases}\tag{1}$$

$$\mathfrak{S}:\begin{cases}\dot{y}(t)=Ay(t)+Bf(Cy(t))+u(t)\\ q(t)=Hy(t)\end{cases}\tag{2}$$

$$\mathfrak{G}:u(t)=K(x(t)-y(t))+M(p(t-\tau_1)-q(t-\tau_1)).\tag{3}$$

with master system $\mathfrak{M}$, slave system $\mathfrak{S}$ and controller $\mathfrak{G}$, where the time delay $\tau_1>0$ is constant, state vectors $x,y\in R^n$, outputs of subsystems $p,q\in R^l$, H,A,B,C are real matrices, $f(.)$ is a sector condition. They are derived some delay-independent and delay-dependent synchronization criteria. In [13], Jinde Cao , H.X.Li b, Daniel W.C. Ho, studied the systems (1)–(3), employed model transformation, which leads to some conservative synchronization criteria for inducing additional terms. In [14], Ji Xiang, Yanjun Li, WeiWei used Integral inequality approach studied the same systems and again improved synchronization condition. In [15], Tao Li, Jianjiang Yu, and Zhao Wang, they considered the time-varying delay which often arises and may vary in a range, they studied the system of the form

$$\mathfrak{M}:\begin{cases}\dot{x}(t)=Ax(t)+Bf(Cx(t))\\ p(t)=Hx(t)\end{cases}\tag{4}$$

$$\mathfrak{S}:\begin{cases}\dot{y}(t)=Ay(t)+Bf(Cy(t))+u(t)\\ q(t)=Hy(t)\end{cases}\tag{5}$$

$$\mathfrak{G}:u(t)=M(p(t-d(t))-q(t-d(t))).\tag{6}$$

where the time-delay $h_1\le d(t)\le h_2$ and $\dot{d(t)}<\mu$. And derived the delay-range-dependent asymptotical synchronization criteria.

The hybrid systems combine a part of the state that takes values continuously and another part of the state that takes discrete values, an important class of hybrid systems is the semi-linear stochastic differential equation with Markovian switching of the form

$$\mathfrak{M}:\begin{cases}\dot{x}(t)=A(r(t))x(t)+B(r(t))f(Cx(t))\\ p(t)=Hx(t)\end{cases}\tag{7}$$

$$\mathfrak{S}:\begin{cases}\dot{y}(t)=A(r(t))y(t)+B(r(t))f(Cy(t))+u(t)\\ q(t)=Hy(t)\end{cases}\tag{8}$$

$$\mathfrak{G}:u(t)=M(p(t-d(t))-q(t-d(t))).\tag{9}$$

where $r(t)$ is a Markov chain taking values in $S=\{1,2,...,N\}$. Continuous-time Marlov chains are used to model the abrupt changes in system structure and parameters. If we also take the environmental noise into account, the systems (7)–(9)becomes

$$\mathfrak{M}:\begin{cases}dx(t)=[A(r(t))x(t)+B(r(t))f(Cx(t))]dt\\ \qquad +D(r(t))x(t)dw(t)\\ p(t)=Hx(t)\end{cases}\tag{10}$$

$$\mathfrak{S}:\begin{cases}dy(t)=[A(r(t))y(t)+B(r(t))f(Cy(t))\\ \qquad +u(t)]dt+D(r(t))y(t)dw(t)\\ q(t)=Hy(t)\end{cases}\tag{11}$$

$$\mathfrak{G}:u(t)=M(p(t-\delta(t))-q(t-\delta(t))).\tag{12}$$

In this paper, we will discuss (10)–(12), which is advanced system, and takes all the features of interval systems, Itô equations, Markovian switching, time-varying delay, as well as the environmental noise, into account. Then we will give the delay-range-dependent exponential synchronization criteria.

The rest of this paper is organized as follows. In section 2, we introduce the basic notation, lemma's and some definitions. In section 3, give our main results and corollary's.

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In section 4, an example is given to show the effectiveness and less conservatism of the proposed criterion.

II. Notation and preliminaries

Throughout this article, unless otherwise specified, we use the following notations. Let $|.|$ be the Euclidean norm in R^n. If A is a vector or matrix, its transpose is denoted by A^T. If A is a matrix, its trace norm is denoted by $|A| = \sqrt{trace(A^TA)}$ while its operator norm is denoted by $\|A\| = \sup\{|Ax| : |x| = 1\}$. If A is a symmetric matrix, denote by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ its largest and smallest eigenvalue, respectively.

