Conjugate Heat and Mass Transfer for MHD Mixed Convection with Viscous Dissipation and Radiation Effect for Viscoelastic Fluid past a Stretching Sheet

Kai-Long Hsiao, and BorMing Lee

Abstract—In this study, an analysis has been performed for conjugate heat and mass transfer of a steady laminar boundary-layer mixed convection of magnetic hydrodynamic (MHD) flow with radiation effect of second grade subject to suction past a stretching sheet. Parameters E Nr, Gr, Gc, Ec and Sc represent the dominance of the viscoelastic fluid heat and mass transfer effect which have presented in governing equations, respectively. The similar transformation and the finite-difference method have been used to analyze the present problem. The conjugate heat and mass transfer results show that the non-Newtonian viscoelastic fluid has a better heat transfer effect than the Newtonian fluid. The free convection with a larger G_r or G_c has a good heat transfer effect better than a smaller G_r or G_c, and the radiative convection has a good heat transfer effect better than non-radiative convection.

Keywords—Conjugate heat and mass transfer, Radiation effect, Magnetic effect, Viscoelastic fluid, Viscous dissipation, Stretching sheet.

I. Introduction

THE study of visco-elastic fluids had become of increasing importance in the last few years. Qualitative analyses of these studies have significant bearing on several industrial applications such as polymer sheet extrusion from a dye, drawing of plastic films etc. When the manufacturing process at high temperature and need cooling the stretching sheet. The flows may need visco-elastic fluids to produce a good effect to reduce the temperature from the sheet. It is a well-known fact in the studies of non-Newtonian fluid flows by Hartnett [1].

Rajagopal et al. [2] studied a Falkner-Skan flow field of a second-grade visco-elastic fluid. Massoudi and Ramezan [3] studied a wedge flow with suction and injection along walls of a wedge by the similarity method and finite-difference calculations. An excellent review of boundary layers in non-linear fluids was recently written by Rajagopal [4]. These are related studies to the present investigation about second-grade fluids. All of above are dealing with forced convection problems. Recently, Vajravelu and Soewono [5] had solved the fourth order non-linear systems arising in combined free and forced convection flow of a second order fluid, over a stretching sheet. The stretching sheet flow of a non-Newtonian fluid is also one of important flow fields in real world, Raptis [6] had studied heat transfer of a visco-elastic fluid. On the other hand, researches in connection with visco-elastic fluid or second grade non-Newtonian fluids, but there are not the mixed convection flow [7]. Recently, Sanjayanand et al. [8], Cortell, Rafael [9,10] and Seddeek [11] had studied the heat and mass transfer problems about the viscoelastic boundary layer flow over a stretching sheet with magnetic effect. The related boundary layer flow or heat and mass transfer problems are studied by Hsiao et al. [12-13]. From above studies are still not considered the conjugate heat and mass mixed convection with radiation effect.

There are some related conjugate problems concerning a fin in a Newtonian flow, for instance, a complete model study about the forced convection on a rectangular fin has been investigated by Sparrow and Chyu [14]; the effect of the Prandtl number on the heat transfer from a rectangular fin has been studied by Sunden [15]. In addition, Luikov and his co-workers solved the conjugate forced convective problem along a flat-plate both numerically [16] and analytically [17] by Luikov. Lately, relative researches in connection with mixed convection almost all were working for Newtonian fluid [18,19] by Seddeek et al. On the other hand, researches in connection with viscoelastic fluid or second grade non-Newtonian fluids, but there are not the mixed convection flow by Siddheshwar et al. [20,21], therefore the plan proceed especially toward this ways. Hsiao and Chen [22,23] have studied conjugate heat transfer problems about a second grade fluid adjacent to a stretching sheet, but not have toward the conjugate heat and mass transfer for electrical conducting magnetic mixed convection past a stretching sheet. There are some different features than before about the conjugate heat transfer problem, the first difference is the momentum equation

