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Contact Stress on the Surface of Gear Teeth with Different Profile

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***Abstract*—**Contact stress is an important problem in industry. This is a problem that in the first attention may be don't appears, but disregard of these stresses cause a lot of damages in machines. These stresses occur at locations such as gear teeth, bearings, cams and between a locomotive wheel and the railroad rail. These stresses cause failure by excessive elastic deformation, yielding and fracture. In this paper we intend show the effective parameters in contact stress and ponder effect of curvature. In this paper we study contact stresses on the surface of gear teeth and compare these stresses for four popular profiles of gear teeth (involute, cycloid, epicycloids, and hypocycloid). We study this problem with mathematical and finite element methods and compare these two methods on different profile surfaces.

***Keywords*—**Contact stress, Cycloid, Epicycloids, Finite element, Gear, Hypocycloid, Involute, Radius of curvature.

I. Introduction

CONTACT stresses are caused by the pressure of one solid on another over limited areas of contact. Contact stresses created when surfaces of two bodies are pressed together by external loads are the significant stresses; that is, the stresses on or somewhat beneath the surface of contact are the major cause of failure of one or both of the bodies. For example, contact stresses may be significant at the area (1) between a locomotive wheel and the railroad rail (2) between a roller or ball and its race in a bearing (3) between the teeth of a pair of gears in mesh (4) between the cam and valve tappets of a gasoline engine; etc. [1].

Contact stresses are significant in industry, for example, a railroad rail sometimes fails as a result of "contact stresses". The failure start as a localized fracture in the form of a minute transverse crack at a point in the head of the rail and progresses outwardly under the influence of the repeated wheel loads until the entire rail cracks or fractures. This fracture is called a transverse fissure failure [1].

H. Hertz (1895) was the first to obtain a satisfactory solution, although his solution gives only principal stresses in the contact area [1].

The solution of the problem of the contact stresses in the neighborhood of the point of contact of two bodies is based on the following two assumptions:

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A.

Properties of materials: the material of each body homogeneous, isotropic, and elastic in accordance with Hook's law, but the two bodies is not necessarily made of the same material.

B.

Contact surface area, before loading. If two bodies are in contact at a point, there is a common tangent plane to the surfaces at the point of contact [1].

In this paper we intend to ponder effect of radius of curvature on the contact stresses, and study contact stresses on the surface of gear teeth and compare these stresses for four popular profiles of gear teeth; involute, cycloid, epicycloids, hypocycloid; At the next sections we introduce the four profiles of gear teeth, their mathematical equations and calculate the radius of curvature at the contact point on the pitch circle. Also we use finite element model for calculation these stresses, we do this by ANSYS. At the end we compare the mathematical results and ANSYS results.

II. Gear Teeth Profiles Type

In this section we should introduce gear teeth profiles and represent the needed information.

A. Involute

The involute curve is defined as the locus of an end of a taut string as it unwinds from a base circle [7].

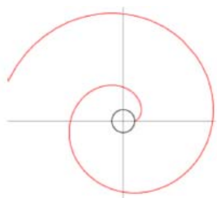

Fig. 1 Involutes curve

B. Cycloid

The cycloid is the locus of a point on the rim of a circle of radius a rolling along a straight line [7].

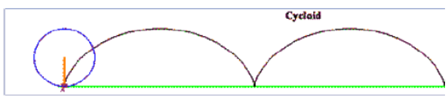

Fig. 2. Cycloid curve

C. Hypocycloid

A hypocycloid is defined to be the path traced out by a point P on the edge of a circle of radius b rolling on the inside of a fixed circle of radius a [7].

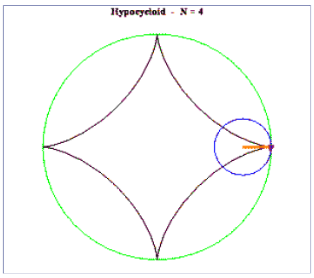

Fig. 3 Hypocycloid curve

D. Epicycloid

An epicycloid is defined to be the path traced out by a point P on the edge of a circle of radius b rolling on the outside of a fixed circle of radius a [7].