Let $R_+ = [0,\infty)$ and $\tau > 0$. Let $C([-\tau,0];R^n)$ denote the family of continuous functions φ from $[-\tau,0]$ to R^n with the norm $\|\varphi\| = sup_{-\tau\leq\theta<0}|\varphi(\theta)|$. Let $\delta(t) : R_+ \to [0,\tau]$ be a continuous function which will stand for the time delay of the systems discussed in this paper. As a standing hypothesis, we shall always assume that $\delta(t)$ is differentiable and its derivative is bounded by a constant less than one, namely $\dot{\delta}(t) \leq \delta_0 < 1, \forall t \geq 0$. Let $\{\Omega,\mathcal{F},\mathbb{P}\}$ be a complete probability space equipped with some filtration $\{\mathcal{F}_t\}_{t\geq0}$ satisfying the usual conditions, i.e., the filtration is right continuous and $\mathcal{F}_0$-contains all $\mathbb{P}$-null sets. $C^b_{\mathfrak{F}_0}([-\tau,0];R^n)$: the family of all bounded, $C([-\tau,0];R^n)$-valued, $\mathfrak{F}_0$measurable random variables. Let $\mathbb{W}(t)$ be a standard n-dimensional Brownian motion defined on the probability space. Let $r(t), t \geq 0$ be a right-continuous Markov chain on the probability space taking values in a finite state space $S = \{1,2,...,N\}$ with generator $\Gamma = (r_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta) = j|r(t) = i\} = \begin{cases} r_{ij}\Delta + o(\Delta), & \text{if } i \neq j \\ 1 + r_{ii}\Delta + o(\Delta), & \text{if } i = j \end{cases}$$

where $\Delta > 0$. Here $r_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$, while $r_{ii} = -\sum_{j\neq i} r_{ij}$. We assume that the Markov chain $r(t)$ is independent of the Brownian motion $w(t)$. It is well known that almost every sample path of $r(t)$ is a right-continuous step function with a finite number of simple jumps in any finite subinterval of R_+. In other words, there is a sequence of stopping times $0 = \tau_0 < \tau_1 < ... < \tau_k \to \infty$ almost surely such that $r(t) = \sum_{k=0}^{\infty} r(\tau_k)I_{[}\tau_k,\tau_{k+1})$, where I_A denotes the indicator function of set A.

If A and B are symmetric matrix, by $A > B$ and $A \geq B$ we means that $A - B$ is positive and nonnegative definite, respectively. If A_1 is a vector or matrix, by $A_1 \gg 0$ we mean all elements of A_1 are positive. If A_1 and A_2 are vectors or matrices with same dimensions, we write $A_1 \gg A_2$ if and only if $A_1 - A_2 \gg 0$. Moreover, we also adopt here the traditional notation by letting $Z^{N\times N} = \{A = [a_{ij}]_{N\times N} : a_{ij} \leq 0, i \neq j\}$. Now defining the synchronization error as $e(t) = y(t) - x(t)$, (10)–(12) has the error-dynamics system of the form

$$\begin{cases} de(t) = [A(r(t))e(t) + B(r(t))\eta(Ce,x) \\ \qquad + MHe(t-\delta(t))]dt \\ \qquad + D(r(t))e(t)dw(t), \forall t \geq 0 \\ e(t) = \xi(t), t \in [-\tau,0] \end{cases} \tag{13}$$

where $\xi(t) = \psi(t) - \phi(t), t \in [-\tau,0]$, $\eta(Ce,x) := f(Ce + Cx) - f(Cx)$ and assume $\eta(Ce,x) \leq KCe(t)$. State vectors $x,y \in R^n$, outputs of subsystems $p,q \in R^l$, and matrices $H \in R^{l\times n}, A_r \in R^{n\times n}, B_r \in R^{n\times n_h}, C \in R^{n_h\times n}, D_r \in R^{n\times n}, K \in R^{n\times n_h}$, $r(t)$ is a Markov chain taking values in $S = \{1,2,...,N\}$.

The purpose of this paper is to find the condition made the erro-dynamics systems (13) exponential stable in mean square, which means that the system described by (10)–(12) exponential synchronization in mean square.

