Crack Width Evaluation for Flexural RC Members with Axial Tension

Sukrit Ghorai

Abstract—Proof of controlling crack width is a basic condition for securing suitable performance in serviceability limit state. The cracking in concrete can occur at any time from the casting of time to the years after the concrete has been set in place. Most codes struggle with offering procedure for crack width calculation. There is lack in availability of design charts for designers to compute crack width with ease. The focus of the study is to utilize design charts and parametric equations in calculating crack width with minimum error. The paper contains a simplified procedure to calculate crack width for reinforced concrete (RC) sections subjected to bending with axial tensile force following the guidelines of Euro code [DS EN-1992-1-1 & DS EN-1992-1-2]. Numerical examples demonstrate the application of the suggested procedure. Comparison with parallel analytical tools supports the validity of result and show the percentage deviation of crack width in both the procedures. The technique is simple, user friendly and ready to evolve for a greater spectrum of section sizes and materials.

Keywords—Concrete structures, crack width calculation, serviceability limit state, structural design, bridge engineering.

NOTATION

P   – Axial Force (Compressive (+)/Tensile (-))
M   – Moment
b   – Width of section
d   – Effective depth of section
h   – Depth of section
L   – Length of section
A_s – Area of tensile steel
w_k – Characteristic Crack width
$s_{r,max}$ – Maximum crack spacing
x_{na} – Depth of neutral axis from top of section
ε_{sm} – Mean strain of reinforcement in the length $s_{r,max}$
ε_{cm} – Mean strain of concrete in the length $s_{r,max}$
$f_{ct,eff}$ – Mean value of tensile strength in concrete
α – Modular ratio E_s/E_c
$A_{c,eff}$ – Effective tension area = b x $h_{c,eff}$
$\rho_{p,eff}$ – Effective reinforcement ratio = $A_s/A_{c,eff}$
$H_{c,eff}$ – Min. of 2.5(h-d), (h-x_{na})/3, h/2
ϕ – Diameter of reinforcement bar
c   – Clear cover of the reinforcement
P/M(%)– Percent ratio of Axial force to Moment (always positive value)

I. INTRODUCTION

LIMITING crack width is one of the two basic conditions for securing suitable performance of structure in serviceability limit state: deformation and crack width limitations. The Eurocode has pursued in providing procedures for calculation of crack width, however the focus of the code is for sections under bending or pure tensile force. The code is silent for design of section under bending with tensile force. An eccentrically loaded section is important in design as it illustrates the solution to the real world problems.

The method specified in the code for computation of crack width is tedious. In addition, there exists a method for crack control without direct calculation, not recommended for bridge design in the code.

Lack of simplified design approach motivated the study of a method to help the engineers calculate the crack width with ease. A simplified procedure offered in the literature will allow an engineer to compute crack width without going into rigorous calculations of the code. One can find the solution to the problem with the help of design charts and parametric equations mentioned in the paper.

Furthermore, to check the validity of results from the method, the crack width from the method is compared with results obtained from design procedure mentioned in [1]-[3].

II. CALCULATION OF CRACK WIDTH ACCORDING TO [4] AND [5]

The basis of the crack width calculation to [4] and [5] presented here is considering the simplified case of a reinforced concrete prism in tension. The member will first crack when the tensile strength of the weakest section is reached. Cracking leads to a local redistribution of stresses adjacent to the crack by strain distributions. At the crack, the entire tensile force is carried by the reinforcement. Moving away from the crack, tensile stress is transferred from the reinforcement by bond to the surrounding concrete and, therefore, at some distance from the crack, the distribution of stress is unaltered from that before the crack formed. At this location, the strain in concrete and reinforcement is equal and the stress in the concrete is just below its tensile strength.

