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Delay-dependent H_∞ performance analysis for Markovian jump systems with Time-varying delays

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Abstract—This paper considers H_∞ performance for Markovian jump systems with Time-varying delays. The systems under consideration involve disturbance signal, Markovian switching and time-varying delays. By using a new Lyapunov-Krasovskii functional and a convex optimization approach, a delay-dependent stability condition in terms of linear matrix inequality (LMI) is addressed, which guarantee asymptotical stability in mean square and a prescribed H_∞ performance index for the considered systems. Two numerical examples are given to illustrate the effectiveness and the less conservatism of the proposed main results. All these results are expected to be of use in the study of stochastic systems with time-varying delays.

Keywords—H_∞ performance; Markovian switching; Delay-dependent stability; Linear matrix inequality (LMI)

I. Introduction

STOCHASTIC modeling has come to play an important role in many branches of science and industry. Markovian jump systems (MJSs) are a special class of stochastic hybrid systems. Loosely speaking, a MJS is a hybrid system with state vector that has two components $x(t)$ and $r(t)$. The first one is in general referred to as the state, and the second one is regarded as the mode. In its operation, the jump system will switch from one mode to another in a random way, based on a Markovian chain with finite state space $\mathcal{S} = \{1, 2, \cdots, N\}$ [1]. Many dynamical systems subject to random abrupt variations can be modeled by MJS such as a manufacturing system, a networked control system (NCS) etc. Due to their extensive applications in many files, much research has investigated such a class of stochastic systems and lots of significant results have been reported. For more details on such systems we refer the readers to [2-4] and the references therein.

With the maturity of H_∞ control theory, many works have been devoted to H_∞ control of time delayed Markovian jump linear systems. Based on the stochastic version of bounded real lemma, sufficient conditions for the existence of H_∞ controllers for continuous stochastic systems were presented in terms of linear matrix inequalities in the work [5-10].

The stability and stabilization conditions for stochastic systems classified into two types: the one is delay-independent conditions which are applicable to delays of arbitrary size; the other is delay-dependent conditions which include information on the size of delays. Since the stability of systems depends explicitly on the time-delay, the delay-independent conditions are more conservative, especially for small delays. Compared with delay-independent conditions, the delay-dependent conditions are usually less conservative. It is worth pointing out that recent research efforts in the study of delay systems are towards developing less conservative delay-dependent results. Delay-dependent stability and H_∞ control results were presented by resorting to some bounding techniques for some cross terms and using model transformation to the original delay system. It is worth pointing out that recent research efforts in the study of delay systems are towards developing less conservative delay-dependent results[5]. It has been shown that the conservatism in the existing delay-dependent results are mainly caused by using model transformation to the original delay system or resorting to bounding techniques for some cross terms. Recently, some new methods have been provided to reduce the conservative without using model transformation, such as convex analysis method[11-13], delay decomposition approach [13,14].

This article deals with the H_∞ performance of Markovian jump systems with time-varying delays. The method used is based on Lyapunov-Krasovskii approach. Novel delay-dependent sufficient conditions are obtained to guarantee the considered systems are asymptotically stable in mean square and guarantee a prescribed H_∞ performance index in terms of linear matrix inequality. The presented results are derived by exploiting a new Lyapunov-Krasovskii functional and a convex optimization approach. Two numerical examples are provided to show the effectiveness of the proposed results.

