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Curvature Ductility Factor of Rectangular Sections Reinforced Concrete Beams

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Abstract**—**The present work presents a method of calculating the ductility of rectangular sections of beams considering nonlinear behavior of concrete and steel. This calculation procedure allows us to trace the curvature of the section according to the bending moment, and consequently deduce ductility. It also allowed us to study the various parameters that affect the value of the ductility. A comparison of the effect of maximum rates of tension steel, adopted by the codes, ACI [1], EC8 [2] and RPA [3] on the value of the ductility was made. It was concluded that the maximum rate of steels permitted by the ACI [1] codes and RPA [3] are almost similar in their effect on the ductility and too high. Therefore, the ductility mobilized in case of an earthquake is low, the inverse of code EC8 [2]. Recommendations have been made in this direction.

Keywords**—**Ductility, beam, reinforced concrete, seismic code, relationship, time bending, resistance, non-linear behavior.

I. Introduction

IN seismic zones, it is important to design structures, with power ranging deformation beyond the elastic deformations without losing its ability to stay in service, in other words designing structures with ductile behavior. The current philosophy used in the seismic design of reinforced concrete frames auto-stable is based on the hypothesis of the formation of plastic hinges at critical sections, the ability of the latter to resist several cycles of inelastic deformations without significant loss in bearing capacity is evaluated in terms of available ductility.

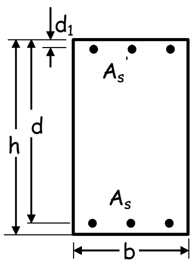

Fig. 1 Rectangular beam section

The ductility of a reinforced concrete beam section is measured by the expression $\mu = \emptyset_u/\emptyset_e$ [4], [5], [6], [7], [8], $\emptyset_u$ represents the curvature of the section when the concrete reaches its ultimate limit state and $\emptyset_e$ is the curvature of the section when steels in tension reaches the elastic limit state Fig. 12.

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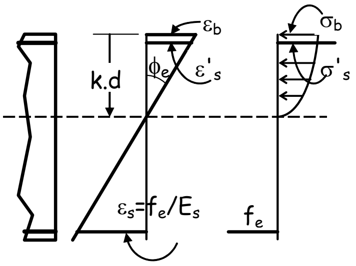

Fig. 2 Deformation and stress at the elastic limit state

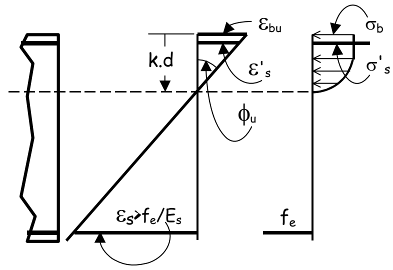

Fig. 3 Deformation and stress at the ultimate limit state

II. Concrete and Steel Model

The stress-strain curve (see Fig. 4) for unconfined concrete is assumed to be composed of a branch of the second degree ascending parabolic followed by a linear horizontal branch represented by the following expressions:

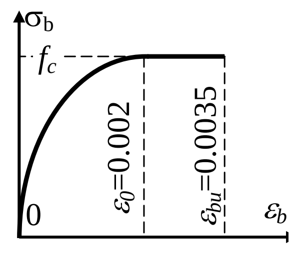

Fig. 4 Stress-strain curve of concrete

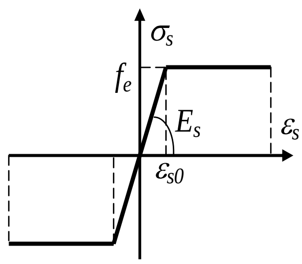

Fig. 5 Stress-strain curve of steel

$$\sigma_b = f_c\left[\frac{2\varepsilon_b}{0.002} - \left(\frac{2\varepsilon_b}{0.002}\right)^2\right]\text{if}\varepsilon_b \le 0.002 \quad (1)$$

$$\sigma_b = f_c\text{if}0.002 \le \varepsilon_b \le 0.0035 \quad (2)$$

The stress-strain curve of steel shown in Fig. 5, suppose an elastic-plastic behavior identical in compression and tension, represented by the following expressions:

$$\sigma_s = \varepsilon_s E_s\text{if}-f_e/E_s \le \varepsilon_s \le f_e/E_s \quad (3)$$

$$\sigma_s = f_e \text{if} \varepsilon_s \geq f_e / E_s \quad (4)$$

$$\sigma_s = -f_e \text{if} \varepsilon_s \leq -f_e / E_s \quad (5)$$

III. The Homogeneous Section Behavior Laws

A. Resistant Forces Developed by the Compressed Concrete, Tensile and Compression Steels

Possible distributions of compressive stresses in the concrete are shown in Fig. 6, according to the deformation of the farthest compressed fiber ε_{bS} from the neutral axis.