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about the mass transfer force item and the second difference is the mass transfer equation adding into the study. Keeping this in view, the analysis of conjugate heat transfer problem encompasses simultaneous solutions for the heat conduction equation for the fin and the boundary layer equations for the adjacent fluid. By taking the importance of mathematical equivalence of the thermal boundary layer problem with the concentration analogue, results obtained for heat transfer characteristics can be carried directly to the mass transfer by replacing Prandtl number by Schmidt number. In this study, dealing the flow and heat and mass transfer in an incompressible second-grade fluid caused by a stretching sheet with a view to examining the influence of viscous elasticity on flow and heat transfer characteristics of free convection phenomena. The system to analyze in the present study is a stretching sheet in a second-grade viscoelastic fluid flow. The objective of the present analysis is to study the heat and mass transfer of a stretching sheet cooled or heated by a high or low Prandtl-number Pr, the buoyancy parameter Gr and Gc, the magnetic parameter Mn, the radiation parameter Nr and conduction-convection coefficient Ncc, for second-grade viscoelastic fluid. A schematic diagram of the stretching sheet is shown in Fig. 1 to illustrate the physical situation and symbols of parameters needed for the analysis.
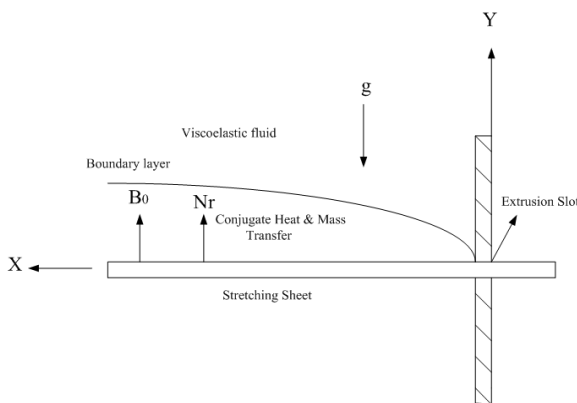

Fig. 1 A sketch of the physical model for conjugate mixed convection heat and mass transfer pass a stretching sheet with magnetic and radiation effects

II. THEORETICAL AND ANALYSIS

An incompressible, homogeneous, non-Newtonian, second-grade fluid having a constitutive equation based on the postulate of gradually fading memory suggested by Rivlin and Ericksen [24] is used for the present flow. The model equation is express as follows:

$$\mathbf{T} = -P\mathbf{I} + \mu\mathbf{A}_1 + \alpha_1\mathbf{A}_2 + \alpha_2\mathbf{A}_1^2 \qquad (1)$$

$\mathbf{T}$ is the stress tensor, P is the pressure, $\mathbf{I}$ is the unit tensor, $\mathbf{A}_1^2$ is a second order tensor, μ is the dynamic viscosity, α_1 and α_2 are first and second normal stress coefficients that are related to the material modulus and for the present second-grade fluid:

$$\mu \geq 0, \quad \alpha_1 > 0, \quad \alpha_1 + \alpha_2 = 0 \qquad (2)$$

The kinematic tensors $\mathbf{A}_1$ and $\mathbf{A}_2$ are defined as:

$$\mathbf{A}_1 = \nabla\mathbf{V} + (\nabla\mathbf{V})^{\mathbf{T}} \qquad (3)$$

$$\mathbf{A}_2 = \frac{d\mathbf{A}_1}{dt} + \mathbf{A}_1\square(\nabla\mathbf{V}) + (\nabla\mathbf{V})^{\mathbf{T}}\square\mathbf{A}_1 \qquad (4)$$

Where $\mathbf{V}$ is velocity and d/dt is the material time derivative. The steady boundary-layer equations for this flow, heat transfer and mass transfer, in usual notations, are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (5)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 v}{\partial y^2} + k_1\left[\frac{\partial}{\partial x}\left(u\frac{\partial^2 u}{\partial y^2}\right) + \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2} + v\frac{\partial^3 u}{\partial y^3}\right] \qquad (6)$$

$$+ g\beta^*(T - T_\infty) + g\beta^{**}(C - C_\infty) - \frac{\sigma B_0^2}{\rho}u$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} + \mu\frac{\partial^2 u}{\partial y^2} + q(T - T_\infty) - \frac{\partial q_r}{\partial y} \qquad (7)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \qquad (8)$$

The well-known Boussinesq approximation is used to represent the buoyancy mixed term. Where u, v are the velocity components in the x and y directions, T is the temperature, g is the magnitude of the gravity, υ is the kinematic viscosity, $k_1 = -\frac{\alpha_1}{\rho}$ is the visco-elastic parameter, β^* is the coefficient of thermal expansion, β^{**} is the concentration coefficient, T_∞ is the temperature of the ambient fluid, ρ is the density, c_p is the specific heat at constant pressure, k is the conductivity, σ is the electrical conductivity, B_0 is the magnetic field factor, q is the specific heat generation rate, D is mass diffusivity, respectively. By using Rosseland approximation the radiation heat flux. is given by $q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}$; Where σ^* and k^* respectively, the Stefan-Boltzmann constant and the mean absorption coefficient. Further we assume that the temperature difference within the flow is such that T^4 may be expanded in a Taylor series. Hence, expanding T^4 about T_∞ and neglecting higher order terms we get