The design procedure for calculating crack width is as follows:

$$w_k = s_{r,max}(\epsilon_{sm} - \epsilon_{cm}) \tag{1}$$

The maximum crack spacing is defined as:

$$s_{r,max} = k_3 c + k_1 k_2 k_4 \emptyset / \rho_{p,eff} \tag{2}$$

The mean strain in reinforcement is defined as:

$$\epsilon_{sm} - \epsilon_{cm} = \frac{\sigma_s - k t \frac{f_{ct,eff}}{\rho_{p,eff}}(1 + \alpha \rho_{p,eff})}{E_s} \geq 0.6 \frac{\sigma_s}{E_s} \tag{3}$$

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Note:

k_1 = 0.8 (high bond bars), 1.6 (bars with plain surface)

k_2 = 0.5 (bending), 1.0 (pure tension)

$k_3 = 3.4 \times (25/_c)^{0.667}$

k_4 = 0.425

K_t = 0.6 (short term loading), 0.4 (long term loading)

The values of the co-efficient considered are relevant to bridge structures.

The calculation of crack width by using [4] and [5] requires lot of time and complicity. Engineers require understanding in the evaluation of neutral axis in RC section subjected to bending with axial tensile force. Hence, there is no simplified method available for the calculation of crack width.

III. Proposed Method for Calculation of Crack Width for RC Sections with Axial Tension

The proposed herein method, utilizes the calculation procedure given in [4] and [5] with the help of design graphs and empirical equations to calculate crack width of a RC section under tensile force. The main intent of the paper is to validate the procedure of the literature. Thus, only one section size and material is used to calculate the crack width to assure the authenticity of the method.
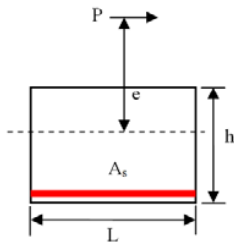

Fig. 1 Eccentric normal force in tension acting on a section

Fig. 1 shows a typical rectangular section subjected to bending and axial tension. The paper focuses on section and material properties mentioned in
TABLE I.

TABLE I
SECTION AND MATERIAL PROPERTIES

Parameter	Value	Unit
f_{ck}	35	MPa
f_{cm}	43	MPa
$f_{ct,eff}$	3.2	MPa
E_{cm}	34000	MPa
f_{yk}	500	MPa
E_s	200000	MPa
c	50	mm
γ_c	1.38	-
γ_s	1.14	-
f_{cd}	25.41	MPa
f_{yd}	438.60	MPa
h	1000	mm
b	1000	mm
k_1	0.8	-
k_2	0.5	-
k_3	2.1419	-
k_4	0.425	-
k_t	0.4	-

In order to understand the procedure in a simply way, a systematic procedure to calculate the crack width is as follows:

1. The percentage ratio of tensile force to moment (P/M) is calculated and linear equation is found for the corresponding moment from Fig. 2 in the form y = Ax + B where 'x' is the P/M ratio. For clarity only moment with magnitude of 250KNm, 500KNm, 750KNm and 1000KNm are shown but for intermediate equations can also be found by interpolation of P/M percentage ratio from the graph as well.
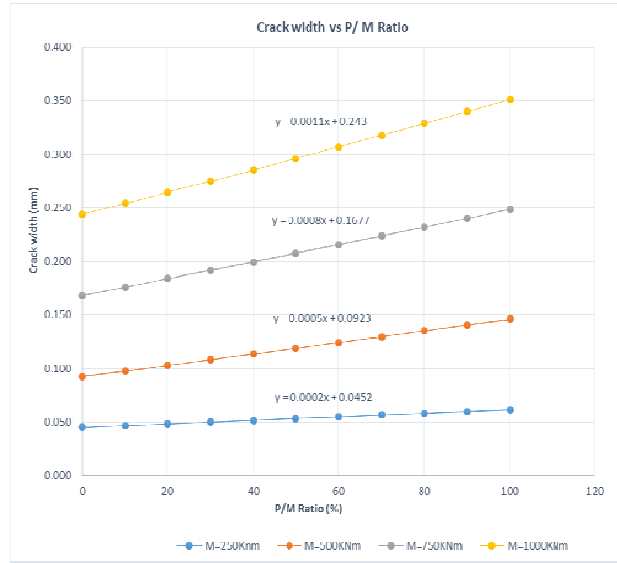
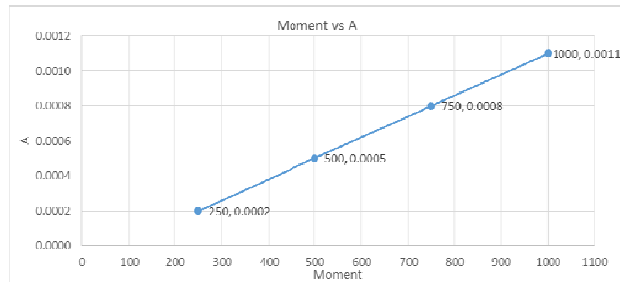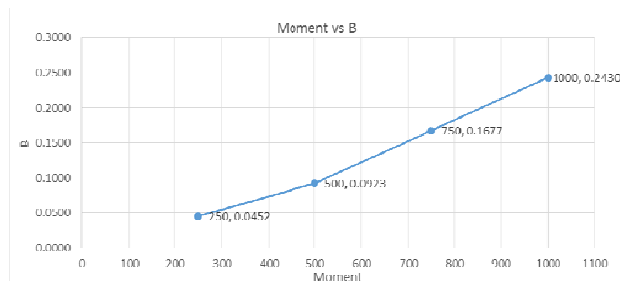

Fig. 2 Crack width vs P/M Ratio

Fig. 3 Moment vs A

Fig. 4 Moment vs B

2. It is difficult to find precisely the co-efficient A and B using Fig. 2. So, it is recommended to find out the co-efficients by using Figs. 3 and 4 respectively for the corresponding moment. As the co-efficients and variable 'x' are known, crack width can be determined. However, the crack width calculated is for a standard tensile steel area of Y25 @ 100mm c/c.

3. The calculation of crack width with different tensile steel area other than standard can be easily computed. This can be done by finding out a modification factor to the crack width with standard reinforcement. A factor is found out with the help of Fig. 5 and empirical equations. Multiplying the modification factor with crack width obtained from Fig. 2 gives the required crack width for any area of steel.

$$\alpha_1 = \beta_1 + \beta_2 \frac{\emptyset}{\rho_{P\,eff}} \qquad (4)$$

$$\alpha_2 = \beta_3 \frac{(1 + \beta_4 \rho_{P,eff})}{\rho_{P,eff}} \qquad (5)$$

$$\alpha_3 = \frac{\varepsilon_{sm} - \varepsilon_{cm}}{\varepsilon} \qquad (6)$$

$$\varepsilon_{sm} - \varepsilon_{cm} = \max(\varepsilon_{sm} - \alpha_2 \times \eta, \frac{0.6\sigma_s}{E_s}) \qquad (7)$$

$$w_{k,mod} = \alpha_1 \times \alpha_3 \times w_k \qquad (8)$$

Note:

$\beta_1 = 0.44$

$\beta_2 = 7.01E\text{-}04$

$\beta_3 = 0.027$

$\beta_4 = 5.88$

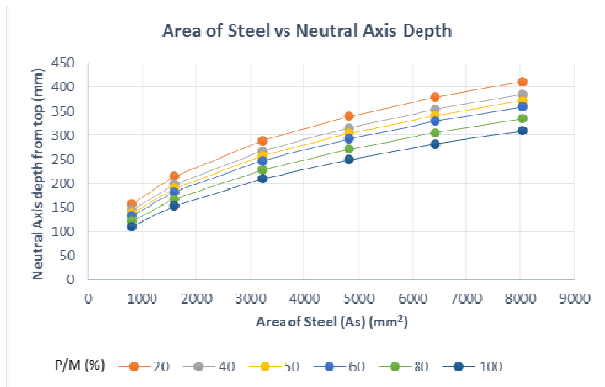$\eta = 2.41E\text{-}04$

$\varepsilon = 0.0002$

Fig. 5 Area of Steel vs Neutral Axis Depth

IV. NUMERICAL EXAMPLES

Further to the detailed procedure, examples shown in the section aim to cover a variety of problems that may rise applying the offered procedure. In the examples, section and material properties correspond to the basic parameters mentioned in

TABLE I.