We still denote by $x(t,\xi)$, it is known that $\{x(t,\xi), r(t)\}_{t\geq0}$ is a $C([-\tau,0];R^n)\times S$-valued Markov process. Its infinitesimal operator $\mathfrak{L}$, acting on functional $V{:}C([-\tau,0];R^n)\times S\times R_+ \to R$, is defined by

$$\mathfrak{L}V(x(t),i,t) = \lim_{\Delta\to0^+}\frac{1}{\Delta}[E(V(x(t+\Delta),r(t+\Delta), \\ t+\Delta)|x(t),r(t)=i) - V(x(t),i,t)]. \tag{14}$$

Definition 2.1. The master-slave system (10)–(12) can be exponentially synchronized in mean square, if the trivial solution of equation (13) is exponentially stable in mean square, i.e.

$$\lim_{t\to\infty}\sup\frac{1}{t}\ln(E(|e(t;\xi)|^2) < 0 \tag{15}$$

for any initial data $\xi \in C^b_{\mathfrak{F}_0}([-\tau,0];R^n)$

Definition 2.2. A square matrix $A = [a_{ij}]_{N\times N}$ is called a nonsingular M-matrix if A can be expressed in the form $A = sI - B$ with $s > \rho(B)$ while all the elements of B are nonnegative, where I is the identity matrix and$\rho(B)$ the spectral radius of B. It is easy to see that a nonsingular M-matrix A has nonpositive off-diagonal and positive-diagonal entries, that is $a_{ii} > 0$, while $a_{ij} \leq 0, i \neq j$.

lemma 2.1. (See Ref.[16]) *If $A \in Z^{N\times N}$, then the following statements are equivalent.*
1) A is a nonsingular M-matrix.
2) A is semipositive; that is, there exists $x \gg 0$ in R^N such that $Ax \gg 0$.
3) A^{-1} exists and its elements are all nonnegative.
4) All the leading principal minors of A are positive; that is

$$\begin{vmatrix} a_{11} & \cdots & a_{1k} \\ \vdots & & \vdots \\ a_{k1} & \cdots & a_{kk} \end{vmatrix} > 0, \text{ for every } k = 1,2,...,N$$

lemma 2.2. Let $x \in R^n$,$y \in R^n$, matrix B has the suitable dimensional, constant $\varepsilon > 0$ and $Q > 0$. Then, we have

$$2x^TQBy \leq \varepsilon x^TQx + \varepsilon^{-1}y^TB^TQBy$$

III. Main results

Theorem 3.1. Assume that there are two constants λ_1 and λ_2 such that

$$\lambda_1 > \tau\lambda_2 \tag{16}$$

Assume also that there are symmetric matrices $Q_i > 0$,$E_i \geq 0$ and constants $\varepsilon_i > 0 (1 \leq i \leq N)$ such that

$$\lambda_{\max}(Q_iA_i + A_i^TQ_i + 2Q_iB_iKC + D_i^TQ_iD_i + \sum_{j=1}^{N} r_{ij}Q_j + E_i + \varepsilon_iQ_i) \leq -\lambda_1 \quad (17)$$

$$\lambda_{\max}(\sum_{j=1}^{N} r_{ij}E_j) \leq \lambda_2 \quad (18)$$

$$E_i \geq \frac{1}{1-\delta_0}\varepsilon_i^{-1}H^TM^TQ_iMH \quad (19)$$

for all $i \in S$. Then, for any initial data $\xi \in C^b_{\mathfrak{F}_0}([-\tau, 0]; R^n)$, the solution of (13) has the property that

$$\lim_{t\to\infty} \sup \frac{1}{t}\ln(E(|e(t;\xi)|^2) < -\lambda < 0 \quad (20)$$

In other words, (13) is exponentially stable in mean square and Lyapunov exponent is not greater than $-\lambda$. Moreover, the positive number λ is the unique root to

$$\alpha\lambda + (\lambda_2 + \alpha_1\lambda)\tau e^{\lambda\tau} = \lambda_1 \quad (21)$$

where $\alpha = \max_{1\leq i\leq N} \lambda_{\max}(Q_i)$,
$\alpha_1 = \max_{1\leq i\leq N} \lambda_{\max}(E_i)$