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II. System description and definitions

Consider the following stochastic hybrid systems:

$$\begin{aligned} \dot{x}(t) =& A(r_t)x(t) + A_d(r_t)x(t-\tau(t)) \\ &+ D_1(r_t)\omega(t) \end{aligned} \tag{1a}$$

$$\begin{aligned} z(t) =& C(r_t)x(t) + C_d(r_t)x(t-\tau(t)) \\ &+ D_2(r_t)\omega(t) \end{aligned} \tag{1b}$$

$$x(t) = \phi(t), \quad t \in [-\tau_2, 0], \tag{1c}$$

where $x(t) \in \mathbf{R}^n$ is the state vector; $\omega(t)$ is the noise signal which is assumed to be an arbitrary signal in $\mathcal{L}_2[0\ \infty)$; $\phi(t)$ is a compatible vector-valued initial function defined on $[-\tau_2, 0]$; $A(r_t)$, $A_d(r_t)$, $D_1(r_t)$, $C(r_t)$, $C_d(r_t)$ and $D(r_t)$ are real constant matrices with appropriate dimensions. $\{r_t, t \geqslant 0\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{S} = \{1, 2, \cdots, N\}$ with transition probability matrix $\Pi = \pi_{ij}$ given by

$$Pr\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij} h + o(h), & i \neq j, \\ 1 + \pi_{ii} h + o(h), & i = j, \end{cases}$$

where $h > 0$ and $\lim_{h \to 0} \frac{o(h)}{h} = 0$; $\pi_{ij} \geqslant 0$ for $i \neq j$ is the transition rate from mode i at time t to mode j at time $t+h$ and $\pi_{ii} = -\sum_{j=1, j \neq i}^{N} \pi_{ij}$. In system (1), $\tau(t)$ denotes the time-varying delay when the mode is in r_t and satisfies

$$0 \leqslant \tau(t) \leqslant \tau, \quad \dot{\tau}(t) \leqslant \mu < 1, \tag{2}$$

where τ_1 and τ_2 are the time delay lower and upper bounds, respectively.

For simplicity, for each possible $r_t = i,\ i \in \mathcal{S}$, a matrix $R(r_t)$ will be denoted by R_i, for example, $A(r_t)$ is denoted by A_i, $A_d(r_t)$ is denoted by A_{di}, and so on.

III. Main results

1) Stability analysis: In this section, We first propose a delay-dependent sufficient condition for stability of system $(1a)$ with $\omega(t) = 0$.

Theorem 1. *Given scalars τ, μ, the free nominal system $(1a)$ is asymptotically stable in mean square for any time delay $\tau(t)$ satisfying (2), if there exist symmetric positive-definite matricesP_i, Q_{1i}, Q_2, Q_3, R and such that for every $i \in \mathcal{S}$,*

$$\Phi_{11} = \begin{bmatrix} \Phi_1 & -\frac{\tau}{2} N \\ * & -(\frac{\tau}{2})^2 Q_2 \end{bmatrix} < 0, \tag{3a}$$

$$\Phi_{12} = \begin{bmatrix} \Phi_1 & -\frac{\tau}{2} L \\ * & -(\frac{\tau}{2})^2 Q_2 \end{bmatrix} < 0, \tag{3b}$$

$$\Phi_{21} = \begin{bmatrix} \Phi_2 & -\frac{\tau}{2} U \\ * & -(\frac{\tau}{2})^2 Q_3 \end{bmatrix} < 0, \tag{3c}$$

$$\Phi_{22} = \begin{bmatrix} \Phi_2 & -\frac{\tau}{2} V \\ * & -(\frac{\tau}{2})^2 Q_3 \end{bmatrix} < 0, \tag{3d}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_{1j} < R, \tag{3e}$$

$$\begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0, \tag{3f}$$

where

$$\Phi_1 = \begin{bmatrix} \Pi_1^{11} & \Pi_1^{12} & \Pi_1^{13} & L_4^T & \Pi_1^{15} \\ * & \Pi_1^{22} & \Pi_1^{23} & \Pi_1^{24} & \Pi_1^{25} \\ * & * & \Pi_1^{33} & \Pi_1^{34} & -N_5^T \\ * & * & * & -X_{22} - Q_3 & 0 \\ * & * & * & * & \Pi_1^{55} \end{bmatrix}$$