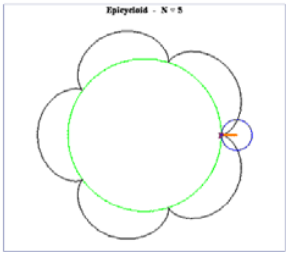

Fig. 4. Epicycloids curve

III. The Mathematical Equations for Different Profile's Curve

A. Involute profile:

$$R(t)=a\left[(\cos t+t\times\sin t)\hat{i}+(\sin t-t\times\cos t)\hat{j}\right]$$

Where, a is radius of base circle and t is angle between horizontal line and line that connect the circle center and point that string separate from base circle [7].

B. Cycloid profile:

$$R(t)=a\left[(t-\sin t)\hat{i}+(1-\cos t)\hat{j}\right]$$

Where, a is radius of generator circle and t is angle between vertical line and radius line at generator point [7].

C. Hypocycloid profile:

$$R(t)=\left[(a-b)\cos t+b\times\cos(\frac{a-b}{b}t)\right]\hat{i}$$
$$+\left[(a-b)\sin t-b\times\sin(\frac{a-b}{b}t)\right]\hat{j}$$

Where, b is radius of generator circle, a is radius of circle that generator circle rolling on the inside of it and t is angle between horizontal line and line that connect centers of generator and fixed circle [7].

D. Epicycloids profile:

$$R(t)=\left[(a+b)\cos t-b\times\cos(\frac{a+b}{b}t)\right]\hat{i}$$
$$+\left[(a+b)\sin t-b\times\sin(\frac{a+b}{b}t)\right]\hat{j}$$

Where, b is radius of generator circle, a is radius of circle that generator circle rolling on the outside of it and t is angle between horizontal line and line that connect centers of generator and fixed circle [7].

In all above equations, t is in radian, a, b and R have same unit. Now we intend to find an equation for curvature radius for these profiles.

If $R(t)$ be mathematical equation for curve, by derivative of it with respect to t we can obtain $V(t)$ and by derivative of $V(t)$, $A(t)$ is obtained [3].

$$\kappa=\frac{|V\times A|}{|V|^3},\qquad \rho=\frac{1}{\kappa}$$

Where, κ is curvature and ρ is radius of curvature.

Involute:

$$R(t)=a\begin{bmatrix}(\cos t+t\times\sin t)\hat{i}\\+(\sin t-t\times\cos t)\hat{j}\end{bmatrix}$$

$$V(t)=a\begin{bmatrix}(-\sin t+\sin t+t\times\cos t)\hat{i}\\+(\cos t-\cos t+t\times\sin t)\hat{j}\end{bmatrix}$$
$$=a\left[(t\times\cos t)\hat{i}+(t\times\sin t)\hat{j}\right]$$

$$A(t)=a\begin{bmatrix}(\cos t-t\times\sin t)\hat{i}\\+(\sin t+t\times\cos t)\hat{j}\end{bmatrix}$$

$$V\times A=a^2\begin{vmatrix}i & j & k\\ t\cos t & t\sin t & 0\\ \cos t-t\sin t & \sin t+t\cos t & 0\end{vmatrix}$$
$$=a^2(t\sin t\cos t+t^2\cos^2 t$$
$$-t\sin t\cos t+t^2\sin^2 t)=a^2t^2$$

$$|V|=a\sqrt{t^2\cos^2 t+t^2\sin^2 t}=at$$

$$\kappa=\frac{|V\times A|}{|V|^3}=\frac{a^2t^2}{a^3t^3}=\frac{1}{at}$$

$$\Rightarrow \rho(t)=at \lhd\lhd$$

Cycloid:

$$R(t)=a\left[(t-\sin t)\hat{i}+(1-\cos t)\hat{j}\right]$$

$$V(t)=a\left[(1-\cos t)\hat{i}+(\sin t)\hat{j}\right]$$

$$A(t)=a\left[(\sin t)\hat{i}+(\cos t)\hat{j}\right]$$

$$V\times A=a^2\begin{vmatrix} i & j & k \\ 1-\cos t & \sin t & 0 \\ \sin t & \cos t & 0\end{vmatrix}$$

