Complex Wavelet Transform Based Image Denoising and Zooming Under the LMMSE Framework

T. P. Athira, Gibin Chacko George

Abstract—This paper proposes a dual tree complex wavelet transform (DT-CWT) based directional interpolation scheme for noisy images. The problems of denoising and interpolation are modelled as to estimate the noiseless and missing samples under the same framework of optimal estimation. Initially, DT-CWT is used to decompose an input low-resolution noisy image into low and high frequency subbands. The high-frequency subband images are interpolated by linear minimum mean square estimation (LMMSE) based interpolation, which preserves the edges of the interpolated images. For each noisy LR image sample, we compute multiple estimates of it along different directions and then fuse those directional estimates for a more accurate denoised LR image. The estimation parameters calculated in the denoising processing can be readily used to interpolate the missing samples. The inverse DT-CWT is applied on the denoised input and interpolated high frequency subband images to obtain the high resolution image. Compared with the conventional schemes that perform denoising and interpolation in tandem, the proposed DT-CWT based noisy image interpolation method can reduce many noise-caused interpolation artifacts and preserve well the image edge structures. The visual and quantitative results show that the proposed technique outperforms many of the existing denoising and interpolation methods.

Keywords—Dual-tree complex wavelet transform (DT-CWT), denoising, interpolation, optimal estimation, super resolution.

I. INTRODUCTION

RESOLUTION enhancement of an image plays an important role in many image and video processing applications due to the recent advances in low-cost imaging solutions and increasing storage capacities. An improvement in the spatial resolution for the images directly improves the ability to distinguish important features in images with a better precision. Most of the existing interpolation schemes assume that the original image is noise free. This assumption, however, is invalid in practice because noise will be inevitably introduced in the image acquisition process. Usually denoising and interpolation are treated as two different problems and they are performed separately. However, this may not be able to yield satisfying result because the denoising process may destroy the edge structure and introduce artifacts, which can be further amplified in the interpolation stage.

The interpolation based image resolution enhancement has been used for a long time. Interpolation is a method to increase the number of pixels in a digital image. There are three main linear interpolation techniques namely nearest neighbor, bilinear and bicubic interpolation as in [1]-[3]. Among the linear interpolation methods, bicubic interpolation is more sophisticated than the other two techniques and produces sharper edges. These linear interpolation techniques suffer from some inherent defects, including block effects, blurred details and ringing artifacts around edges [1]. In the linear interpolation methods, filtering is done alternatively in horizontal and vertical directions without considering the edges. In the presence of a sharp edge, if a missing sample is interpolated not along the edge direction, large and visually disturbing artifacts will be introduced [5].

To overcome the drawbacks of linear interpolation methods, nonlinear interpolation techniques such as edge guided interpolation [4], NEDI [5] were presented in this literature. The basic idea of techniques such as edge guided interpolation [4], NEDI [5] were presented in this literature. The basic idea of NEDI is to first estimate local covariance coefficients from a low-resolution image and then use these covariance estimates to adapt the interpolation at a higher resolution based on the geometric duality between the low-resolution covariance and the high-resolution covariance. In edge guided interpolation [4], two observation sets are defined in two orthogonal directions to interpolate a pixel. From each subset, a directional interpolation is made, and then the two interpolated values are fused by the linear minimum mean square-error estimate (LMMSE) of the missing sample.

Wavelets are playing a significant role in many image resolution enhancement applications as compared to spatial domain approaches. In wavelet zero padding (WZP) [7] the HR image is reconstructed using zero padding of high-frequency subbands (i.e. setting all elements of these subbands to zeros) followed by inverse wavelet transform. Hidden Markov tree (HMT) models [8] are used to interpolate images predicting coefficients at finer scales. Generally HMT model suffers due to the sign changes between the scales. The main drawbacks of the above wavelet based methods introduce ringing artifacts in the neighbourhood due to the suppression of wavelet coefficients and it is shift variant [6]. Cycle-spinning has been shown to be an effective method against ringing artifacts when used for resolution enhancement purposes in the wavelet domain [7].