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \qquad (9)$$

The problem for equation (9) can only be used for T_w near-by the ambient temperature T_∞. There will be accuracy problem if $(T_w - T_\infty)/T_w$ is greater than 0.04, since $\Delta\eta = 0.01$, if $(T_w - T_\infty)/T_w = 0.04$ then $(T^4 - 4T_\infty^3 T + 3T_\infty^4)/T_w^4 <= 0.00906$. $(T_w - T_\infty)/T_w$ is greater than 0.04. So that it is one of the similarities transformations restrict. Sometime may

control the $\Delta \eta$ values to obtain a accuracy result. Now using Eqs. (8), (9), Eq. (7) becomes:

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \frac{\partial^2 u}{\partial y^2} + q(T - T_\infty) - \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial^2 T}{\partial y^2}$$

(10)

Both u and T were assumed to be linearly dependent on x. The boundary conditions to the problem for Eqs. (5), (6), (8) and (10) are

$$u = Bx, v = v_w = -(B\upsilon)^{1/2}(m - 1/m),$$ (11)

$$B > 0 \text{ at } y \to 0$$

$$u \to 0, \ \partial u / \partial y \to 0 \text{ at } y \to \infty$$

$$T = T_w = T_\infty + A(\frac{x}{L}) \text{ at } y \to 0$$

$$T \to T_\infty \text{ at } y \to \infty$$

The item $u = Bx$, the velocity component u is assumed linear with x at the boundary, just a simple approach method. Where T_w and T_∞ are constant wall temperature and ambient fluid temperature, A and B is the proportional constant. The item $T_w = T_\infty + A(\frac{x}{L})$ has the relationship about the wall temperature varies along the x with a linear approximation, this is a simplify model. Where $v_w = -(B\upsilon)^{1/2}(m - 1/m)$, m is suction constant and L is the characteristic length, respectively. It should be noted that m>1 corresponds to suction (v_w <0).

Where m<1 corresponds to blowing (v_w >0). In the case when the parameter m=1, the stretching sheet is impermeable. A similarity solution for velocity will be obtained if introduce a set of transformations, such that:

$$u = Bxf'(\eta), \ v = -(B\upsilon)^{1/2}f(\eta),$$ (12)

$$\eta = (B/\upsilon)^{1/2}y$$

Equation (12) has satisfied the continuity equation (5), Substituting (12) into (6), we have:

$$f'^2 - ff'' = f''' + E(2f'f'')$$

$$+E(2f'f''' - f''^2 - ff^{IV}) + G_r\theta + G_c\phi - Mf'$$ (13)

$E = \alpha_1 B / \mu$ is the viscoelastic parameter, $G_r = g_x\beta^*(T_w - T_\infty)/B^2x$ is the free convection parameter, $G_c = g_x\beta^{**}(\phi_w - \phi_\infty)/B^2x$ is the free mass convection parameter and $M = \sigma B_0^2 / \sigma B$ is the magnetic parameter. L is the wall thickness of the stretching sheet. The corresponding boundary conditions become:

$$f=0 \ \ f' = 1 \text{ at } \eta = 0$$ (14)

$$f' \to 0, \ f'' \to 0, \text{ at } \eta \to \infty$$

for the prescribed surface temperature. We introduce the dimensionless temperature $\theta(\eta)$:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad \text{and} \quad k = k_\infty(1 + \varepsilon\theta)$$ (15)

And combine the transformations from equation (12), the energy equation (10) becomes:

$$\theta'' + \frac{3 \Pr N_r}{(3N_r + 4)} f\theta' - \frac{3 \Pr N_r}{(3N_r + 4)} (2f_\eta - a1)\theta$$

$$+ \frac{3 \Pr N_r}{(3N_r + 4)} E_c(f'')^2 = 0$$ (16)