$$\Phi_2 = \begin{bmatrix} \Pi_2^{11} & \Pi_2^{12} & \Pi_2^{13} & -U_1 & \Pi_2^{15} \\ * & \Pi_2^{22} & \Pi_2^{23} & \Pi_2^{24} & \Pi_2^{25} \\ * & * & \Pi_2^{33} & \Pi_2^{34} & V_5^T \\ * & * & * & \Pi_2^{44} & -U_5^T \\ * & * & * & * & \Pi_2^{55} \end{bmatrix}$$

$$\begin{aligned}
\Pi_1^{11} =& P_i A_i + A_i^T P_i + \sum_{j=1}^{N} \pi_{ij} P_j + X_{11} + Q_{1i} + \tau R \\
&+ A_i^T T_{1i}^T + T_{1i} A_i + L_1 + L_1^T, \\
\Pi_1^{12} =& P_i A_{di} + T_{1i} A_{di} + N_1 - L_1 + L_2^T, \\
\Pi_1^{13} =& X_{12} - N_1 + L_3^T,\ \Pi_1^{15} = A_i^T T_{2i}^T + L_5^T - T_{1i}, \\
\Pi_1^{22} =& -(1-\mu) Q_{1i} - L_2 - L_2^T + N_2 + N_2^T, \\
\Pi_1^{23} =& -N_2 - L_3^T + N_3^T,\ \Pi_1^{24} = N_4^T - L_4^T, \\
\Pi_1^{25} =& A_{di}^T T_{2i}^T + N_5^T - L_5^T, \\
\Pi_1^{33} =& X_{22} - X_{11} - N_3 - N_3^T - Q_3, \\
\Pi_1^{34} =& -X_{12} - N_4^T + Q_3^T, \\
\Pi_1^{55} =& (\frac{\tau}{2})^2 (Q_2 + Q_3) - T_{2i} - T_{2i}^T, \\
\Pi_2^{11} =& P_i A_i + A_i^T P_i + \sum_{j=1}^{N} \pi_{ij} P_j + X_{11} + Q_{1i} + \tau R \\
&+ A_i^T T_{1i}^T + T_{1i} A_i - Q_2, \\
\Pi_2^{12} =& P_i A_{di} + T_{1i} A_{di} + U_1 - V_1, \\
\Pi_2^{13} =& X_{12} + V_1 + Q_2,\ \Pi_2^{15} = A_i^T T_{2i}^T - T_{1i}^T, \\
\Pi_2^{22} =& -(1-\mu) Q_{1i} + U_2 + U_2^T - V_2 - V_2^T, \\
\Pi_2^{23} =& U_3^T + V_2 - V_3^T,\ \Pi_2^{24} = -U_2 - V_4^T + U_4^T, \\
\Pi_2^{25} =& A_{di}^T T_{2i}^T + U_5^T - V_5^T, \\
\Pi_2^{33} =& X_{22} - X_{11} + V_3 + V_3^T - Q_2, \\
\Pi_2^{34} =& -X_{12} - U_3 + V_4^T, \\
\Pi_2^{44} =& -X_{22} - U_4 - U_4^T, \\
\Pi_2^{55} =& (\frac{\tau}{2})^2 (Q_2 + Q_3) - T_{2i} - T_{2i}^T,
\end{aligned}$$

Proof. Consider a Lyapunov-Krasovskii functional candidate for the free nominal system as

$$V = V_1 + V_2 + V_3 + V_4 + V_5 \tag{5}$$

where

$$\begin{aligned}
V_1 =& x^T(t) P(r_t) x(t), \\
V_2 =& \int_{t-\frac{h}{2}}^{t} \begin{bmatrix} x(s) \\ x(s-\frac{h}{2}) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ x(s-\frac{h}{2}) \end{bmatrix} \mathrm{d}s, \\
V_3 =& \int_{t-\tau(t)}^{t} x^T(s) Q_1(r_t) x(s) \mathrm{d}s, \\
V_4 =& \frac{\tau}{2} \int_{-\frac{\tau}{2}}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) Q_2 \dot{x}(s) \mathrm{d}s \mathrm{d}\theta, \\
&+ \frac{\tau}{2} \int_{-\tau}^{-\frac{\tau}{2}} \int_{t+\theta}^{t} \dot{x}^T(s) Q_3 \dot{x}(s) \mathrm{d}s \mathrm{d}\theta, \\
V_5 =& \int_{-\tau}^{0} \int_{t+\theta}^{t} x^T(s) R x(s) \mathrm{d}s \mathrm{d}\theta.
\end{aligned}$$