Proof. Let us first show that (21) does has a unique root $\lambda > 0$. Choose i for $\lambda_{\min}E_i$ to be the smallest of $\lambda_{\min}E_j (1 \leq i \leq N)$, i.e., $\lambda_{\min}E_i = \min_{1\leq i\leq N} \lambda_{\min}E_j$. Let $v \neq 0$ be the corresponding eigenvector of E_i, i.e., $E_iv = \lambda_{\min}(E_i)v$. Then $v^TE_iv = \lambda_{\min}(E_i)|v|^2$. Moreover

$$\begin{aligned} v^T(\sum_{j=1}^{N} r_{ij}E_j)v &= \sum_{j\neq i}^{N} r_{ij}v^TE_jv + r_{ii}v^TE_iv \\ &\geq \sum_{j\neq i}^{N} r_{ij}\lambda_{\min}(E_j)|v|^2 + r_{ii}\lambda_{\min}(E_i)|v|^2 \\ &\geq \lambda_{\min}(E_i)|v|^2 \sum_{j=1}^{N} r_{ij} \\ &= 0. \end{aligned}$$

Thus

$$\lambda_{\max}(\sum_{j=1}^{N} r_{ij}E_j)|v|^2 \geq v^T(\sum_{j=1}^{N} r_{ij}E_j)v \geq 0$$

Since $|v| > 0$, we have

$$\lambda_{\max}(\sum_{j=1}^{N} r_{ij}E_j) \geq 0$$

so we obtain $\lambda_2 \geq 0$, with (16), we see that $\lambda_1 > 0$, so (21) does has a unique root $\lambda > 0$. Next, we show that the solution of (13) has the property of (20). Fix any initial data ξ and write $e(t;\xi) = e(t)$. Let us define the Lyapunov functional $V : C([-\tau, 0]; R^n) \times S \times R_+ \to R$ by

$$V(e(t), i, t) = e(t)^TQ_ie(t) + \int_{t-\delta(t)}^{t} e(s)^TE_ie(s)ds$$

while by (14)

$$\begin{aligned} \mathfrak{L}V(e(t), i, t) = {} & 2e(t)^TQ_i[A(r(t))e(t) + B(r(t))\eta(Ce, x) \\ & + MHe(t-\delta(t))] + \sum_{j=1}^{N} r_{ij}e(t)^TQ_je(t)) \\ & + (D(r(t))e(t))^TQ_iD(r(t))e(t) \\ & - (1-\dot{\delta}(t))e(t-\delta(t))^TE_ie(t-\delta(t)) \\ & + \sum_{j=1}^{N} r_{ij}\int_{t-\delta(t)}^{t} e(\theta)^TE_je(\theta)d\theta \\ & + e(t)^TE_ie(t) \end{aligned}$$

Using the assumptions,(17), (18), (19), and lemma 2.2, we compute

$$\begin{aligned} \mathfrak{L}V(e(t), i, t) \leq {} & e(t)^T(Q_iA_i + A_i^TQ_i)e(t) + e(t)^TE_ie(t) \\ & + 2e(t)^TQ_iB_iKCe(t) + e(t)^T(D_i^TQ_iD_i)e(t) \\ & + 2e(t)^TQ_iMHe(t-\delta(t)) \\ & + e(t)^T(\sum_{j=1}^{N} r_{ij}Q_j)e(t) \\ & - (1-\delta_0)e(t-\delta(t))^TE_ie(t-\delta(t)) \\ & + \int_{t-\delta(t)}^{t} e(\theta)^T(\sum_{j=1}^{N} r_{ij}E_j)e(\theta)d\theta \\ \leq {} & e(t)^T(Q_iA_i + A_i^TQ_i + 2Q_iB_iKC \\ & + D_i^TQ_iD_i\sum_{j=1}^{N} r_{ij}Q_j + E_i + \varepsilon_iQ_i)e(t) \\ & + e(t-\delta(t))^T[\varepsilon_i^{-1}H^TM^TQ_iMH \\ & - (1-\delta_0)E_i]e(t-\delta(t)) \\ & + \int_{t-\delta(t)}^{t} e(\theta)^T(\sum_{j=1}^{N} r_{ij}E_j)e(\theta)d\theta \\ \leq {} & -\lambda_1|e(t)|^2 + \lambda_2\int_{t-\delta(t)}^{t} |e(\theta)|^2d\theta \end{aligned} \quad (22)$$