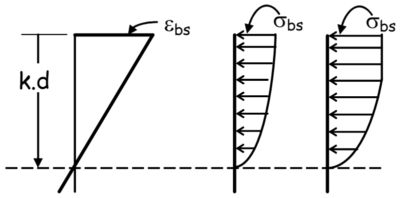

Fig. 6 Diagrams of possible compressive stresses distributions

The compressive force in the concrete is calculated from:

$$F_b = \int_0^{k.d} \sigma_b . b . dx = \alpha . f_c . b . k . d \quad (6)$$

The moment of this force with respect to the neutral axis is written as follow:

$$M_b = \int_0^{k.d} \sigma_b . b . x . dx = \beta . f_c . b . (k . d)^2 \quad (7)$$

From expressions (6) and (7) are deduced for the particular values

- ε_b= 0.002.
  - α = 2/3 = 0.6666
  - β = 5/12 = 0.4166
- $\varepsilon_b = 0.0035$
  - α = 17/21 = 0.8095
  - β = 143/147 = 0.9727
  -

Knowing the tensile steels deformation εs, the tension force $F_s$developed by them will be

$$F_s = A_s \varepsilon_s E_s \text{if} \varepsilon_s \leq f_e / E_s \quad (8)$$

$$F_s = A_s f_e \text{if} \varepsilon_s > f_e / E_s \quad (9)$$

The moment M_s of the force $F_s$relative to the neutral axis is written as follow:

$$M_s = F_s . (d - k . d) \quad (10)$$

The force F_s'developed by the compressed steels, knowing their deformation ε's will be:

$$F_s' = A_s' . \varepsilon_s' . E_s \text{if} -f_e / E_s \leq \varepsilon_s' \leq f_e / E_s \quad (11)$$

$$F_s' = A_s' . f_e \text{if} \varepsilon_s' > f_e / E_s \quad (12)$$

$$F_s' = -A_s' . f_e \text{if} \varepsilon_s' < -f_e / E_s \quad (13)$$

The moment M_s' of the force F_s'relative to the neutral axis is written as follow:

$$M_s' = F_s' . (k . d - d_1) \quad (14)$$

B. Calculation of the Section Equilibrium

The homogeneous section, must verify the following equilibrium equations:

$$\Sigma F_i = 0 \text{ and} \Sigma M_i = 0 \quad (15)$$

We define the rate of tension reinforcement by:

$$\rho = A_s / b . d \quad (16)$$

And the rate of compression reinforcement by:

$$\rho' = A_s' / b . d \quad (17)$$

1) Section without compression reinforcement

a) Case where $0 < \emptyset < \emptyset_e$.

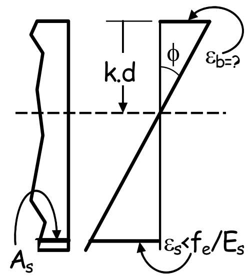

Fig. 7 Deformations diagram for $\emptyset < \emptyset_e$

For a value of ε_s such that $\varepsilon_s < f_e / E_s$ and for a supposed deformation of ε_b of the compressed fiber, most remote from the neutral axis, the coefficient k is written as follow:

$$k = \frac{\varepsilon_b}{\varepsilon_s + \varepsilon_b} \quad (18)$$

$$\varepsilon_b = \varepsilon_s \frac{k}{1-k} \quad (19)$$

The forces equilibrium gives:$F_b - F_s = 0$, consequently:

$$\alpha . f_c . b . k . d = A_s \varepsilon_s E_s \quad (20)$$

$$k = (\rho . \varepsilon_s E_s) / \alpha . f_c \quad (21)$$

At the elastic limit:

$$k_e = (\rho . f_e) / \alpha . f_c \quad (22)$$

b) Case where $\emptyset_e < \emptyset < \emptyset_u$.