Let $\mathcal{L}$ be the weak infinitesimal generator of the random process $\{x_t,\ r_t\}$. Then, for each $i \in \mathcal{S}$, we have

$$\mathcal{L}V = \mathcal{L}V_1 + \mathcal{L}V_2 + \mathcal{L}V_3 + \mathcal{L}V_4 + \mathcal{L}V_5, \tag{6}$$

where

$$\begin{aligned}
\mathcal{L}V_1 =& 2x^T(t) P_i (A(t, r_t) x(t) + A_d(t, r_t) x(t - \tau(t))) \\
&+ x^T(t) \left(\sum_{j=1}^{N} \pi_{ij} P_j \right) x(t),
\end{aligned}$$

$$\mathcal{L}V_2 = \begin{bmatrix} x(t) \\ x(t-\frac{\tau}{2}) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\frac{\tau}{2}) \end{bmatrix} - \begin{bmatrix} x(t-\frac{\tau}{2}) \\ x(t-\tau) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \begin{bmatrix} x(t-\frac{\tau}{2}) \\ x(t-\tau) \end{bmatrix},$$

$$\mathcal{L}V_3 \le \int_{t-\tau(t)}^{t} x^T(s) \left(\sum_{j=1}^{N} \pi_{ij} Q_{1j}\right) x(s)\mathrm{d}s + x^T(t)Q_{1i}x(t) - (1-\mu)x^T(t-\tau(t))Q_{1i}x(t-\tau(t)),$$

$$\mathcal{L}V_4 = (\frac{\tau}{2})^2 \dot{x}^T(t)(Q_2+Q_3)\dot{x}(t) - \frac{\tau}{2}\int_{t-\frac{\tau}{2}}^{t} \dot{x}^T(s)Q_2\dot{x}(s)\mathrm{d}s - \frac{\tau}{2}\int_{t-\tau}^{t-\frac{\tau}{2}} \dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s,$$

$$\mathcal{L}V_5 = \tau x^T(t)Rx(t) - \int_{t-\tau}^{t} x^T(s)Rx(s)\mathrm{d}s \le \tau x^T(t)Rx(t) - \int_{t-\tau(t)}^{t} x^T(s)Rx(s)\mathrm{d}s.$$

To obtain the main results, we consider the following two cases: $0 \le \tau(t) \le \frac{\tau}{2}$ and $\frac{\tau}{2} \le \tau(t) \le \tau$.

When $0 \le \tau(t) \le \frac{\tau}{2}$, we have

$$-\frac{\tau}{2}\int_{t-\tau}^{t-\frac{\tau}{2}} \dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s \le -\left[x(t-\frac{\tau}{2}) - x(t-\tau)\right]^T Q_3 \left[x(t-\frac{\tau}{2}) - x(t-\tau)\right],$$

and

$$-\frac{\tau}{2}\int_{t-\frac{\tau}{2}}^{t} \dot{x}^T(s)Q_2\dot{x}(s)\mathrm{d}s = -\frac{\tau}{2}\int_{t-\frac{\tau}{2}}^{t-\tau(t)} \dot{x}^T(s)Q_2\dot{x}(s)\mathrm{d}s - \frac{\tau}{2}\int_{t-\tau(t)}^{t} \dot{x}^T(s)Q_2\dot{x}(s)\mathrm{d}s \le -\frac{\tau}{2}\left(\frac{\tau}{2} - \tau(t)\right) f_1^T(t)Q_2f_1(t) - \frac{\tau}{2}\tau(t)g_1^T(t)Q_2g_1(t).$$

where

$$f_1(t) = \frac{1}{\frac{\tau}{2} - \tau(t)}\int_{t-\frac{\tau}{2}}^{t-\tau(t)} \dot{x}(s)\mathrm{d}s, \quad g_1(t) = \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \dot{x}(s)\mathrm{d}s$$