Let us define the Lyapunov functional $V_1 : C([-\tau, 0]; R^n) \times S \times R_+ \to R$ by

$$V_1(e(t), i, t) = e^{\lambda t}V(e(t), i, t)$$

By the generalized Itô formula, we have

$$EV_1(e(t), i, t) = EV_1(\xi, r(0), 0) + E\int_0^t \mathfrak{L}V_1(e(s), r(s), s)ds \quad (23)$$

and it is straightforward to see that

$$\mathfrak{L}V_1(e(t), i, t) = e^{\lambda t}[\lambda V(e(t), i, t) + \mathfrak{L}V(e(t), i, t)] \quad (24)$$

we note that

$$V(e(t), i, t) \leq \alpha|e(t)|^2 + \alpha_1\int_{t-\delta(t)}^{t} |e(\theta)|^2d\theta \quad (25)$$

Substituting (22), (24), (25) into (23) we obtain that

$$\begin{aligned}EV_1(e(t),i,t) &\le EV_1(\xi,r(0),0) + E\int_0^t e^{\lambda s}\lambda(\alpha|e(s)|^2 \\ &+\alpha_1\int_{s-\delta(s)}^{s}|e(\theta)|^2d\theta)ds \\ &+E\int_0^t e^{\lambda s}(-\lambda_1|e(s)|^2 \\ &+\lambda_2\int_{s-\delta(s)}^{s}|e(\theta)|^2d\theta)ds \\ &=(\lambda_2+\alpha_1\lambda)E\int_0^t e^{\lambda s}(\int_{s-\delta(s)}^{s}|e(\theta)|^2d\theta)ds \\ &-(\lambda_1-\alpha\lambda)E\int_0^t e^{\lambda s}|e(s)|^2 \\ &+EV_1(\xi,r(0),0)\end{aligned} \tag{26}$$

We compute

$$\begin{aligned}\int_0^t e^{\lambda s}(\int_{s-\delta(s)}^{s}|e(\theta)|^2d\theta)ds &\le \int_0^t e^{\lambda s}(\int_{s-\tau}^{s}|e(\theta)|^2d\theta)ds \\ &\le \int_{-\tau}^{t}\int_{\theta}^{\theta+\tau}e^{\lambda s}ds|e(\theta)|^2d\theta \\ &\le \tau e^{\lambda\tau}\int_{-\tau}^{t}e^{\lambda\theta}|e(\theta)|^2d\theta\end{aligned} \tag{27}$$

Substituting (27) into (26), and using (21) we get

$$\begin{aligned}EV_1(e(t),i,t) &\le EV_1(\xi,r(0),0) \\ &+(\lambda_2+\alpha_1\lambda)\tau e^{\lambda\tau}\int_{-\tau}^{0}e^{\lambda s}|\xi(s)|^2ds\end{aligned} \tag{28}$$

On the other hand, we also note that

$$\begin{aligned}EV_1(e(t),r(t),t) &= e^{\lambda t}E[e(t)^TQ_{r(t)}e(t) \\ &+\int_{t-\delta(t)}^{t}e(\theta)^TE_{r(t)}e(\theta)d\theta] \\ &\ge e^{\lambda t}E[e(t)^TQ_{r(t)}e(t)] \\ &\ge e^{\lambda t}\min_{1\le i\le N}\lambda_{\min}(Q_i)E|e(t)|^2\end{aligned} \tag{29}$$

By (28), (29), we know that

$$\lim_{t\to\infty}\sup\frac{1}{t}\ln(E(|e(t;\xi)|^2)<-\lambda<0$$

In other words, (13) is exponentially stable in mean square and Lyapunov exponent is not greater than $-\lambda$.

If we let $\varepsilon_i=1,\forall i\in S$ in Theorem 3.1, we obtain the following useful result.

Corollary 3.1. Assume that there are two constants λ_1 and λ_2 such that

$$\lambda_1>\tau\lambda_2$$

Assume also that there are symmetric matrices $Q_i>0$, and $E_i\ge 0(1\le i\le N)$ such that

$$\lambda_{\max}(Q_iA_i+A_i^TQ_i+2Q_iB_iKC+ D_i^TQ_iD_i+\sum_{j=1}^{N}r_{ij}Q_j+E_i+Q_i)\le-\lambda_1$$

$$\lambda_{\max}(\sum_{j=1}^{N}r_{ij}E_j)\le\lambda_2$$

$$E_i\ge\frac{1}{1-\delta_0}H^TM^TQ_iMH$$

for all $i\in S$. Then, for any initial data $\xi\in C^b_{\mathfrak{F}_0}([-\tau,0];R^n)$, the solution of (13) is exponentially stable in mean square and Lyapunov exponent is not greater than $-\lambda$.