Where $\Pr = \mu c_p / k$ is the Prandtl number, Nr= $\frac{16\sigma^* T_\infty^3}{3k^* k_\infty}$ is the radiation parameter, $E_c = \frac{B^2 t^2}{c_p}$ is the Eckert number, t is the characteristic length of the stretching sheet and $a1 = \frac{q}{\rho c_p B}$ is the heat source/sink parameter. The corresponding thermal boundary conditions are:

$$\theta = 1 \text{ at } \eta = 0$$ (17)

$$\theta \to 0 \text{ as } \eta \to \infty$$

For the solutions of heat and mass transfer equations, it can be defined non-dimensional temperature and concentration variables as:

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}$$ (18)

This leads to the non-dimensional form of temperature and concentration equations as follows:

$$\phi'' + S_c f\phi' - S_c f'\phi = 0$$ (19)

Where Sc=υ/D is the Schmidt number. The corresponding boundary conditions are:

$$\phi = 1 \text{ at } \eta = 0$$ (20)

$$\phi = 0 \text{ as } \eta \to \infty$$

The relationship between, the density ρ and the concentration C is $\rho = -DA\frac{C}{L}$, where D is the diffusing coefficient, A is control surface area, and L is concentration adjacent regions length. In terms of similarity parameters and dimensionless quantities defined by Equations (9) and (12), the heating rate on the wall is:

$$q_w = -k(\frac{\partial T}{\partial y})_{y=0} - \frac{4\sigma^*}{3k^*}(\frac{\partial T^4}{\partial y})_{y=0}$$ (21)

In addition, the local Nusselt number Nu_x is defined by:

$$Nu_x = \frac{hx}{k} = \frac{q_w}{T_w - T_\infty} \frac{x}{k}$$ (22)

This expression has written as:

$$Nu_x = \frac{hx}{k} = \frac{q_w}{T_w - T_\infty} \frac{x}{k} = -\theta'(0)[1 + Nr(\theta(0) + N)^3]G^{1/4}$$ (23)

(Where, $N = \frac{T_\infty}{T_w - T_\infty}$, $G = Gr + Gc$)

The formulation of the first analysis principle for free convection along a stretching sheet involves the energy conservation for the stretching sheet and the boundary layer equations for the flow. For a slender stretching sheet, ample

evidence based on finite difference solutions shows that a one-dimensional model is adequate. The stretching sheet temperature at any x location serves as the wall temperature for the adjacent fluid and has denoted as $T_f(x)$. The energy equation for the stretching sheet may be written in two different forms, depending on how the coupled-fin/boundary-layer problem is solved. The stretching sheet energy equation can be expressed as:

$$\frac{d^2 T_f}{dx^2} = \frac{q}{k_f t} \qquad (24)$$

or

$$\frac{d^2 T_f}{dx^2} = \left(\frac{h}{k_f t}\right)(T_f - T_e) \qquad (25)$$

In which, k_f is the thermal conductivity of the stretching sheet. For the solutions of either equation (24) or (25) at a given cycle of the iterative procedure, h and q can be regarded as known quantities. At first glance, it appears advantageous to use equation (25) rather than equation (24) because it is easier to solve; however, equation (25) has been employed in the solution scheme. Equation (25) recasts in a dimensionless form by the substitutions:

$$X = x/L, \quad Y = y/L, \quad \theta_f = (T_f - T_e)/(T_0 - T_e) \qquad (26)$$

where T_0 is the base temperature of the stretching sheet, so that:

$$\frac{d^2 \theta_f}{dx^2} = \hat{h} N_{cc} \theta_f \qquad (27)$$

with boundary conditions:

$$\theta_f = 1 \ (X=0), \quad \frac{d\theta_f}{dX} = 0 \ (X=1) \qquad (28)$$

where N_{cc} is the conduction-convection number and is defined as:

$$N_{cc} = (kL/k_f t)[1 + Nr(\theta(0) + N)^3]G^{1/4} \qquad (29)$$

The quantity $\hat{h}$ is a dimensionless form of the local convective heat transfer coefficient and can be written as:

$$\hat{h} = \left(\frac{hL}{k}\right)[1 + Nr(\theta(0) + N)^3]^{-1}G^{-1/4} \qquad (30)$$

III. NUMERICAL TECHNIQUE

In the present problem, the set of similar equations (13) to (20) and (27) to (30) are solved by a finite difference method. These ordinary differential equations have discretized by an accurate finite difference method, and a computer program has been developed to solve these equations. A suitable η range and a direct gauss elimination method with Newton's method [25] is used in the computer program to obtain solutions of these difference equations.