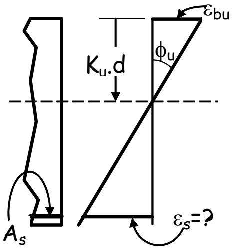

Fig. 8 Deformations diagram for $\emptyset_{\mathbf{e}} < \emptyset < \emptyset_{\mathbf{u}}$

Knowing ε_{be} be corresponding to $\varepsilon_s = f_e/E_s$ such that $\varepsilon_b < \varepsilon_{bu}$, we assume a value of ε_b such that $\varepsilon_{be} < \varepsilon_b \leq \varepsilon_{bu}$. The deformation of tension steel can be written as follow:

$$\varepsilon_s = \varepsilon_b \frac{1-k}{k} \quad (23)$$

Thus

$$\alpha.f_c.b.k.d = A_s f_e \quad (24)$$

$$k = (\rho.f_e)/\alpha.f_c \quad (25)$$

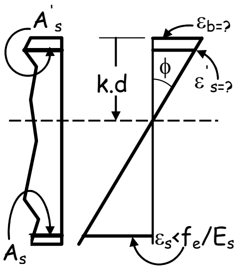

Fig. 9 Deformations diagram for $\emptyset < \emptyset_e$

2) Section with compression reinforcement

a) Case where $\mathbf{0} < \emptyset < \emptyset_e$.

The curvature corresponding to the elastic limit is written as follow: $\emptyset = \frac{\varepsilon_s}{d(1-k)}$

For a supposed deformation ε_b such as $\varepsilon_b = \varepsilon_s \frac{k}{(1-k)}$, the coefficient k is written as: $k = \frac{\varepsilon_b}{\varepsilon_b+\varepsilon_s}$, This allows to write the expression which allows to calculate the deformation of compressed steel:

$$\varepsilon_s' = \varepsilon_s \frac{k.d-d_1}{d-k.d} = \varepsilon_s \frac{k-\delta}{1-k} \text{with} \delta = d_1/d \quad (26)$$

On the assumption that the deformation is elastic: $\varepsilon_s' < f_e/E_s'$, the forces equilibrium is written $F_b + F_s' = F_s$, we deduced of this the following expressions:

$$\alpha.f_c.b.k.d + E_s.\varepsilon_s'.A_s' = A_s \varepsilon_s E_s \quad (27)$$

$$\alpha.f_c.k + E_s.\varepsilon_s'.\rho' = \rho.\varepsilon_s E_s \quad (28)$$

$$\alpha.f_c.k + E_s.\frac{f_e}{E_s}\frac{k-\delta}{1-k}.\rho' = \rho.\varepsilon_s E_s \quad (29)$$

Rearranging the expression (28) we obtain the following quadratic equation:

$$k^2.[\alpha.f_c] + k.[-\alpha.f_c - \varepsilon_s E_s(\rho+\rho')] + [\varepsilon_s E_s(\rho'.\delta+\rho)] = 0 \quad (30)$$

$$\Delta = [\alpha.f_c - \varepsilon_s E_s.(\rho+\rho')]^2 - 4.\alpha.f_c.[\varepsilon_s E_s(\rho'.\delta+\rho)] \quad (31)$$

$$k = \left[\alpha.f_c + \varepsilon_s E_s.(\rho+\rho') - \sqrt{\Delta}\right]/[2.\alpha.f_c] \quad (32)$$

If $\varepsilon_s' > f_e/E_s$ then:

$$\sigma_s' = f_e \quad (33)$$

$$\alpha.f_c.k + f_e.\rho' = \rho.\varepsilon_s.E_s \quad (34)$$

We obtain:

$$k = [\rho.\varepsilon_s.E_s - f_e.\rho']/\alpha.f_c \quad (35)$$

b) In case where $\emptyset_e < \emptyset < \emptyset_u$.

