Delay-Dependent Stability Analysis for Neural Networks with Distributed Delays

Qingqing Wang, Shouming Zhong

Abstract—This paper deals with the problem of delay-dependent stability for neural networks with distributed delays. Some new sufficient condition are derived by constructing a novel Lyapunov-Krasovskii functional approach. The criteria are formulated in terms of a set of linear matrix inequalities, this is convenient for numerically checking the system stability using the powerful MATLAB LMI Toolbox. Moreover, in order to show the stability condition in this paper gives much less conservative results than those in the literature, numerical examples are considered.

Keywords—Neural networks, Globally asymptotic stability , LMI approach, Distributed delays.

I. INTRODUCTION

STABILITY analysis for neural networks have attracted many researchers attention due to the fact that in many applications the designed neural networks is required to have a unique and stable equilibrium point [1-3]. the occurrence of time delays is unavoidable during the processing and transmission of the signals because of the finite switching speed of amplifiers in electronic networks or finite speed for signal propagation in biological networks ,the existence of time delay may cause instability and oscillation of neural networks.Therefore stability analysis of delayed neural networks has been extensively investigated and reported in the literature; see [4-15], and the references cited therein.

When bounded distributed delay appear in a neural network,stability results for such a class of delayed neural networks were reported in [16-19]. In the case when unbounded distributed delayed appear in a neural network, stability results were provided in [20-22] by using the M-matrix theory and the Lyapunov functional method. Usually, delay-dependent stability results are less conservative than delay-dependent ones, especially when the delay size is small [23,24]. In this paper, we concerned with the problem of stability analysis for neural networks with distributed delay. The distributed delay is assumed to be unbounded. Delay-dependent stability conditions are obtained,which can be easily checked by MATLAB LMI Toolbox.Finally, in order to show the stability condition in this paper gives much less conservative results than those in the literature, numerical examples are considered.

Notations: The notations in this paper are quite standard. I denotes the identity matrix with appropriate dimensions,R^n

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denotes the n dimensional Euclid space, and $R^{m \times n}$is the set of all $m \times n$ real matrices, $*$ denotes the elements below the main diagonal of a symmetric block matrix. For symmetric matrices A and B,the notation $A > B$(respectively,$A \geq B$) means that the matrix $A - B$ is positive definite (respectively, nonnegative).

II. PROBLEM STATEMENT

Consider a class of delay neural networks described by the following equation

$$\dot{\mu}_i(t) = -c_i\mu_i(t) + \sum_{j=1}^{n} d_{ij}g_j(\mu_j(t)) + \sum_{j=1}^{n} a_{ij}g_j(\mu_j(t-\varsigma))$$
$$+ \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_j(t-s)g_j(\mu_j(s))ds + I_i$$

(1)

$$\mu_i(t) = \phi_i(t), \quad -\infty \leq t \leq 0 \tag{2}$$

for $i = 1, 2, \ldots, n$, $\mu_i(t)$ is the state of the ith unit at time t;$c_i > 0$ denotes the passive decay rate; a_{ij}, b_{ij}, d_{ij} are the interconnection matrices representing the weight coefficients of the neurons; ς is a constant scalar representing the delay of the neural network; $\phi_i(t)$, $i = 1, 2, \ldots, n$, is the initial condition of the neural network; I_i, $i = 1, 2, \ldots, n$, is the external constant inputs; $g_i(\cdot)$, $i = 1, 2, \ldots, n$, is the activation function; the delay kernel k_i is a real valued continuous nonnegative function defined on $[0, +\infty)$,which is assumed to satisfy $\int_0^\infty k_i(s)ds = 1$, $i = 1, 2, \ldots, n$. The following assumptions are adopted throughout the paper.