Hsiao et al. [26-29] Vajravelu. [30] are also using analytical and numerical solutions to solve the related problems. So, some numerical technique methods will be applied to the same area in the future. In this study, the program to compute finite difference approximations of derivatives for equal spaced discrete data. The code employ centered differences of O(h^2) for the interior points and forward and backward differences of O(h) for the first and last points, respectively. See Chapra and Canale, Numerical Methods for Engineers [31].

IV. RESULTS AND DISCUSSION

The model for grade-two fluids is used in this study. The effects of dimensionless parameters, the Prandtl number (Pr), the magnetic parameter (M), the radiation parameter (Nr), the elastic number (E), the free-convection parameter (Gr), the free-convection mass transport parameter (Gc) are main parameters of the study. Flow and temperature fields of the stretching sheet flow are analyzed by utilizing the boundary layer concept to obtain a set of coupled momentum equation, energy equation and mass equation. A similarity transformation is then used to convert the nonlinear, coupled partial differential equations to a set of nonlinear, coupled ordinary differential equations. A second-order accurate finite difference method is used to obtain solutions of these equations. Table 1 shows that the different values of skin friction $-f''(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\varphi'(0)$ for different values of physical parameters. Table II shows that the different values of physical parameters for Newtonian flow E=0 and for non-Newtonian flow E=5 its $-\theta'(0)$ for different values of physical parameters.

TABLE I

VALUES OF $-f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ FOR DIFFERENT VALUES OF PHYSICAL PARAMETERS E=0.1, Gc=0.5, Pr=1.0, Nr=1

Gr	M	al	Ec	Sc	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.5	0.0	0.0	0.0	0.1	0.3332	0.9684	0.3379
0.0	0.0	0.0	0.0	0.1	0.5564	0.9416	0.3274
0.5	0.0	0.0	0.1	0.5	0.4588	0.9282	0.7792
0.0	0.5	0.2	0.1	0.5	1.0183	0.7491	0.6871
0.2	0.5	0.2	0.0	1.0	0.9748	0.7613	1.0456
0.2	0.5	0.0	0.0	1.0	0.9801	0.8304	1.0435

TABLE II

VALUES OF $-\theta'(0)$ AND $-\phi'(0)$ FOR DIFFERENT VALUES OF PHYSICAL PARAMETERS M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1

$-\theta'(0)$	$-\theta'(0)$	$\hat{h}$	$\hat{h}$
(E=0)	(E=5)	(E=0)	(E=5)
0.9006	1.4833	5.3192	8.7607

TABLE III

VALUES OF $-\theta'(0)$ and $-\phi'(0)$ FOR DIFFERENT VALUES OF PHYSICAL PARAMETERS M=0.1, al=0.1, E=0.1, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1

$-\theta'(0)$	$-\theta'(0)$	$\hat{h}$	$\hat{h}$
(G=1)	(G=5)	(G=1)	(G=5)
0.9122	1.1283	5.3877	6.6640

TABLE IV

VALUES OF $-\theta'(0)$ AND $-\phi'(0)$ FOR DIFFERENT VALUES OF PHYSICAL

PARAMETERS M=0.1, E=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Ec=0.1, Sc=0.1

$-\theta'(0)$	$-\theta'(0)$	$\hat{h}$	$\hat{h}$
(Nr=0)	(Nr=5)	(Nr=0)	(Nr=5)
0.1429	1.2615	0.8440	7.4507