It is easy to see $\lim_{\tau(t)\to\frac{\tau}{2}} f_1(t) = \dot{x}(t-\frac{\tau}{2})$, $\lim_{\tau(t)\to 0} g_1(t) = \dot{x}(t)$. Adding the following terms to (6)

$$0 = 2\xi^T(t)N\left[x(t-\tau(t)) - x(t-\frac{\tau}{2}) - (\frac{\tau}{2} - \tau(t))f_1(t))\right],$$
$$0 = 2\xi^T(t)L\left[x(t) - x(t-\tau(t)) - \tau(t)g_1(t))\right],$$
$$0 = 2\left[x^T(t)T_{1i} + \dot{x}^T(t)T_{2i}\right]\left[-\dot{x}(t) + A_ix(t) + A_{di}x(t-\tau(t))\right],$$

where

$$\xi^T(t) = [x^T(t)\ x^T(t-\tau(t))\ x^T(t-\frac{\tau}{2})\ x^T(t-\tau)\ \dot{x}^T(t)],$$
$$N = [N_1^T\ N_2^T\ N_3^T\ N_4^T\ N_5^T]^T$$
$$L = [L_1^T\ L_2^T\ L_3^T\ L_4^T\ L_5^T]^T$$

Hence, we can obtain

$$\mathcal{L}V \le \zeta(t)^T\Phi\zeta(t)$$

where

$$\Phi = \begin{bmatrix} \Phi_1 & -(\frac{\tau}{2} - \tau(t))N & -\tau(t)L \\ * & -\frac{\tau}{2}(\frac{\tau}{2} - \tau(t))Q_2 & 0 \\ * & * & -\frac{\tau}{2}\tau(t)Q_2 \end{bmatrix},$$
$$\zeta^T(t) = [\xi^T(t)\ f_1^T(t)\ g_1^T(t)].$$

It is easy to see Φ_{11} and Φ_{12} result from $\Phi_{\tau(t)\to 0}$ and $\Phi_{\tau(t)\to\frac{\tau}{2}}$, respectively, where we have deleted the zero row and the zero column. Denoting: $\zeta_1^T(t) = [\xi^T(t)\ f_1^T(t)]$ and $\zeta_2^T(t) = [\xi^T(t)\ g_1^T(t)]$. The LMIs $(3a)$ and $(3b)$ imply $\Phi < 0$ because

$$\frac{\frac{\tau}{2} - \tau(t)}{\frac{\tau}{2}}\zeta_1^T(t)\Phi_{11}\zeta_1(t) + \frac{\tau(t)}{\frac{\tau}{2}}\zeta_2^T(t)\Phi_{12}\zeta_2(t) = \zeta^T(t)\Phi\zeta(t).$$

and Φ is convex in $\tau(t) \in [0\ \frac{\tau}{2}]$.

When $\frac{\tau}{2} \le \tau(t) \le \tau$, we have

$$-\frac{\tau}{2}\int_{t-\tau}^{t-\frac{\tau}{2}} \dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s = -\frac{\tau}{2}\int_{t-\tau}^{t-\tau(t)} \dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s - \frac{\tau}{2}\int_{t-\tau(t)}^{t-\frac{\tau}{2}} \dot{x}^T(s)Q_3\dot{x}(s)\mathrm{d}s \le -\frac{\tau}{2}\left(\tau - \tau(t)\right) f_2^T(t)Q_3f_2(t) - \frac{\tau}{2}(\tau(t) - \frac{\tau}{2})g_2^T(t)Q_3g_2(t),$$