$$=a^2(\cos t-\cos^2 t-\sin^2 t)$$

$$=a^2(\cos t-1)$$

$$|V|=a\sqrt{(1+\cos^2 t-2\cos t)+\sin^2 t}$$

$$=a\sqrt{2(1-\cos t)}$$

$$\kappa=\frac{|V\times A|}{|V|^3}=\frac{a^2(1-\cos t)}{a^3 2\sqrt{2}(1-\cos t)^{3/2}}$$

$$=\frac{1}{2\sqrt{2}a\sqrt{(1-\cos t)}}$$

$$\Rightarrow \rho(t)=2\sqrt{2}a\sqrt{(1-\cos t)}$$

$$=4a\sin(t/2)\ \triangleleft\triangleleft$$

Hypocycloid:

$$R(t)=\left[(a-b)\cos t+b\times\cos(\frac{a-b}{b}t)\right]\hat{i}$$

$$+\left[(a-b)\sin t-b\times\sin(\frac{a-b}{b}t)\right]\hat{j}$$

$$V(t)=\left[-(a-b)\sin t-(a-b)\sin(\frac{a-b}{b}t)\right]\hat{i}$$

$$+\left[(a-b)\cos t-(a-b)\cos(\frac{a-b}{b}t)\right]\hat{j}$$

$$A(t)=\left[-(a-b)\cos t-\frac{(a-b)^2}{b}\cos(\frac{a-b}{b}t)\right]\hat{i}$$

$$+\left[-(a-b)\sin t+\frac{(a-b)^2}{b}\sin(\frac{a-b}{b}t)\right]\hat{j}$$

$$V\times A=(a-b)^2\times$$

$$\begin{vmatrix} i & j & k \\ -\sin t-\sin(\frac{a-b}{b}t) & \cos t-\cos(\frac{a-b}{b}t) & 0 \\ -\cos t & -\sin t & \\ -\frac{a-b}{b}\cos(\frac{a-b}{b}t) & +\frac{a-b}{b}\sin(\frac{a-b}{b}t) & 0\end{vmatrix}$$

$$\kappa=\frac{|V\times A|}{|V|^3}=\frac{1}{(a-b)}\times\frac{\frac{a-2b}{b}}{2\sqrt{2}}$$

$$\times\frac{1}{\sqrt{1+\sin t\sin(\frac{a-b}{b}t)-\cos t\cos(\frac{a-b}{b}t)}}$$

$$\sin t\sin(xt)-\cos t\cos(xt)=-\cos(1+x)t$$

$$=-\cos(1+\frac{a-b}{b})t=-\cos(\frac{a}{b})t$$

$$\Rightarrow \rho(t)=\frac{2\sqrt{2}(a-b)\times\sqrt{1-\cos(\frac{a}{b}t)}}{-1+\frac{a-b}{b}}$$

$$=\frac{2\sqrt{2}b(a-b)\times\sqrt{1-\cos(\frac{a}{b}t)}}{a-2b}\ \triangleleft\triangleleft$$

Epicycloid:

$$R(t)=\left[(a+b)\cos t-b\times\cos(\frac{a+b}{b}t)\right]\hat{i}$$

$$+\left[(a+b)\sin t-b\times\sin(\frac{a+b}{b}t)\right]\hat{j}$$

$$V(t)=\left[-(a+b)\sin t+(a+b)\sin(\frac{a+b}{b}t)\right]\hat{i}$$

$$+\left[(a+b)\cos t-(a+b)\cos(\frac{a+b}{b}t)\right]\hat{j}$$

$$A(t)=\left[-(a+b)\cos t+\frac{(a+b)^2}{b}\cos(\frac{a+b}{b}t)\right]\hat{i}$$

$$+\left[-(a+b)\sin t+\frac{(a+b)^2}{b}\sin(\frac{a+b}{b}t)\right]\hat{j}$$

$$V \times A = (a+b)^2 \times \begin{vmatrix} i & j & k \\ -\sin t + \sin(\frac{a+b}{b}t) & \cos t - \cos(\frac{a+b}{b}t) & 0 \\ -\cos t + \frac{a+b}{b}\cos(\frac{a+b}{b}t) & -\sin t + \frac{a+b}{b}\sin(\frac{a+b}{b}t) & 0 \end{vmatrix}$$