Table III shows that the different values of physical parameters for free convection flow Gr=2.5, Gc=2.5 and for mixed convection Gr=0.5, Gc=0.5 flow its $-\theta'(0)$ for different values of physical parameters. Table IV shows that the different values of physical parameters for non-radiative flow Nr=0 and for radiative flow Nr=5 its $-\theta'(0)$ for different values of physical parameters. Fig. 2 depicts conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0, $\hat{h}$ =25.3192, Ncc=0.1, 0.5, 2. Figure 3 depicts conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0, $\hat{h}$ =8.7607, Ncc=0.1, 0.5, 2. Fig. 2 and 3 show the conjugate stretching sheet temperature distributions. The results obtained from the present computation for different Ncc values and different viscoelastic coefficients E by centered finite difference methods. From the results, we find that the larger Ncc parameters reduce the stretching sheet temperature effect is better than the lower Ncc parameters. On the other hand, compared the two Figs. 2 and 3, find that the larger parameters $\hat{h}$ reduce the stretching sheet temperature effect is better than the lower $\hat{h}$ parameters. From Figs. 2 and 3, we find an important result that the non-Newtonian fluid heat transfer effect is better than the Newtonian fluid flow for about 27% at current conditions, and produce a good effect in heat conduction for about 5% to 10% at these conditions.
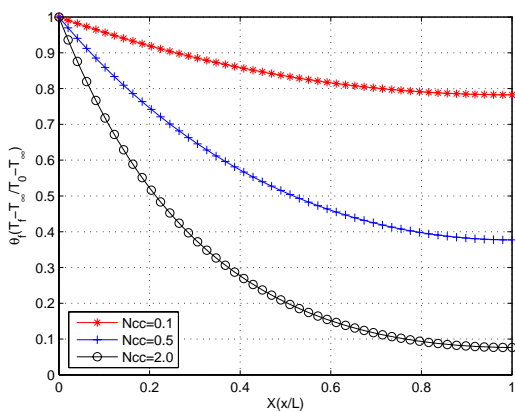

Fig. 2 Conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0, $\hat{h}$ =5.3192

Fig. 4 depicts conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0.1, $\hat{h}$ =5.3877, Ncc=0.1, 0.5, 2.

Fig. 5 depicts conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=2.5, Gc=2.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0.1, $\hat{h}$ =6.6640, Ncc=0.1, 0.5, 2.
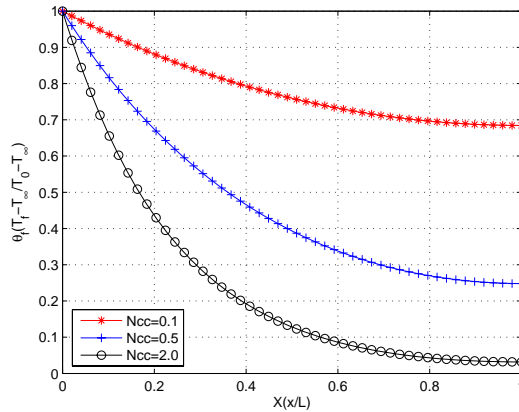

Fig. 3 Conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=5
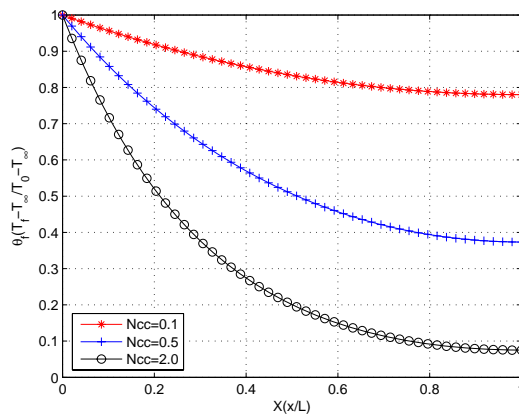
$\hat{h}$ =8.7607

Fig. 4 Conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0.1,
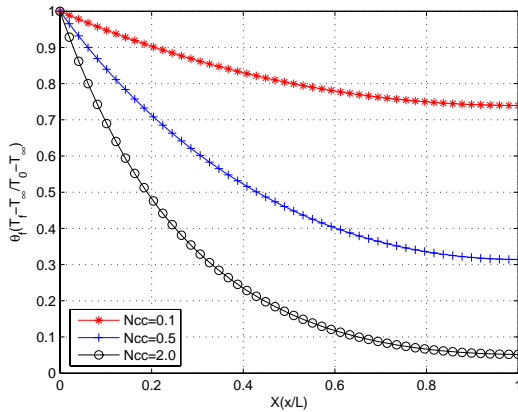
$\hat{h}$ =5.3877

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Fig. 5 Conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=2.5, Gc=2.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0.1, $\hat{h}$ =6.6640