and

$$-\frac{\tau}{2}\int_{t-\frac{\tau}{2}}^{t} \dot{x}^T(s)Q_2\dot{x}(s)\mathrm{d}s \le -\left[x(t) - x(t-\frac{\tau}{2})\right]^T Q_2 \left[x(t) - x(t-\frac{\tau}{2})\right],$$

where

$$f_2(t) = \frac{1}{\tau - \tau(t)}\int_{t-\tau}^{t-\tau(t)} \dot{x}(s)\mathrm{d}s, \quad g_2(t) = \frac{1}{\tau(t) - \frac{\tau}{2}}\int_{t-\tau(t)}^{t-\frac{\tau}{2}} \dot{x}(s)\mathrm{d}s$$

It is easy to see $\lim_{\tau(t)\to\tau} f_2(t) = \dot{x}(t-\tau)$, $\lim_{\tau(t)\to\frac{\tau}{2}} g_2(t) = \dot{x}(t-\frac{\tau}{2})$. Adding the following terms to (6)

$$0 = 2\xi^T(t)U\left[x(t-\tau(t)) - x(t-\tau) - (\tau - \tau(t))f_2(t))\right],$$
$$0 = 2\xi^T(t)V\left[x(t-\frac{\tau}{2}) - x(t-\tau(t)) - (\tau(t) - \frac{\tau}{2})g_2(t))\right],$$
$$0 = 2\left[x^T(t)T_{1i} + \dot{x}^T(t)T_{2i}\right]\left[-\dot{x}(t) + A_ix(t) + A_{di}x(t-\tau(t))\right],$$

where

$$U = [U_1^T\ U_2^T\ U_3^T\ U_4^T\ U_5^T]^T,$$
$$V = [V_1^T\ V_2^T\ V_3^T\ V_4^T\ V_5^T]^T.$$

Hence, we can obtain

$$\mathcal{L}V \leq \bar{\zeta}(t)^T \bar{\Phi} \bar{\zeta}(t)$$

where

$$\bar{\Phi} = \begin{bmatrix} \Phi_2 & -(\tau - \tau(t))U & -(\tau(t) - \frac{\tau}{2})V \\ * & -\frac{\tau}{2}(\tau - \tau(t))Q_3 & 0 \\ * & * & -\frac{\tau}{2}(\tau(t) - \frac{\tau}{2})Q_3 \end{bmatrix},$$
$$\bar{\zeta}^T(t) = [\xi^T(t)\ f_2^T(t)\ g_2^T(t)].$$

It is easy to see Φ_{21} and Φ_{22} result from $\bar{\Phi}_{\tau(t)\to\frac{\tau}{2}}$ and $\bar{\Phi}_{\tau(t)\to\tau}$, respectively, where we have deleted the zero row and the zero column. Denoting: $\bar{\zeta}_1^T(t) = [\xi^T(t)\ f_2^T(t)]$ and $\bar{\zeta}_2^T(t) = [\xi^T(t)\ g_2^T(t)]$. The LMIs $(3c)$ and $(3d)$ imply $\bar{\Phi} < 0$ because

$$\frac{\tau - \tau(t)}{\frac{\tau}{2}} \bar{\zeta}_1^T(t)\Phi_{21}\bar{\zeta}_1(t) + \frac{\tau(t) - \frac{\tau}{2}}{\frac{\tau}{2}} \bar{\zeta}_2^T(t)\Phi_{22}\bar{\zeta}_2(t)$$
$$= \bar{\zeta}^T(t)\bar{\Phi}\bar{\zeta}(t).$$

and $\bar{\Phi}$ is convex in $\tau(t) \in [\frac{\tau}{2}\ \tau]$.

Hence, if $(3a)$-$(3f)$ are satisfied , then the considering free nominal system is guaranteed to be asymptotically stable in mean square. This completes the proof.