Assumption 1:Each neuron activation function $g_i(\cdot)$, in (1) satisfies the following condition:

$$0 \leq \frac{g_j(r_1) - g_j(r_2)}{r_1 - r_2} \leq l_j, \ \forall r_1, r_2 \in R, i = 1, 2, \ldots, n.$$

(3)

where $l_i > 0$ and assume that $L = diag\{l_1, l_2, \ldots, l_n\}$.
Based on Assumption 1, it can be easily proven that there exists one equilibrium point for (1) by Brouwer's fixed-point theorem. Assuming that $\mu^* = [\mu_1^*, \mu_2^*, \ldots, \mu_n^*]^T$ is the equilibrium point of (1) and using the transformation $x(\cdot) = \mu(\cdot) - \mu^*$,the system (1) can be converted to the

following system :

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} d_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t-\varsigma))$$
$$+ \sum_{j=1}^{n} b_{ij} \int_{-\infty}^{t} k_j(t-s) f_j(x_j(s)) ds$$

(4)

which can be re-written as

$$\dot{x}(t) = -Cx(t) + Df(x(t)) + Af(x(t-\varsigma))$$
$$+ B \int_{-\infty}^{t} K(t-s) f(x(s)) ds$$

(5)

where
$C = diag\{c_i\}, A = [a_{ij}]_{n \times n}, B = [b_{ij}]_{n \times n}, D = [d_{ij}]_{n \times n}$
and vector $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T, f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T,$
$f_i(x_i(t)) = g_i(x_i(t) + \mu_i^*) - g_i(\mu_i^*), K(s) = diag\{k_i(s)\}.$
By Assumption 1, it is easy to see that

$$0 \le \frac{f_j(r)}{r} \le l_i, \ \forall r \in R, r \neq 0, \ i = 1, 2, \ldots, n.$$

(6)

Lemma 1 [25].For any positive semi-definite matrices $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \ge 0$,the following integral integral inequality holds:

$$-\int_{t-\varsigma}^{t} \dot{x}^T(s) X_{33} \dot{x}(s) ds$$
$$\le \int_{t-\varsigma}^{t} \begin{bmatrix} x(t) \\ x(t-\varsigma) \\ \dot{x}(s) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\varsigma) \\ \dot{x}(s) \end{bmatrix} ds$$

(7)

Lemma 2 [26].Let $\zeta \in R^n, \Gamma = \Gamma^T \in R^{n \times n}$, and $B \in R^{m \times n}$ such that $rank(G) < n$. Then, the following statements are equivalent:

(1) $\zeta^T \Gamma \zeta < 0, \ G\zeta = 0, \zeta \neq 0,$
(2) $(G^\perp)^T \Gamma G^\perp < 0,$

(8)

where $G^\perp$ is a right orthogonal complement of G.

III. MAIN RESULTS

In this section,a new Lyapunov functional is constructed and a less conservative delay-dependent stability criterion is obtained.

Theorem 1 Given that the Assumption 1 hold, the system (5) is globally asymptotic stability if there exist matrices $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0$, diagonal matrices $E = diag\{e_i\} > 0, S = diag\{s_i\} > 0, \Lambda_1 > 0, \Lambda_2 > 0$ and matrix $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \ge 0$, such that the following LMI holds:

$$R_2 - X_{33} \ge 0$$

(9)

$$(\Gamma^\perp)^T \Omega \Gamma^\perp < 0$$

(10)

where

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 \\ * & * & \Omega_{33} & \Omega_{34} & -\Lambda_2 L \\ * & * & * & \Omega_{44} & 0 \\ * & * & * & * & \Omega_{55} \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} -C & D & A & B & 0 \end{bmatrix}, \ \bar{R} = R_1 + \varsigma R_2$$

$$\Omega_{11} = -2PC + Q_1 + \varsigma X_{11} + X_{13} + X_{13}^T + C^T \bar{R} C$$