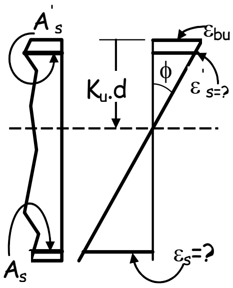

Fig. 10 Deformations diagram for $\emptyset_{\mathbf{e}} < \emptyset < \emptyset_{\mathbf{u}}$

Both the curvature and the curvature coefficient are written respectively:

$\emptyset = \frac{\varepsilon_b}{k.d}$, $k = \frac{\varepsilon_b}{\varepsilon_b+\varepsilon_s}$.

The tensile steel and compressed steel deformation are written respectively:

$$\varepsilon_s = \varepsilon_b \frac{1-k}{k}$$

$$\varepsilon_s' = \varepsilon_b \frac{k.d-d_1}{k.d} = \varepsilon_b \frac{k-\delta}{k} \quad (36)$$

With $\delta = d_1/d$

- CASE 1: $\varepsilon_s' < f_e/E_s$

The forces equilibrium can be written $F_b + F_s' = F_s$ consequently we obtain:

$$\alpha.f_c.b.k.d + E_s.\varepsilon_s'.A_s' = A_s.f_e$$

$$\alpha.f_c.k + E_s.\varepsilon_b \frac{k-\delta}{k}\rho' = \rho.f_e \quad (37)$$

,

Rearranging this expression (36) we obtain the following second degree equation:

$$k^2.[\alpha.f_c] + k.[E_s.\varepsilon_b.\rho' - \rho.f_e] + [-\varepsilon_b E_s \rho'.\delta] = 0 \quad (38)$$

$$\Delta= [\varepsilon_b E_s \rho' - \rho . f_e]^2 - 4.\alpha . f_{c.}[-\varepsilon_b E_s . \rho' . \delta] \quad (39)$$

$$k = [\rho . f_e - \varepsilon_b E_s \rho' - \sqrt{\Delta}]/[2.\alpha . f_{c.}] \quad (40)$$

- CASE 2: $\varepsilon_s' > f_e/E_s$

$$\sigma_s' = f_e \quad (41)$$

$$\alpha . f_{c.} k + . f_e . \rho' = \rho . f_e \quad (42)$$

We obtain:

$$k = f_e(\rho - \rho')/\alpha . f_c \quad (43)$$

- CASE 3: $\mathrm{E}_S' < -\mathrm{F}_E/\mathrm{E}_S$

$$\sigma_s' = -f_e \quad (44)$$

$$\alpha . f_{c.} k - . f_e . \rho' = \rho . f_e \quad (45)$$

We obtain:

$$k = f_e(\rho + \rho')/\alpha . f_c \quad (46)$$

C. Resistant Moment M Variation according to Curvature ϕ

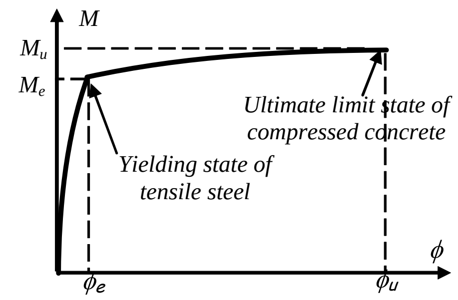

Fig. 11 Moment curvature beam model

For each equilibrium state of the section we can calculate the corresponding resistant bending moment $M(\phi)$. The calculation procedure of M, for a given curvature ϕ is necessarily iterative. We can deduce the ductility of a beam section bent according to the parameters: ρ, ρ', f_c et f_e.

D. Section Balanced State

By definition, a reinforced concrete section is said to be in a balanced state when the compressed concrete reaches its ultimate limit state deformation $\varepsilon_{bu} = 0.0035$ at the same time the deformation of tensile steel reached its elastic limit $\varepsilon_s = f_e/E_s$.

Applying the equilibrium equations mentioned earlier to calculate the state of the balanced section, we get the following results:

We define the quantity ρ_{b0} by:

$$\rho_{b0} = \left[\frac{566.66}{700+f_e}\right]\frac{f_c}{f_e} \quad (46)$$

Then:

$$\rho' = 0 \text{ and } \rho_b = \rho_{b0} = \left[\frac{566.66}{700+f_e}\right]\frac{f_c}{f_e} \quad (47)$$

where ρ_b is the tensile steel rate.