Edge preservation is also a crucial issue in denoising. A general principle is to smooth the noise and interpolate the missing samples along the edge direction, instead of across the edge direction. Although denoising and interpolation have been studied for many years as two independent problems, they can be modeled under the same framework of signal

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estimation. In paper [15], a directional denoising and interpolation algorithm is proposed to estimate both the noiseless and missing samples from the noisy image under the LMMSE framework. For each noisy pixel, a local window centered at it is used to analyze the local statistics. To calculate and preserve the directional information for image interpolation, we first compute multiple estimates of a noisy pixel along different directions. Those directional estimates are then fused for a more accurate output. The data fusion is adaptive to the statistics of the directional estimates to ensure the denoising being along the main direction of the local window. Such a directional denoising process naturally provides a directional interpolator, which consequently enlarges the image to a higher resolution.

The drawbacks of DWT can be overcome by using complex wavelets since it has the properties of shift invariance and good directionality [10], [11]. DT-CWT decomposes an image into two complex-valued low-frequency subband images and six complex-valued high frequency subband images. The high frequency subband images are highly oriented at six different directions which are +75º, +45º, +15º, −15º, −45º, and −75º [12].

Recently, DT-CWT with bicubic interpolation method [9] is introduced for image resolution enhancement. Since it uses linear bicubic interpolation, edge preservation in the resolution enhanced image is poor. To avoid this problem and to use it for noisy images, a dual tree complex wavelet transform (DT-CWT) based directional interpolation and denoising scheme is proposed. Initially, DT-CWT is used to decompose an input low-resolution noisy image into low and high frequency subbands. The high-frequency subband images are interpolated by linear minimum mean square estimation (LMMSE) based interpolation, which preserves the edges of the interpolated images. For each noisy LR image sample, we compute multiple estimates of it along different directions and then fuse those directional estimates to get a more accurate denoised LR image. The estimation parameters calculated in the denoising processing can be readily used to interpolate the missing samples. The inverse DT-CWT is applied on the

denoised input and interpolated high frequency subband images to obtain the high resolution image.

The rest of the letter is organized as follows: Section II introduces the proposed DT-CWT based image denoising and zooming under the LMMSE frame work. Section III provides the experimental results of the proposed approach and comparisons with the existing methods. Section IV concludes this letter.

II. Proposed DT-CWT Based Image Denoising and Zooming under the LMMSE Framework

The main aim of the image resolution enhancement of noisy images is to preserve the edge details. The proposed method is a combination of DT-CWT with LMMSE based image denoising and interpolation which preserves the edge details in the resolution enhanced image. The block diagram of the proposed method is shown in Fig. 1. The algorithm is explained in the following steps.

Step 1. Obtain a low resolution image by low-pass filtering and downsampling of the high resolution image.

Step 2. Perform the Dual Tree Complex Wavelet Transform (DTCWT) on the low resolution image to get low and high frequency subbands.

Step 3. The edge guided interpolation with an interpolation factor of α is applied to the high-frequency sub-band images obtained by DTCWT.

Step 4. The input low resolution image is denoised and interpolated by half of the interpolation factor (α) which is used in the interpolation of the high frequency sub-bands. For each noisy LR image sample, we compute multiple estimates of it along different directions and then fuse those directional estimates for a more accurate denoised LR image. The estimation parameters calculated in the denoising processing can be readily used to interpolate the missing samples as in [15].

Step 5. The inverse DT-CWT is applied on the denoised input and interpolated high frequency subband images to obtain the high resolution image.
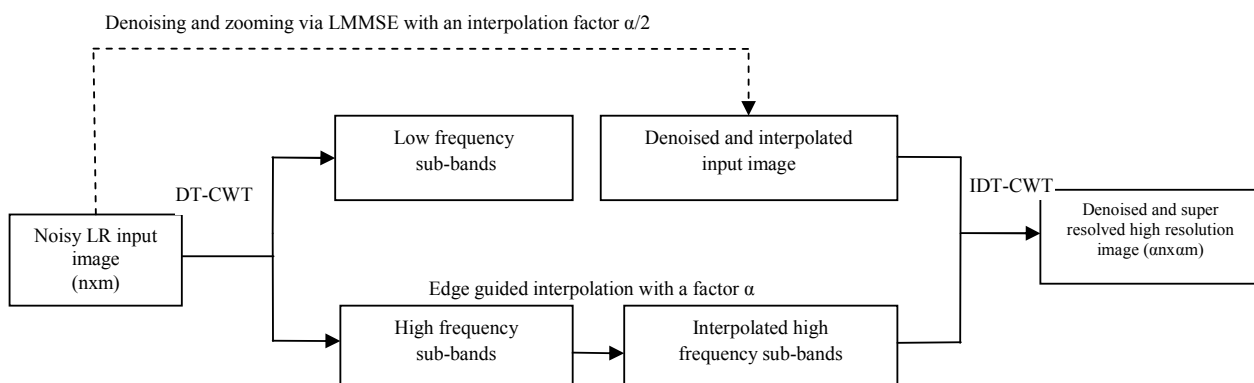