2) H_∞ performance: In the sequel, we shall deal with the H_∞ performance of the system (1). For this purpose, we consider stochastic Lyapunov functional (5) and the following index:

$$J_{z\omega}(t) = \mathcal{E}\left\{\int_0^t \left[z^T(s)z(s) - \gamma^2\omega^T(s)\omega(s)\right] \mathrm{d}s\right\}.$$

Under zero initial condition, it is easy to see that

$$J_{z\omega}(t) \leqslant \mathcal{E}\left\{\int_0^t \left[z^T(s)z(s) - \gamma^2\omega^T(s)\omega(s) + \mathcal{L}V\right] \mathrm{d}s\right\}$$

Theorem 2. *Given scalars τ, μ, $\gamma > 0$, (1) is asymptotically stable in mean square with γ-disturbance attenuation for any time delay $\tau(t)$ satisfying (2), if there exist symmetric positive-definite matrices P_i, Q_{1i}, Q_2, Q_3, R and such that for every $i \in \mathcal{S}$,*

$$\begin{bmatrix} \Phi_1 & -\frac{\tau}{2}N & \mathcal{W}_1 & \mathcal{W}_2 \\ * & -(\frac{\tau}{2})^2 Q_2 & 0 & 0 \\ * & * & -\gamma^2 I & D_{2i}^T \\ * & * & * & -I \end{bmatrix} < 0, \tag{7a}$$

$$\begin{bmatrix} \Phi_1 & -\frac{\tau}{2}L & \mathcal{W}_1 & \mathcal{W}_2 \\ * & -(\frac{\tau}{2})^2 Q_2 & 0 & 0 \\ * & * & -\gamma^2 I & D_{2i}^T \\ * & * & * & -I \end{bmatrix} < 0, \tag{7b}$$

$$\begin{bmatrix} \Phi_2 & -\frac{\tau}{2}U & \mathcal{W}_1 & \mathcal{W}_2 \\ * & -(\frac{\tau}{2})^2 Q_3 & 0 & 0 \\ * & * & -\gamma^2 I & D_{2i}^T \\ * & * & * & -I \end{bmatrix} < 0, \tag{7c}$$

$$\begin{bmatrix} \Phi_2 & -\frac{\tau}{2}V & \mathcal{W}_1 & \mathcal{W}_2 \\ * & -(\frac{\tau}{2})^2 Q_3 & 0 & 0 \\ * & * & -\gamma^2 I & D_{2i}^T \\ * & * & * & -I \end{bmatrix} < 0, \tag{7d}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_{1j} < R, \tag{7e}$$

$$\begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} > 0, \tag{7f}$$

where

$$\mathcal{W}_1 = [D_{1i}^T P_i \quad 0 \quad 0 \quad 0 \quad 0]^T,$$
$$\mathcal{W}_2 = [C_i \quad C_{di} \quad 0 \quad 0 \quad 0]^T.$$

Proof. By Theorem 1 and Lemma 1, the desired results can be obtained.

IV. Numerical examples

In this section, some numerical examples will be presented to show the validity and the advantages of the main results derived above. In the following two examples, we assume the transition probability matrix Π is given by the following expression:

$$\Pi = \begin{bmatrix} -\pi_{11} & \pi_{11} \\ \pi_{22} & -\pi_{22} \end{bmatrix}.$$

Example 1. Let us consider the Markovian jump time-delay system with the following parameters, which is borrowed from [15].

Mode 1

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & 3.2684 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix}.$$

Mode 2

$$A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix}.$$

To compare the stochastic stability condition in [15], we choose $\pi_{22} = 0.8$, $\mu = 0.9$. Using Theorem 1 of this paper, the allowable upper bounder τ_2 for different π_{11} can be found in Table 1, which show that our result is less conservative. For simulation purposes, let the initial condition $x(0) = [-0.5\ \ 0.5]^T$ and $\tau = 0.5586$. Fig.1 and Fig.2 show the simulation results of state $x(t)$ of Mode 1 and Mode 2, respectively.