IV. THE DUCTILITY FACTOR PARAMETRIC STUDY $M_\emptyset$

$$\mu_\emptyset = \emptyset_u/\emptyset_e \quad (48)$$

When the section of tensile steel reaches the elastic limit the curvature of the section is:

$$\emptyset_e = \frac{f_e/E_s}{d.(1-k_e)} \quad (49)$$

At the ultimate limit state this curvature is written as follow:

$$\emptyset_u = \frac{\varepsilon_{bu}}{k_u . d} \quad (50)$$

Where, k_e and k_u are respectively the depth coefficients from the neutral axis at the elastic limit state of steels, and at ultimate limit state of the concrete.

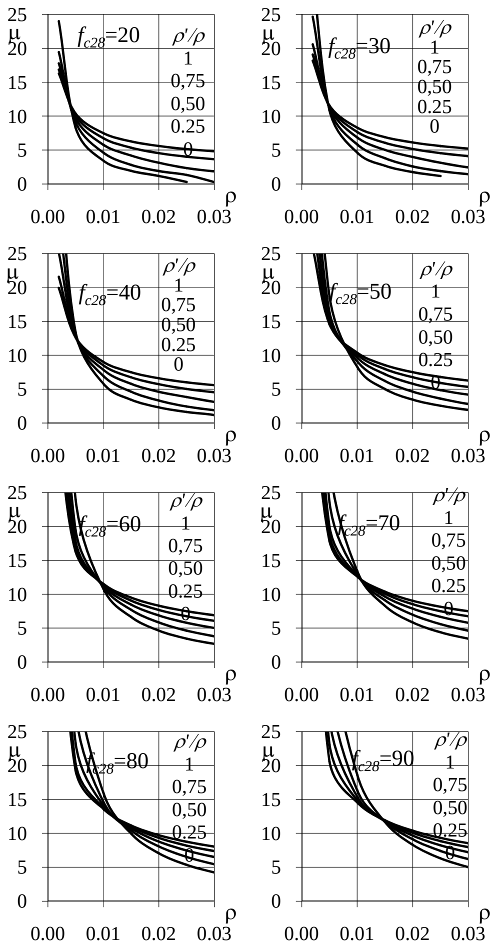

Fig. 12 Variation of μ_ϕ according to the tensile steel ρ and compressed steel ρ' areas for $20MPa < f_{c28} < 90MPa$

The curves in Fig. 12, have allowed us to make the following observations:

For a given strength of the concrete f_{c28}.

- The ductility μ is high if the rate of tensile steel ρ is small.
- The curves shown in Fig. 12, all interfere at the same point, which corresponds to a certain value of $\rho = \rho_i$ thus forming two distinct areas. In the first domain, the rate of tensile steel are small, the increase of compressedsteel ρ' for a constant value of ρ has the effect of reducing the value of ductility. Otherwise, in the second domain ($\rho > \rho_i$), ductility, μ is improved if the amount of compressed steel is increased.
- The resistances value f_{c28} of the concrete affect significantly the value of the ductility μ.
    - For a 20 MPa concrete ductility varies from 20 to 7 for $0 < \rho < 0.01$ and from 7 to 2 for $0.01 < \rho < 0.02$ and from 6 to 0 for $0.02 < \rho < 0.03$.
    - For a 30 MPa concrete ductility varies from 45 to 6 for $0 < \rho < 0.01$ and from 9 to 3 for $0.01 < \rho < 0.02$ and from 7 to 1 for $0.02 < \rho < 0.03$.
    - For a 50 MPa concrete ductility varies from 78 to 12 for $0 < \rho < 0.01$ and from 12 to 5 for $0.01 < \rho < 0.02$ and from 8 to 3 for $0.02 < \rho < 0.03$.
    - For an 80 MPa concrete ductility varies from 130 to 15 for $0 < \rho < 0.01$ and from 19 to 7 for $0.01 < \rho < 0.02$ and from 11 to 5 for $0.02 < \rho < 0.03$.