$$\Omega_{12} = -L\Lambda_1 - CS + PD - C^T \bar{R} D$$

$$\Omega_{13} = PA - C^T \bar{R} A, \ \Omega_{14} = PB - C^T \bar{R} B$$

$$\Omega_{15} = \varsigma X_{12} - X_{13} + X_{13}^T$$

$$\Omega_{22} = Q_2 + E - 2\Lambda_1 + 2SD + D^T \bar{R} D$$

$$\Omega_{23} = SA + D^T \bar{R} A, \ \Omega_{24} = SB + D^T \bar{R} B$$

$$\Omega_{33} = -Q_2 + A^T \bar{R} A - 2\Lambda_2, \ \Omega_{34} = A^T \bar{R} B$$

$$\Omega_{44} = -E + B^T \bar{R} B$$

$$\Omega_{55} = -Q_1 + \varsigma X_{22} - X_{23} - X_{23}^T$$

Proof: Construct a new class of Lyapunov functional candidate as follow:

$$V(x_t) = \sum_{i=1}^{4} V_i(x_t)$$

with

$$V_1(x_t) = x^T(t) P x(t) + 2 \sum_{i=1}^{n} s_i \int_{0}^{x_i(t)} f_i(s) ds$$

$$V_2(x_t) = \int_{t-\varsigma}^{t} x^T(s) Q_1 x(s) ds + \int_{t-\varsigma}^{t} f^T(x(s)) Q_2 f(x(s)) ds$$

$$V_3(x_t) = \sum_{i=1}^{n} \int_{0}^{\infty} \int_{t-\sigma}^{t} k_i(\sigma) f_i^2(x_i(s)) ds d\sigma$$

$$V_4(x_t) = \int_{t-\varsigma}^{t} \dot{x}^T(s) R_1 \dot{x}(s) ds + \int_{-\varsigma}^{0} \int_{t+\beta}^{t} \dot{x}^T(s) R_2 \dot{x}(s) ds d\beta$$

Then, taking the time derivative of V(t) with respect to t along the system (5) yield

$$\dot{V}(x_t) = \sum_{i=1}^{4} \dot{V}_i(x_t)$$

where

$$\dot{V}_1(x_t) = 2x^T(t) P \dot{x}(t) + 2 \sum_{i=1}^{n} s_i f_i(x_i(t)) \dot{x}_i(t)$$
$$= 2x^T(t) P \dot{x}(t) + 2 f^T(x(t)) S \dot{x}(t)$$

(11)

$$\dot{V}_2(x_t) = x^T(t)Q_1 x(t) - x^T(t-\varsigma)Q_1 x(t-\varsigma)$$
$$+ f^T(x(t))Q_2 f(x(t)) - f^T(x(t-\varsigma))Q_2 f(x(t-\varsigma)) \tag{12}$$

$$\dot{V}_3(x_t) = f^T(x(t))E f(x(t)) - \sum_{i=1}^{n} e_i \int_0^\infty k_i(\sigma) f_i^2(x_i(t-\sigma))d\sigma \tag{13}$$

In the derivative of $V_3(x_t)$, we use Cauchy's inequality $(\int p(s)q(s)ds)^2 \le (\int p^2(s)ds)(\int q^2(s)ds)$ and assumption $\int_0^\infty k_i(s)ds = 1$, we can obtain that

$$\dot{V}_3(x_t) \le f^T(x(t))E f(x(t))$$
$$-(\int_{-\infty}^{t} K(t-s)f(x(s))ds)^T E(\int_{-\infty}^{t} K(t-s)f(x(s))ds) \tag{14}$$

$$\dot{V}_4(x_t) = \dot{x}^T(t)R_1\dot{x}(t) - \dot{x}^T(t-\varsigma)R_1\dot{x}(t-\varsigma)$$
$$+ \varsigma\dot{x}^T(t)R_2\dot{x}(t) - \int_{t-\varsigma}^{t}\dot{x}^T(\beta)R_2\dot{x}(\beta)d\beta$$
$$= \dot{x}^T(t)\bar{R}\dot{x}(t) - \dot{x}^T(t-\varsigma)R_1\dot{x}(t-\varsigma)$$
$$- \int_{t-\varsigma}^{t}\dot{x}^T(\beta)(R_2-X_{33})\dot{x}(\beta)d\beta - \int_{t-\varsigma}^{t}\dot{x}^T(\beta)X_{33}\dot{x}(\beta)d\beta \tag{15}$$