Fig. 1 Block diagram of the proposed denoising and resolution enhancement technique

| (a) | (b) | (c) | (d) |

Fig. 2 (a) Original HR image (b) LR noisy image (c) Denoised and interpolated image via LMMSE (d) Denoised and interpolated image by the proposed scheme

III. RESULTS AND DISCUSSION

This section performs experiments to verify the proposed DT-CWT based directional denoising and interpolation algorithm. For comparison, we employ the sophisticated wavelet based denoising scheme [13] and the anisotropic diffusion denoising scheme [14] to denoise the LR image and then interpolate the denoised images using the state-of-the-art directional interpolation schemes [4], [5] respectively. Four test images, *Lena*, *Butterfly*, *House* and *Peppers* are used in the experiments. The size of all the original HR images is 512×512. In the experiments, the original images are first downsampled to 256×256 and then added Gaussian white noise. Two noise levels with standard deviation σ=15 and σ=25 are tested respectively. Different denoising and interpolation schemes are then applied to the noisy LR image to compute the HR images. In the proposed DT-CWT based directional denoising and interpolation scheme, the

interpolation factor α used is 2. All the methods presented in this letter are implemented in MATLAB 7.0 and run in64-bit Windows 7 with 2.53-GHz Intel Core i3 CPU and 3-GB RAM.

The peak signal to noise ratio (PSNR) is a commonly used metric to evaluate the reconstructed image quality. However, it is well known that PSNR may not be able to faithfully reflect the image perceptual quality. SSIM and MSSIM have been used as new IQA measures in many image processing applications. Here we also employ MSSIM to evaluate the image denoising and interpolation results in the experiments. The PSNR and MSSIM values of the denoised interpolated images by various schemes with σ=15 are listed in Tables I & II. However, from Fig. 2, we see that the proposed method leads to less block effects and ring effects. Since the edge directional information is adaptively estimated and employed in the denoising process and consequently in the interpolation process, the proposed directional joint denoising and interpolation algorithm can better preserve the edge structures. Overall the presented joint denoising and interpolation scheme yields encouraging results.

TABLE I
THE PSNR (DB) RESULTS FOR DIFFERENT DENOISING AND INTERPOLATION SCHEMES

Technique/Image σ =15	Lena	House	Butterfly	Peppers
[13]+[5]	25.91	27.34	23.56	26.07
[14]+[5]	25.43	27.18	23.32	26.02
[13]+[4]	26.01	27.58	23.69	26.28
[14]+[4]	25.87	2744	23.51	26.14
[15]	26.25	28.65	24.86	27.23
Proposed method	**27.96**	**29.98**	**26.57**	**28.92**

TABLE II
THE MSSIM RESULTS FOR DIFFERENT DENOISING AND INTERPOLATION SCHEMES

Technique/Image σ =15	Lena	House	Butterfly	Peppers
[13]+[5]	0.9286	0.9399	0.9568	0.9384
[14]+[5]	0.9275	0.9378	0.9554	0.9376
[13]+[4]	0.9311	0.9404	0.9579	0.9418
[14]+[4]	0.9302	0.9401	0.9571	0.9422
[15]	0.9398	0.9408	0.9699	0.9415
Proposed method	**0.9556**	**0.9589**	**0.9758**	**0.9603**

IV. CONCLUSION

This paper presented a DT-CWT based directional interpolation scheme for noisy images. Unlike the conventional schemes that perform denoising first and interpolation later, the proposed method treats both denoising and interpolation as an estimation problem and implements them under a unified framework of linear minimum mean square-error estimation (LMMSE). The proposed algorithm has been tested on four different test images and compared with five existing image denoising and interpolation techniques. The experiment results show that the proposed method outperforms the conventional and state-of-the-art image resolution enhancement methods for noisy images in terms of visual clarity and edge preservation capability.

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