- Generally and whatever the concrete strength, we have the ductility $\mu > 7$ if $\rho < 0.25\rho_b$, it varies from 9 to 3.5 for $0.25\rho_b < \rho < 0.5\rho_b$, and from 7 to 2 for $0.5\rho_b < \rho < 0.75\rho_b$ and from 6 to 1 for $0.75\rho_b < \rho < \rho_b$.

- The balanced state for a concrete strength of 20 MPa corresponds to $\rho = \rho_b = 2.18\%$, for a concrete strength of 30 MPa to $\rho_b = 2.73\%$, for a concrete strength of 50 MPa to $\rho_b = 5.47\%$ and for a concrete strength of 80 MPa the balanced state corresponds to $\rho = \rho_b = 8.75\%$.

- The improvement of compressive strength of the concrete has resulted in increased ductility for tensile steel rates more or less high, for $\rho = 0.03$ with $f_{c28} = 20\,MPa$ we have μ which varies between 0 and 5, with $f_{c28} = 40\,MPa$ we obtain μ which varies between 2.5 and 7 for $f_{c28} = 80\,MPa\,\mu$ qui varies between 5 and 9.

This situation is generally required for its economic character, because it ensures at the same time the load-carrying capacity who requires sometimes high sections of steels (what implies a reduction of ductility) and a sufficient ductility for its deformability which is better if the tensile steel rate ρ is weak (what implies a reduction in resistance), the ideal is to find an intermediate position for maximum strength and maximum ductility.

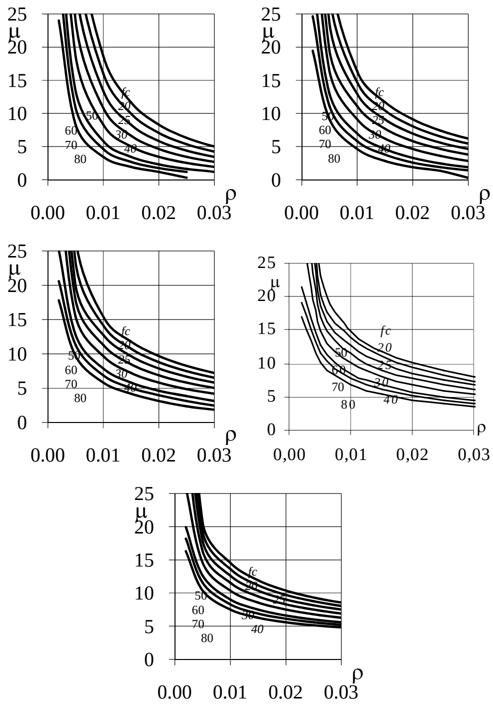

Fig. 13 Variation of μ_ϕ according to the tensile steel ρ, the resistance f_{c28} and the ratio (ρ'/ρ)

From Fig. 13, we deduce that the value of the ductility increases with increases in the ratio (ρ'/ρ).

V. Comparative Study of RPA-99, ACI And EC8 Recommendations Codes

a) Algerian earthquake code recommends the following measures:
- The concrete of the principal elements must have strength of $20\,MPa < f_{c28} < 45\,MPa$.
- The minimum $(\rho' + \rho)_{min}$ total rate, of the longitudinal reinforcement along the entire length of the beam is 0.5%.
- While-the maximum $(\rho' + \rho)_{max}$ total rate is 4% in current zone and 6% in overlapping zone
- No carrying Beams solicited mainly by the seismic loads must have symmetrical steel reinforcements ($\rho = \rho'$) with a steel section at mid span at least equalizes with the section on support.

b) The U.S. regulation ACI building code recommendations are:
- The minimum rate of steel to be disposed on the entire length of the beam is given by:
- $\rho_{min} = \rho'_{min} = \max\left[0.25\sqrt{f_c}/f_e\,,1.38/f_e\right]$
- For bent elements of structure located in seismic zone. $\rho - \rho' \le 0.5\rho_b$.
- The section of tensile steels of the bent elements belonging to ductile beam-column structures resistant to seismic forces shall not exceed 2.5% of the section of concrete.

$$\rho \le \rho_{max} = 0.025$$

c) The EC8 requires

- for beams high ductility to dispose closed stirrups which diameter $\phi \geq 6$ mm with spacing such that s_t :

$$s_t \leq \min[h_p/4, 24\emptyset_t, 150mm, 6\emptyset_l]$$

This confinement allows a considerable increase in ductility.