$$\kappa = \frac{|V \times A|}{|V|^3} = \frac{1}{(a+b)} \times \frac{\frac{a+2b}{b}}{2\sqrt{2}} \times \frac{1}{\sqrt{1-\sin t \sin(\frac{a+b}{b}t) - \cos t \cos(\frac{a+b}{b}t)}}$$

$$\rho = \frac{2\sqrt{2}b(a+b)}{a+2b} \times \sqrt{1-\sin t \sin(\frac{a+b}{b}t) - \cos t \cos(\frac{a+b}{b}t)}$$

$$\sin t \sin(xt) = -\frac{1}{2}(\cos(1+x)t - \cos(x-1)t)$$

$$\cos t \cos(xt) = \frac{1}{2}(\cos(1+x)t + \cos(x-1)t)$$

$$\stackrel{+}{\Rightarrow} \cos(x-1)t = \cos(\frac{a+b}{b}-1)t = \cos(\frac{a}{b}t)$$

$$\Rightarrow \rho(t) = \frac{2\sqrt{2}b(a+b) \times \sqrt{1-\cos(\frac{a}{b}t)}}{a+2b} \quad \triangleleft\triangleleft$$

IV. Calculation Stresses in Different Profiles

After finding the needed equations for calculation radius of curvature, in order to a good compare between different profiles we give examples. In these examples we obtain radius of curvature for all profiles at the contact place on the pitch circle for the same condition (module, number of teeth …). Then we calculate contact stresses.

In these examples we consider two gears with $m = 2.5, n_p = 20, n_g = 50$ consequently,

$$(d_p)_p = 2.5 \times 20 = 50mm$$

$$(d_p)_g = 2.5 \times 50 = 125mm$$

$$(r_b)_p = 0.5 \times 50 \times \cos 20 = 23.5mm$$

$$(r_b)_g = 0.5 \times 125 \times \cos 20 = 58.73mm$$

These are radiuses of base circles and generator circles. Then we calculate radius of curvature on pitch circle:

Involute:

$$\rho = at$$

$$\Rightarrow t = 20^\circ = \frac{\pi}{9} rad$$

$$\Rightarrow \rho_p = (r_b)_p \times t = 23.5 \times \frac{\pi}{9} = 8.2mm$$

$$\Rightarrow \rho_g = (r_b)_g \times t = 58.73 \times \frac{\pi}{9} = 20.5mm$$

Cycloid:

$$\rho = 4a\sin(t/2)$$

$$y = r_p - r_b$$

$$pinion: y = 25 - 23.5 = 1.5mm$$

$$gear: y = \frac{125}{2} - 58.73 = 3.77mm$$

$$y = a(1-\cos t)$$

$$\Rightarrow \frac{y}{a} = 1 - \cos t \quad \Rightarrow \cos t = 1 - \frac{y}{a}$$

$$\Rightarrow t = \cos^{-1}(1-\frac{y}{a}) \quad \Rightarrow t = 0.36 rad$$

$$\rho = \begin{cases} pinion: 4a\sin(t/2) = 16.83mm \\ gear: 4a\sin(t/2) = 42.08mm \end{cases}$$

Hypocycloid:

$$y = a - R \qquad if: a = 4b$$

$$\rho = \frac{2\sqrt{2} \times 3b^2\sqrt{1-\cos(4t)}}{2b}$$

$$R = \sqrt{x^2+y^2} = b\sqrt{(3\cos t - \cos 3t)^2 + (3\sin t - \sin 3t)^2}$$

$$\Rightarrow R = b\sqrt{10 + 6(\cos t \cos 3t - \sin t \sin 3t)} = b\sqrt{10 + 3\cos 4t}$$

$$y = a - R = 4b - R = b(4 - \sqrt{10+3\cos 4t})$$

$$\Rightarrow t = 0.104 rad$$

$$\rho = \begin{cases} pinion: 29.04mm \\ gear: 72.6mm \end{cases}$$

Epicycloids:

$$y = R - a \qquad if: a = 5b$$

$$\rho = \frac{2\sqrt{2} \times 6b^2 \sqrt{1-\cos(5t)}}{7b}$$

$$R = \sqrt{x^2 + y^2}$$

$$= b\sqrt{(6\cos t - \cos 6t)^2 + (6\sin t - \sin 6t)^2}$$

$$\Rightarrow R = b\sqrt{37 - 12(\cos t \cos 6t + \sin t \sin 6t)}$$

$$= b\sqrt{37 - 12\cos 5t}$$

$$y = R - a = R - 5b = b(\sqrt{37 - 12\cos 5t} - 5)$$

$$\Rightarrow t = 0.066 rad$$

$$\rho = \begin{cases} pinion: 13.21mm \\ gear: 33.03mm \end{cases}$$

At here we consider, $F = 100N$, $l = 20mm$ (width of teeth), $E = 200Gpa$, $\nu = 0.3$. [2]

Involute:

$$b = \sqrt{\frac{2F}{\pi l} \times \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

$$b = \sqrt{\frac{2\times 100}{\pi \times 20 \times 10^{-3}} \times \frac{2(1-0.3^2)/200\times 10^9}{(1/16.4 + 1/41)\times 10^{-3}}}$$

$$= 1.842 \times 10^{-5} m$$

$$P_{\max} = \frac{2F}{\pi b l}$$

$$P_{\max} = \frac{2\times 100}{\pi \times 20\times 10^{-3} \times 1.842 \times 10^{-5}}$$

$$= 172.8 Mpa$$

$$(\sigma_{\max})_{y,z} = 172.8 Mpa$$

$$(\sigma_{\max})_x = 2\times 0.3 \times 172.8 = 103.68 Mpa$$

Cycloid:

$$b = \sqrt{\frac{2\times 100}{\pi \times 20 \times 10^{-3}} \times \frac{2(1-0.3^2)/200\times 10^9}{(1/33.66 + 1/84.16)\times 10^{-3}}}$$

$$= 2.64 \times 10^{-5} m$$

$$P_{\max} = \frac{2\times 100}{\pi \times 20\times 10^{-3} \times 2.64 \times 10^{-5}}$$

$$= 120.57 Mpa$$

$$(\sigma_{\max})_{y,z} = 120.57 Mpa$$

$$(\sigma_{\max})_x = 2\times 0.3 \times 120.57 = 72.34 Mpa$$

Hypocycloid:

$$b = \sqrt{\frac{2\times 100}{\pi \times 20 \times 10^{-3}} \times \frac{2(1-0.3^2)/200\times 10^9}{(1/58.1 + 1/144.8)\times 10^{-3}}}$$

$$= 3.47 \times 10^{-5} m$$

$$P_{\max} = \frac{2\times 100}{\pi \times 20\times 10^{-3} \times 3.47 \times 10^{-5}}$$

$$= 91.73 Mpa$$

$$(\sigma_{\max})_{y,z} = 91.73 Mpa$$

$$(\sigma_{\max})_x = 2\times 0.3 \times 91.73 = 55.04 Mpa$$

Epicycloid:

$$b = \sqrt{\frac{2\times 100}{\pi \times 20 \times 10^{-3}} \times \frac{2(1-0.3^2)/200\times 10^9}{(1/26.42 + 1/66.1)\times 10^{-3}}}$$

$$= 2.34 \times 10^{-5} m$$

$$P_{\max} = \frac{2\times 100}{\pi \times 20\times 10^{-3} \times 2.34 \times 10^{-5}}$$

$$= 136.03 Mpa$$

$$(\sigma_{\max})_{y,z} = 136.03 Mpa$$

$$(\sigma_{\max})_x = 2\times 0.3 \times 136.03 = 81.62 Mpa$$

With attention to result for this example, it is obtained contact stresses for involute profile is maximum, for epicycloid profile more than cycloid profile and hypocycloid profile, hypocycloid profile has minimum value of contact stresses.

V. Solution Examples with ANSYS

In this section we represent results of ANSYS at tables. For these examples we model gear teeth at contact place on the pitch circle with two semi-cylinders with radii equal to radii of curvature and loading is in pressure form on the horizontal surface of each semi-cylinders.

For meshing and generation contact pair we use solid 95, targe 170, conta 174 elements. For generating contact pair we consider larger semi-cylinder as target and smaller semi-cylinder as contact. For solution these examples we use Lagrange or penalty method. Then enter material properties ($E = 200Gpa$, $\nu = 0.3$) [4], [5].