Example 1

A reinforced concrete section of a beam is reinforced with $4910mm^2$/m tensile reinforcement in the form of $\Phi25$@100mm. Calculate the crack width of the section for an axial tensile force of 50KN and a moment of 250KNm.

Axial Force, P = -50KN

Moment, M = 250KNm

We know that the variation of P/M percentage vs crack width follows a linear variation. The equation will be in the form of y = Ax + B where x is the P/M percentage and y refers to the crack width of the section. In order to find out the value of A and B, Figs. 3 and 4 are used respectively.

For P/M % = 50/250 x 100 = 20

From Fig. 3, A = 0.0002

From Fig. 4, B = 0.0452

Using y = Ax + B

Crack width = 0.0002 x 20 + 0.0452 = 0.0492mm

Validation: (by rigorous procedure as per DS-EN1992)

Crack width = 0.048mm

% Error = 2.0%, Hence OK

Example 2

A section of a beam is reinforced with $2010mm^2$/m tensile reinforcement in the form of $\Phi16$@100mm. Calculate crack width of the section for axial tensile force of 50KN and moment of 250KNm.

Axial Force, P = -50KN

Moment, M = 250KNm

In order to find out the value of A and B, Figs. 3 and 4 are used respectively.

For P/M % = 50/250 x 100 = 20

From Fig. 3, A = 0.0002

From Fig. 4, B = 0.0452

Using y = Ax + B

Crack width = 0.0002 x 20 + 0.0452 = 0.0492mm

The crack width calculated is for section with reinforcement of $\Phi25$@100mm. However to find out the reinforcement of $\Phi16$@100mm, modification factor needs to be found out.

From Fig. 5, Neutral axis depth from top, $x_{na}= 230mm$

As neutral axis of the section is determined for corresponding steel area of $\Phi16$@100mm, moment of Inertia can be calculated using the standard formula for a rectangular section.

Moment of Inertia about neutral axis, $I_{xx} = 2.8E{+}10mm^4$

With known values of P and M, eccentricity (e) can be calculated (e = M/P). Please note that the eccentricity calculated of axial force P is from the center of the rectangular section. Further to it, a lever arm distance of the axial load from the neutral axis needs to be computed.

Lever arm of force P from neutral axis, z = -5270mm

Finding moment about neutral axis for the axial force:

$M_{na} = 264KNm$

As the moment about neutral axis and moment of Inertia are known, hence stress on any fiber of the section can be calculated.

Tensile stress in steel, $\sigma_s = 157.44MPa$

Tensile strain in steel, $\varepsilon_{sm} = \sigma_s / E_s = 0.0008$

From (5), $\alpha_2 = 2.068$

From (7), $\varepsilon_{sm} - \varepsilon_{cm}$ = 4.72E-04
From (6), α_3 = 2.36
From (4), α_1 = 1.25
Modified Crack width = 1.25 x 2.36 x 0.0492 = 0.143mm
Validation: (by rigorous procedure as per DS-EN1992)
Crack width = 0.142mm
% Error = 1.0%, Hence OK

V. CONCLUSION

A simple procedure presented is in line with the proposed procedure in [4] and [5]. It allows user to calculate crack width with ease and minimum error. However, the paper illustrates the method for one particular section size and material property, but the procedure is applicable for different parameters as well. It is helpful if multiple design charts and corresponding empirical equations can be generated for different variables so that it can be scaled to a larger level. The design examples mentioned in the paper are found to be in good agreement with adopted design methodology in Eurocode and design guides of Eurocode.

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