- In zones of plastic hinges, on a length equal to two times the section height of the beam, to dispose in the compression zone, a section of steel not less than the half of the tensile section.

$$\rho' = 0.5\rho$$

- Along the entire length of the beam, the reinforcement should satisfy the followings conditions :

$$\rho_{min} = 0.05\, f_c/f_e$$
$$\rho_{max} = 0.17\, f_c/f_e$$

These recommendations ensure in an indirect way using the condition of minimum reinforcement for concretes with moderate resistance ($f_{c28} \cong 22\ MPa$), in the case of the RPA an average ductility of $\mu \geq 13$, the ACI code ensures a ductility $\mu \cong 12{,}5$ whereas, l'EC8 an average minimal ductility of $\mu \geq 16{,}5$.

The section of maximum reinforcement not to exceed imposed by the RPA and ACI codes aim at ensuring a quasi-identical minimal ductility value about:

In the case: $\rho' + \rho = 4\%$

- $\mu \cong 2.5$ for $\rho'/\rho = 0.5$
- $\mu \cong 4.2$ for $\rho'/\rho = 0.75$

In the case: $\rho' + \rho = 6\%$

- $\mu \cong 2.5$ for $\rho'/\rho = 0.5$
- $\mu \cong 4.2$ for $\rho'/\rho = 0.75$

On the other hand l'EC8 aim at ensuring a higher ductility about:

- $\mu \cong 7.5$ for $\rho'/\rho = 0.5$
- $\mu \cong 8.3$ for $\rho'/\rho = 0.75$

These results show that the maximum steel section tolerated by the codes ACI and RPA reduced ductility considerably comparatively with the EC8 code.

Researchers have agreed on a minimum and reasonable ductility value what must develop the critical sections of a structure known as ductile, conceived to resist the seism effects; they suggest a minimal value who can vary from 4 to 6, thus 2 to 3 times ductility necessary in the structural design under static loads, Where it is permissible to take $\mu \geq 2$.

VI. Conclusion

The ductility calculation $\mu_\emptyset = \emptyset_u/\emptyset_e$ of a reinforced concrete beam by taking into account the nonlinear character materials of concrete and steels can be calculated easily and precisely, by using the method exposed in this work with computer tools help.

It was shown that ductility $\mu_\emptyset = \emptyset_u/\emptyset_e$ increase with

- Reduction in the quantity ρ of tensile steels
- Reduction in the yield stress f_e of steels.
- Increase in the ratio ρ'/ρ quantity of the steels compressed on the quantity of tensile steels.

Moreover, it was shown, it is possible to define the threshold of ductility, to be achieved by the balanced state of the section, indeed with:

$\rho < 0.25\rho_b$	$\mu \geq 7$
$0.25\rho_b < \rho < 0.5\rho_b$	$9 \geq \mu \geq 3.5$
$0.5\rho_b < \rho < 0.75\rho_b$	$7 \geq \mu \geq 2$
$0.75\rho_b < \rho < \rho_b$	$6 \geq \mu \geq 1$

With $\rho_{b0} = \left[\frac{566.66}{700 + f_e}\right]\frac{f_c}{f_e}$

$\rho_b = \rho_{b0} = \left[\frac{566.66}{700 + f_e}\right]\frac{f_c}{f_e}$ if $\rho' = 0$

$\rho_b = \rho_{b0} + \rho'$ if $\rho' \neq 0$

In the light of these results, it appears that it is adapted to limit the rate of the tensile steels to $\rho < 0.25\rho_b$ in the zones of high seismicity, to reach a ductility $\mu \cong 8$, and to $\rho < 0.5\rho_b$ in the average seismicity zones to reach a ductility $\mu \cong 6$, and to limit the total quantity of steels $(\rho + \rho')$ with a maximum of 1.5% at least in high seismicity zones. This last assumption for a concrete with moderate resistance ($f_c = 22\ MPa$) ensure a ductility of:

$\mu \cong 4.5$ with $\rho'/\rho = 0.25$

$\mu \cong 6.5$ with $\rho'/\rho = 0.50$

$\mu \cong 8.5$ with $\rho'/\rho = 0.75$

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