After explaining the method of modeling examples in ANSYS, represent some figure of modeling process and results.

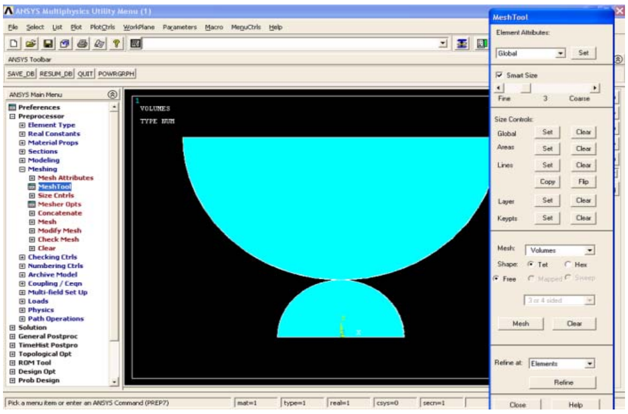

Fig. 5 Meshing model

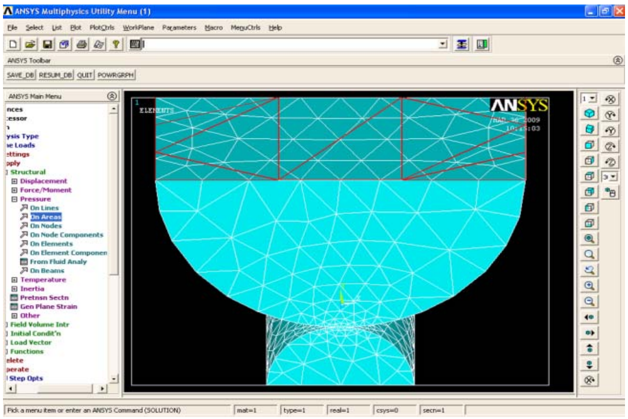

Fig. 6 Imposing pressure

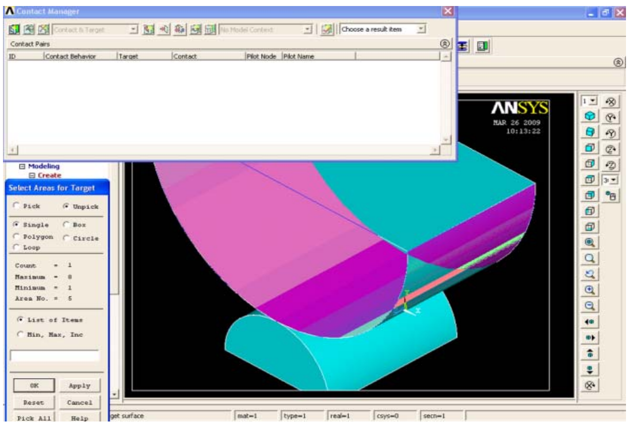

Fig. 7 Creating contact pair

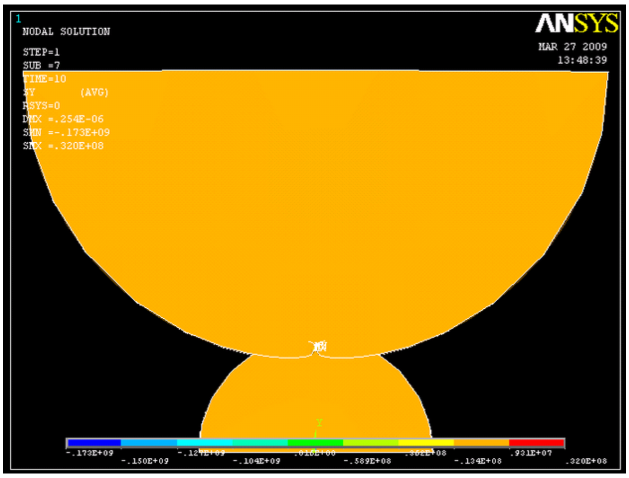

Fig. 8 Stress in vertical direction

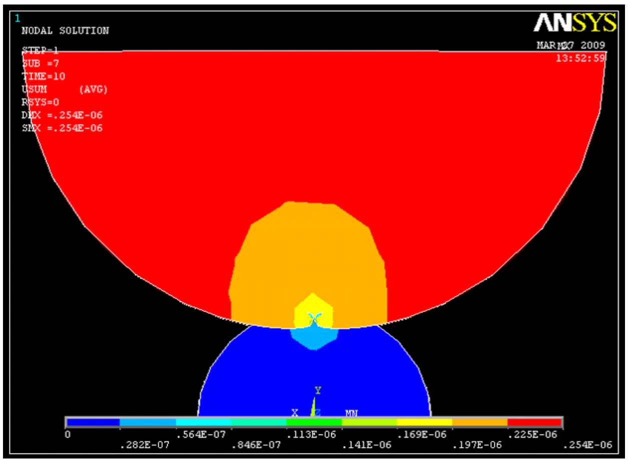

Fig. 9 Displacement in vertical direction

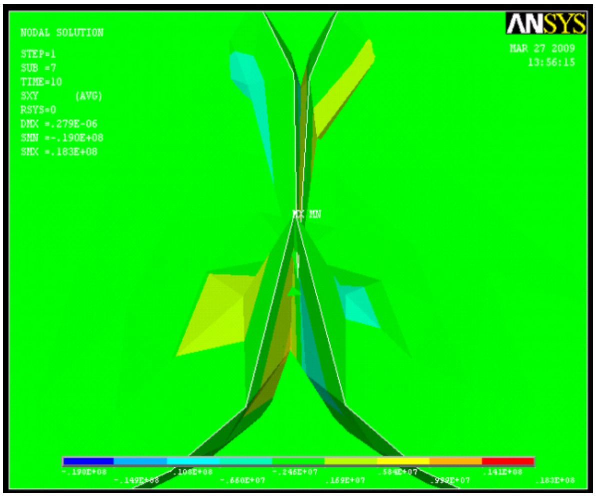

Fig. 10. Stress result

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VI. RESULT

In this section we compare ANSYS results and mathematical results. In order to clear comparing results go in tables.

TABLE I
STRESS IN VERTICAL DIRECTION

profile	Mathematical (Mpa)	ANSYS (Mpa)	Error (percent)
Involute	172.8	175	1.1
Cycloid	120.6	113	6.2
Hypocycloid	136.03	142	4.4
Epicycloid	91.7	94.7	3.2

It is observed that results with a good accuracy are nearly.
Average error = 3.7%

TABLE II
STRESS IN TANGENTIAL DIRECTION

profile	Mathematical (Mpa)	ANSYS (Mpa)	Error (percent)
Involute	172.8	180	4
Cycloid	120.6	122	1.2
Hypocycloid	136.03	125	8
Epicycloid	91.7	86	6.2

Average error = 4.8%

TABLE III
STRESS IN LONGITUDINAL DIRECTION

profile	Mathematical (Mpa)	ANSYS (Mpa)	Error (percent)
Involute	103.7	107	3.2
Cycloid	72.3	66	8.7
Hypocycloid	81.6	85	4
Epicycloid	55	54.1	1.6

Average error = 4.3%

In hertzian equations contact stresses are obtained for cylinders with large length but at here we represent an example with real size so ratio of length to diameter is small and this problem produce some error.