Table 1: Comparisons of max. allowed τ.

π_{11}	-0.1	-0.3	-0.5	-0.7	-0.9
[5]	0.4021	0.4010	0.4001	0.3993	0.3987
[15]	0.4252	0.4250	0.4248	0.4246	0.4242
This paper	0.5586	0.5548	0.5520	0.5497	0.5479

Example 2. Let us consider the Markovian jump time-delay system with the following parameters.

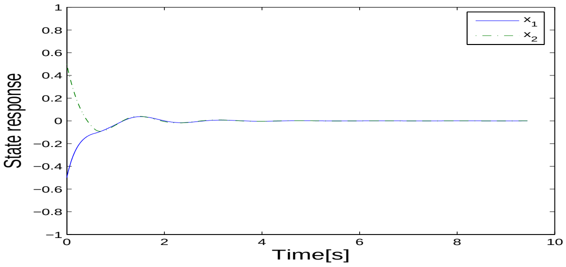

Fig.1 States response of Mode 1 of Example 1

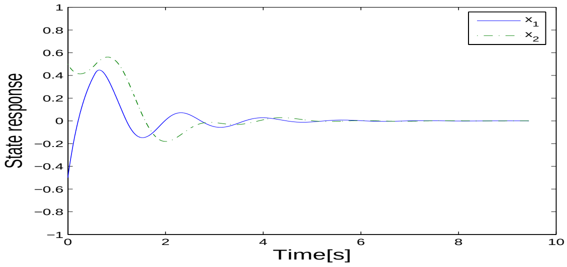

Fig.2 States response of Mode 2 of Example 1

Mode 1

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & 3.2684 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix},$$
$$D_{11} = \begin{bmatrix} 0.0403 & 0.6771 \end{bmatrix}^T,$$
$$C_1 = \begin{bmatrix} -0.3375 & -0.2959 \end{bmatrix},$$
$$C_{d1} = \begin{bmatrix} 0 & 0 \end{bmatrix},$$
$$D_{21} = 0.1184;$$

Mode 2

$$A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix},$$
$$D_{12} = \begin{bmatrix} 0.5689 & -0.2556 \end{bmatrix}^T,$$
$$C_2 = \begin{bmatrix} -1.4751 & -0.2340 \end{bmatrix},$$
$$C_{d2} = \begin{bmatrix} 0 & 0 \end{bmatrix},$$
$$D_{22} = 0.3148.$$

In this example, the parameters are given by $\mu = 0.8$ and $\pi_{22} = 0.8$, For different π_{11}, Table 2 gives the maximum value of τ ensuring the asymptotically stable in mean square for the considered systems with H_∞ performance $\gamma = 2$. Next, we fix $\pi_{11} = 0.3$ and the minimum H_∞ performance γ for different τ can be obtained in Table 3.

Table 2: The max. allowed τ for different π_{11}.

π_{11}	-0.1	-0.3	-0.5	-0.7	-0.9
τ_{max}	0.523	0.521	0.520	0.519	0.518

Table 3: The min. allowed γ for different τ.

τ	0.1	0.2	0.3	0.4	0.5
γ_{min}	0.3149	0.3149	0.3149	0.3778	1.2509

V. Conclusion

In this paper, the asymptotical stability and H_∞ performance for Markovian jump systems with Time-varying delays have been investigated. By using the Lyapunov-Krasovskii functional and a convex optimization approach, novel delay-dependent sufficient conditions are obtained to guarantee the considered systems are asymptotically stable in mean square and guarantee a prescribed H_∞ performance index in terms of linear matrix inequality. The numerical examples demonstrate the effectiveness of the given methods. The foregoing results have the potential to be useful for the study of stochastic systems with Markovian jump parameters and time-varying delays.

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