Remark3.1. Corollary 3.1 is stated without ε_i, so it looks neat, but Theorem 3.1 is more general since it allows to choose different ε_i for different situations in practice, for example, if we choose $\varepsilon_i=\|HM\|$, we can get corollary 3.2.

Corollary 3.2. Assume there are symmetric matrices $Q_i>0$, and $E_i\ge 0(1\le i\le N)$ such that

$$E_i\ge\frac{1}{1-\delta_0}\|HM\|^{-1}H^TM^TQ_iMH$$

for all $i\in S$. Suppose we can verify $\lambda_1>\tau\lambda_2$ where

$$\begin{aligned}\lambda_1=-\max_{1\le i\le N}\{\lambda_{\max}(Q_iA_i+A_i^TQ_i+2Q_iB_iKC \\ +D_i^TQ_iD_i+\|HM\|Q_i+\sum_{j=1}^{N}r_{ij}Q_j+E_i)\}\end{aligned}$$

$$\lambda_2=\max_{1\le i\le N}\lambda_{\max}(\sum_{j=1}^{N}r_{ij}E_j)$$

Then, for any initial data $\xi\in C^b_{\mathfrak{F}_0}([-\tau,0];R^n)$, the solution of (13) is exponentially stable in mean square and Lyapunov exponent is not greater than $-\lambda$. And the λ can be determined in the same way as stated in Theorem 3.1. Let we define

$$\beta=\frac{1}{1-\delta_0}\max_{1\le i\le N}\{\lambda_{\max}\|HM\|^{-1}H^TM^TQ_iMH\}$$

If we choose $E_i=\beta I$ in corollary 3.2, we can easily get corollary 3.3.

Corollary 3.3. Assume there are symmetric matrices $Q_i>0(1\le i\le N)$ such that

$$\begin{aligned}\lambda_{\max}(Q_iA_i+A_i^TQ_i+2Q_iB_iKC \\ +D_i^TQ_iD_i+\|HM\|Q_i+\sum_{j=1}^{N}r_{ij}Q_j)<-\beta\end{aligned}$$

Then the solution of (13) has the property

$$\lim_{t\to\infty}\sup\frac{1}{t}\ln(E(|e(t;\xi)|^2)<-\lambda<0$$

That is the solution of (13) exponentially stable in mean square and Lyapunov exponent is not greater than $-\lambda$. Where the λ

is the unique root to

$$\lambda(\alpha+\beta\tau e^{\lambda\tau})=\lambda_1$$

Where α is the same as defined in Theorem 3.1 but

$$\lambda_1=-\max_{1\le i\le N}\{\lambda_{\max}(Q_iA_i+A_i^TQ_i+2Q_iB_iKC+D_i^TQ_iD_i+\|HM\|Q_i+\sum_{j=1}^{N}r_{ij}Q_j)\}-\beta$$

Theorem 3.2. Define the matrix

$$K=diag(-\lambda_{\max}(A_1+A_1^T+2B_1KC)-\|HM\|-\|D_1\|^2,\ \dots-\lambda_{\max}(A_N+A_N^T+2B_NKC)-\|HM\|-\|D_N\|^2)$$

and the vector $\kappa=(1-\delta_0)\begin{bmatrix}\|HM\|^{-1}\\ \vdots\\ \|HM\|^{-1}\end{bmatrix}$

(Set $a^{-1}=\infty$ when $\alpha=0$ as ussal.)If $K-\Gamma$ is a nonsingular M-matrix and

$$\kappa\gg(K-\Gamma)^{-1}\vec{1} \tag{30}$$

where $\vec{1}=(1,\dots1)^T$, then the solution of (13) exponentially stable in mean square.