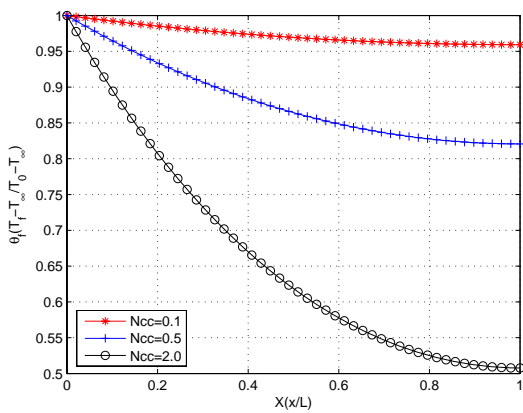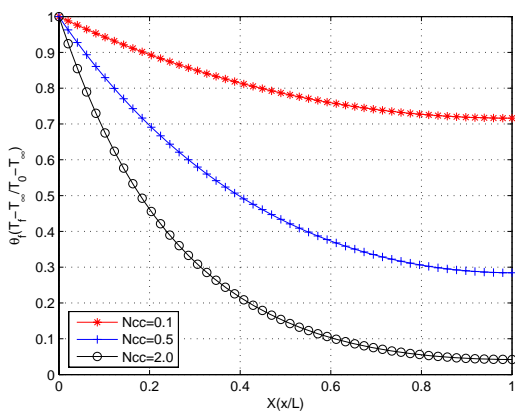Fig. 6 Conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, E=0.1, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, Nr=0, $\hat{h}$ =0.8440

Fig. 7 Conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, E=0.1, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, Nr=5 $\hat{h}$ =7.4507

Figs. 4 and 5 show the conjugate stretching sheet temperature distributions. The results obtained from the present

computation for different Ncc values and different viscoelastic coefficients E by centered finite difference methods. From the results, we find that the larger Ncc parameters reduce the stretching sheet temperature effect is better than the lower Ncc parameters. On the other hand, compared the two Figs. 4 and 5, find that the larger parameters $\hat{h}$ reduce the stretching sheet temperature effect is better than the lower $\hat{h}$ parameters. From Figs. 4 and 5, we find an important result that the free convection fluid heat transfer effect is better than the mixed convection fluid flow for about 23% at current conditions, and produce a good effect in heat conduction for about 3% to 5% at these conditions.

Fig. 6 depicts conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0, $\hat{h}$ =2.0563, Ncc=0.1, 0.5, 2. Fig. 7 depicts conjugate stretching sheet temperature distributions for M=0.1, al=0.1, Gr=0.5, Gc=0.5, Pr=1.0, Nr=1, Ec=0.1, Sc=0.1, E=0, $\hat{h}$ =2.5549, Ncc=0.1, 0.5, 2. Figs. 6 and 7 show the conjugate stretching sheet temperature distributions. The results obtained from the present computation for different Ncc values and different viscoelastic coefficients E by centered finite difference methods. From the results, we find that the larger Ncc parameters reduce the stretching sheet temperature effect is better than the lower Ncc parameters. On the other hand, compared the two Figs. 6 and 7, find that the larger parameters $\hat{h}$ reduce the stretching sheet temperature effect is better than the lower $\hat{h}$ parameters. From Figs. 6 and 7, we find an important result that the radiative convection fluid heat transfer effect is better than the non-radiative convection fluid flow for about 770% at current conditions, and produce a good effect in heat conduction for about 5% to 20% at these conditions.

V. CONCLUSION

The thermal characteristics of conjugate heat and mass transfer on a stretching sheet have been further explored in this study. We have considered a more general variation of the conjugate heat and mass transfer problem than that considered in Refs. [22] and [23]. Nevertheless, the generalized problem admits similarity solutions to be obtained. The numerical solutions of the resulting set of ODEs admit an efficient analysis of the consequences of a stretching sheet. The main conclusions to be drawn are as follows:

1) It is observed that increase in viscoelastic parameter E produces a significant increase in the thickness of the thermal boundary layer of the fluid and so as the temperature increases in presence/absence of thermal conductivity parameter.

2) The effect of Schmidt number Sc on mass transfer process may show that the increase of value of Schmidt number Sc results in the decrease of concentration distribution as a result

of decrease of the concentration boundary layer thickness with the increased values of Sc.

3) The larger Ncc or $\hat{h}$ parameters reduce the stretching sheet temperature effect is better than the lower Ncc or $\hat{h}$ parameters.

4) The larger parameters E reduce the stretching sheet temperature effect is better than the lower E parameters, so that the non-Newtonian viscoelastic fluid has a better heat transfer effect than the Newtonian fluid flow.

5) The larger parameters G reduce the stretching sheet temperature effect is better than the lower G parameters, so that the free convection with a larger G has a better heat transfer effect than a smaller G.

6) The larger parameters Nr reduce the stretching sheet temperature effect is better than the lower E parameters, so that the radiative convection viscoelastic fluid has a better heat transfer effect than the non-radiative fluid flow.

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