Using Lemma 1, we can obtain that

$$-\int_{t-\varsigma}^{t}\dot{x}^T(\beta)X_{33}\dot{x}(\beta)d\beta \le x^T(t)(\varsigma X_{11}+X_{13}+X_{13}^T)x(t)$$
$$+ 2x^T(t)(\varsigma X_{12}-X_{13}+X_{13}^T)x(t-\varsigma)$$
$$+ x^T(t-\varsigma)(\varsigma X_{22}-X_{23}-X_{23}^T)x(t-\varsigma) \tag{16}$$

From (6), we can get that there exist positive diagonal matrices Λ_1, Λ_2 such that the following inequalities holds:

$$-2f^T(x(t))\Lambda_1[f(x(t)) - Lx(t)] \ge 0 \tag{17}$$

$$-2f^T(x(t-\varsigma))\Lambda_2[f(x(t-\varsigma)) - Lx(t-\varsigma)] \ge 0 \tag{18}$$

From (9) and (11)-(18), we can obtain that:

$$\dot{V}(x_t) \le \xi^T(t)\Omega\xi(t) \tag{19}$$

where

$$\xi^T(t) = [x^T(t), f^T(x(t)), f^T(x(t-\varsigma)),$$
$$(\int_{-\infty}^{t} K(t-s)f(x(s))ds)^T, x^T(t-\varsigma)]$$

By Lemma 2, $\xi^T(t)\Omega\xi(t) < 0$ with $\Gamma\xi(t) = 0$ is equivalent to $(\Gamma^\perp)^T\Omega\Gamma^\perp < 0$. Therefore, if LMIs (9),(10) hold, then the neural networks (5) is asymptotically stable. This completes the proof. ∎

Remark 1 Theorem 1 provides an delay-dependent LMI condition,under which the delayed neural network in (5) is guaranteed to have a unique equilibrium point, which is globally asymptotically stable.It is worth pointing out that via a similar approach as in the proof of Theorem 1, we can

deal with the case when the delays in ς are different. When $B = 0$,the delayed neural network in (5) reduces to

$$\dot{x}(t) = -Cx(t) + Df(x(t)) + Af(x(t-\varsigma)) \tag{20}$$

In this case, by Theorem 1, it is easy to have the following result.

Theorem 2 Given that the Assumption 1 hold, the system (5) is globally asymptotic stability if there exist matrices $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0$, diagonal matrices $S = diag\{s_i\} > 0, \Lambda_1 > 0, \Lambda_2 > 0$ and matrix $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix} \ge 0$, such that the following LMI holds:

$$R_2 - X_{33} \ge 0 \tag{21}$$

$$(\Psi^\perp)^T\Phi\Psi^\perp < 0 \tag{22}$$

where

$$\Psi = \begin{bmatrix} -C & D & A & 0 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} \\ * & \Phi_{22} & \Phi_{23} & 0 \\ * & * & \Phi_{33} & -\Lambda_2 L \\ * & * & * & \Phi_{44} \end{bmatrix}$$

$$\Phi_{11} = -2PC + Q_1 + \varsigma X_{11} + X_{13} + X_{13}^T + C^T\bar{R}C$$

$$\Phi_{12} = -L\Lambda_1 - CS + PD - C^T\bar{R}D$$

$$\Phi_{13} = PA - C^T\bar{R}A$$

$$\Phi_{14} = \varsigma X_{12} - X_{13} + X_{13}^T$$

$$\Phi_{22} = Q_2 - 2\Lambda_1 + 2SD + D^T\bar{R}D$$

$$\Phi_{23} = SA + D^T\bar{R}A, \ \Phi_{33} = -Q_2 + A^T\bar{R}A - 2\Lambda_2$$

$$\Phi_{44} = -Q_1 + \varsigma X_{22} - X_{23} - X_{23}^T$$

Proof: The proof of the Theorem 2 is consequence of Theorem 1 by choosing $E = 0$. Hence the proof is omitted. ∎

IV. EXAMPLE

In this section,we provide a numerical example to demonstrate the effectiveness and less conservatism of our delay-dependent stability criteria.