Proof. Since $K-\Gamma$ is a nonsingular M-matrix, by lemma 2.1, we observe that $K-\Gamma^{-1}$ exist and all the elements of $K-\Gamma^{-1}$ are nonnegative. $K-\Gamma^{-1}$ is invertible, its each row must have at least one nonzero, and hence positive element. Let

$$\vec{q}=(q_1,q_2,\dots,q_N)^T=(K-\Gamma)^{-1}\vec{1}$$

then $\vec{q}\gg 0$. By (30)

$$q_i\|HM\|<1-\delta_0,\quad\forall\quad i\in S$$

Let $Q_i=qI$ for $i\in S$. so,

$$\lambda_{\max}(\|HM\|^{-1}H^TM^TQ_iMH)\le q_i(\|HM\|)<1-\delta_0$$

therefore $\beta<1$. On the other hand

$$\begin{aligned}&\lambda_{\max}(Q_iA_i+A_i^TQ_i+2Q_iB_iKC+D_i^TQ_iD_i\\&\quad+\|HM\|Q_i+\sum_{j=1}^{N}r_{ij}Q_j)\\&\le q_i[\lambda_{\max}(A_i+A_i^T+2B_iKC)+\|D_i\|^2I\\&\quad+\|HM\|I]+\sum_{j=1}^{N}r_{ij}q_j\\&=-[(K-\Gamma)\vec{q}]_i\end{aligned}$$

where $[(K-\Gamma)\vec{q}]_i$ stands for the ith element of the vector $(K-\Gamma)\vec{q}$. Then

$$\lambda_{\max}(Q_iA_i+A_i^TQ_i+2Q_iB_iKC+D_i^TQ_iD_i+\|HM\|Q_i+\sum_{j=1}^{N}r_{ij}Q_j)\le-1<-\beta$$

for all $i\in S$. Therefore the result now follows from corollary 3.3.

Remark3.2. In [15], the authors studied the global asymptotically synchronization results for Lur'e systems with delay feedback control. However, the stochastic term and Markovain switching were not taken into account in the models. Therefore, our developed results in this paper are more general than reported in [12].

Remark3.3. In [16], the author studied the exponential stability of stochastic delay interval systems with Markovian switching using Lyapunov-Krasovskii functional and nonsingular M-matrix methods which the same in this paper. But he studied the system is linear system, while we studied the system is nonlinear.

Remark3.4. In [15], the authors studied the global asymptotically synchronization results for Lur'e systems with delay feedback control. But we studied the exponential synchronization in mean square. So, our results are better than reported in [12].

IV. An illustrative example

In this section we shall present one example to illustrate our theory.

Example 4.1. Let $w(t)$ be a 2-dimensional Brownian motion, let $r(t)$ be a right-continuous Markov chain taking values in $S=\{1,2\}$ with generator $\Gamma=(r_{ij})_{2\times 2}$:

$$-r_{11}=r_{12}>0,\quad -r_{22}=r_{21}>0$$

of course $w(t)$ and $r(t)$ are assumed to be independent. Consider the Lur'e systems with delay feedback control and Markovian switching of the form

$$\mathfrak{M}:\begin{cases}dx(t)=A(r(t))x(t)dt+D(r(t))x(t)dw(t)\\ p(t)=Hx(t)\\ x(t)=\phi(t),t\in[-\tau,0]\end{cases}$$

$$\mathfrak{S}:\begin{cases}dy(t)=[A(r(t))y(t)+u(t)]dt+D(r(t))y(t)dw(t)\\ q(t)=Hy(t)\\ y(t)=\psi(t),t\in[-\tau,0]\end{cases}$$

$$\mathfrak{G}:U(t)=M(p(t-\delta(t))-q(t-\delta(t))).$$

where $\delta(t)=0.1\sin^2 t$ and $\dot{\delta(t)}=0.2\sin(t)\cos(t)\le 0.1=\delta_0$, $A_1=\begin{bmatrix}-1&0\\0&-2\end{bmatrix}$, $A_2=\begin{bmatrix}-2&-2\\0&-1\end{bmatrix}$, $D_1=\begin{bmatrix}1&0\\0&1\end{bmatrix}$, $D_2=\begin{bmatrix}\frac{1}{2}&0\\0&\frac{1}{2}\end{bmatrix}$, $M=\begin{bmatrix}\frac{1}{81}&-\frac{1}{81}\\0&\frac{2}{9}\end{bmatrix}$, $H=\begin{bmatrix}9&\frac{1}{2}\\0&\frac{1}{2}\end{bmatrix}$. Then the solution of Lur'e systems with delay feedback control and Markovian switching is exponentially synchronization in mean square.

V. Conclusion

The problem of master−slave exponential synchronization for Lure systems has been addressed by employing time-delay feedback control techniques. By using the methods of Lyapunov-Krasovskii functional and nonsingular M-matrix, some effective criterions for achieving synchronization have been derived. Finally, an example has been given to illustrate the validity theoretical results obtained in this paper.

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