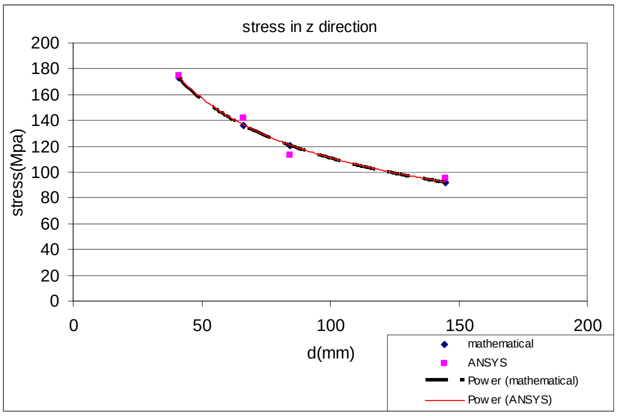

Fig. 11 Stress in vertical direction

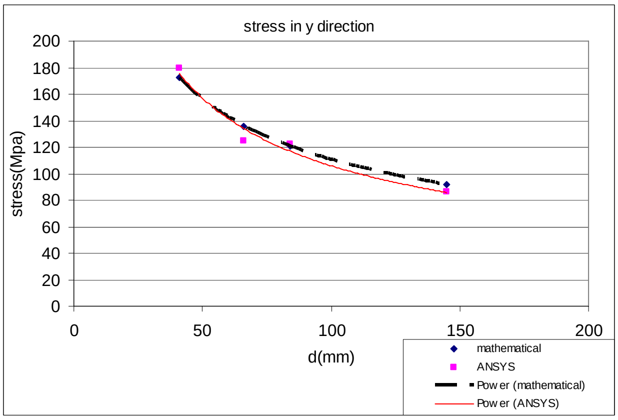

Fig. 12 Stress in tangential direction

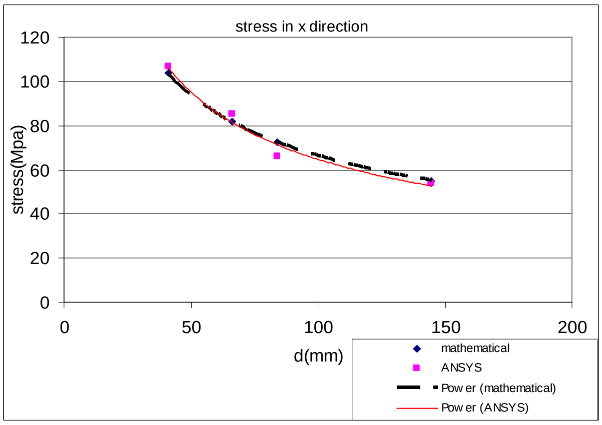

Fig. 13 Stress in longitudinal direction

From diagrams results are close together with a good accuracy.

We observe that by increasing in cylinder's diameter, value of contact stresses decrease. Also by increasing in cylinder's diameter, rate of decrease of contact stresses decreases. We prove this observation in the following.

Derivative of maximum pressure with respect to cylinder's diameter:

In order to pondering effect of cylinder's diameter on contact stresses, we derivative of maximum pressure with respect to cylinder's diameter. We know contact stresses is maximum on the contact surface, also this maximum value relates with maximum pressure.[2].

$$P_{\max} = \frac{2F}{\pi b l} \qquad b = \sqrt{\frac{2F}{\pi l} \times \frac{2(1-\upsilon^2)/E}{1/d_1 + 1/d_2}}$$

$$\frac{\partial P_{\max}}{\partial d_1} = \frac{\partial P_{\max}}{\partial b} \times \frac{\partial b}{\partial d_1}$$

$$= -\frac{E}{4(1-\upsilon^2)} \times \left(\frac{d_2}{d_1 + d_2}\right)^{0.5} \times \frac{1}{d_1^{3/2}}$$

$$\frac{\partial P_{\max}}{\partial d_2} = \frac{\partial P_{\max}}{\partial b} \times \frac{\partial b}{\partial d_2}$$

$$= -\frac{E}{4(1-\upsilon^2)} \times \left(\frac{d_1}{d_1+d_2}\right)^{0.5} \times \frac{1}{d_2^{3/2}}$$

$$d \uparrow \Rightarrow \frac{\partial P_{\max}}{\partial d} \downarrow$$

From above results derivative of maximum pressure with respect to cylinder's diameter has negative mark so by increasing in diameter, maximum pressure and as a result contact stresses decrease. Also, by increasing diameter the value of derivative decreases.

At the end we can say: "In order to decreasing contact stresses we can increase radius of curvature as much as possible."

It is observed that for the same conditions (power transmission, number of gear teeth, and width of teeth…) radius of curvature increases at involute, epicycloid, cycloid, and hypocycloid profile respectively. Consequently, value of contact stresses decrease at involute, epicycloid, cycloid, and hypocycloid profile respectively.

Certainly, these results are obtained for contact stresses, but other parameters such as easy manufacturing and having constant ratio of transmission cause using of involute gear is popular.

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