Example 1 Consider a delayed neural network in (5) with parameters as

$$C = \begin{bmatrix} 1.6305 & 0 & 0 \\ 0 & 1.9221 & 0 \\ 0 & 0 & 2.5973 \end{bmatrix},$$

$$A = \begin{bmatrix} -2.5573 & -1.3813 & 1.9574 \\ -1.0226 & -0.8845 & 0.5045 \\ 1.0378 & 1.5532 & 0.6645 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.0265 & -0.0793 & 0.4694 \\ -0.5955 & 1.3352 & -0.9036 \\ -0.1497 & -0.6065 & -0.1641 \end{bmatrix},$$

$$D = \begin{bmatrix} 0.0265 & 0.1157 & 0.0578 \\ 0.3186 & -0.1363 & -0.0876 \\ -0.2037 & -0.0112 & 0.4225 \end{bmatrix}.$$

Let $L = diag\{0, 1.4, 2\}$.
Then, by the Matlab LMI control toolbox, the maximum allowed delay satsfying the LMI in (9) and (10) can be calculated as $\varsigma = 2.147$. In the case when $\varsigma = 2.147$, a set of solution to the LMI in (9) and (10) can be found as follows:

$$P = \begin{bmatrix} 7.6578 & -2.1251 & 2.8789 \\ -2.1251 & 10.9619 & -0.3941 \\ 2.8789 & -0.3941 & 8.7152 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 14.5700 & -6.9799 & -21.6701 \\ -6.9799 & 17.8915 & -30.0815 \\ -21.6701 & -30.0815 & 19.5754 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 12.8705 & -5.3385 & 4.8151 \\ -5.3385 & 30.7599 & -1.4672 \\ 4.8151 & -1.4672 & 9.9393 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 3.9484 & 0.4578 & -0.1203 \\ 0.4578 & 4.9871 & 0.2531 \\ -0.1203 & 0.2531 & 1.7812 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 0.5124 & -0.1203 & 0.2881 \\ -0.1203 & 0.1611 & -0.1624 \\ 0.2881 & -0.1624 & 0.1627 \end{bmatrix},$$

$$E = \begin{bmatrix} 1.2579 & 0 & 0 \\ 0 & 0.1451 & 0 \\ 0 & 0 & 2.1674 \end{bmatrix},$$

$$S = \begin{bmatrix} 4.1450 & 0 & 0 \\ 0 & 1.2451 & 0 \\ 0 & 0 & 2.7134 \end{bmatrix},$$

$$E = \begin{bmatrix} 0.3170 & 0 & 0 \\ 0 & 0.1756 & 0 \\ 0 & 0 & 1.0904 \end{bmatrix},$$

$$\Lambda_1 = \begin{bmatrix} 1.7890 & 0 & 0 \\ 0 & 0.9681 & 0 \\ 0 & 0 & 0.1245 \end{bmatrix},$$

$$\Lambda_2 = \begin{bmatrix} 0.0126 & 0 & 0 \\ 0 & 0.0686 & 0 \\ 0 & 0 & 0.3412 \end{bmatrix}.$$

V. Conclusion

In this paper, we concerned with the problem of stability analysis for neural networks with distributed delay. The distributed delay is assumed to be unbounded. Delay-dependent stability conditions have been obtained, which can be easily checked by MATLAB LMI Toolbox. Numerical examples have shown the less conservatism and effectiveness of the proposed conditions.

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