International Journal of Earth, Energy and Environmental Sciences
ISSN: 2517-942X
Vol:6, No:8, 2012

Comparison of Stochastic Point Process Modelsof Rainfall in Singapore

Y. Lu, X. S. Qin

Abstract**—**Extensive rainfall disaggregation approaches have been developed and applied in climate change impact studies such as flood risk assessment and urban storm water management.In this study, five rainfall models that were capable ofdisaggregating daily rainfall data into hourly one were investigated for the rainfall record in theChangi Airport, Singapore. The objectives of this study were (i) to study the temporal characteristics of hourly rainfall in Singapore, and (ii) to evaluate the performance of variousdisaggregation models. The used models included: (i) Rectangular pulse Poisson model (RPPM), (ii) Bartlett-Lewis Rectangular pulse model (BLRPM), (iii) Bartlett-Lewis model with 2 cell types (BL2C), (iv) Bartlett-Lewis Rectangular with cell depth distribution dependent on duration (BLRD), and (v) Neyman-Scott Rectangular pulse model (NSRPM). All of these models werefitted using hourly rainfall data ranging from 1980 to 2005 (which was obtained from Changimeteorological station).The study results indicated that the weight scheme of inversely proportional variance could deliver more accurateoutputs for fitting rainfall patterns in tropical areas, and BLRPM performedrelatively better than other disaggregation models.

Keywords**—**rainfall disaggregation, statistical properties, Poisson processed, Bartlett-Lewis model, Neyman-Scott model

I. Introduction

RAINFALL is one of the most important inputs for hydrological modeling, and indicators forclimate change impact studies. The rainfall data at finer timescales is generally more useful for hydrological process research, especiallyin the field ofextreme rainfall-event evaluation and flood risk management [1].Inthe world wide, rainfall data are usually available at timescales of daily or monthly. The short interval records are limited due to various reasons such as costlydata procurement andcomplex geographical conditions. Over the past decades, a large variety of disaggregation methods were proposedand usedto provide possible realization of hourly data which were aggregated up to the given daily data. Rodriguez-Iturbe et al. [2] first introduced the temporal rainfall models based on the clustered Poisson process.Based onsuch atheory, rainfall event was divided into a cluster of rain cells, where the temporal location of the cells relative to the event origin was specified by either Bartlett-Lewis orNeyman-Scottclustering mechanism [3]. The cluster models weresuitedin summarizing rainfall statistical characteristicsby a group ofparameters and regenerating the hierarchical rainfall structures conveniently [3].

Y. Lu is with the School of Civil & EnvironmentalEngineering,Nanyang Technological University, 50 Nanyang Ave., 679798 Singapore (e-mail: ylu11@e.ntu.edu.sg).

X. S. Qin is with the School of Civil & Environmental Engineering and Earth Observatory of Singapore (EOS), Nanyang Technological University, Singapore (*Correspondence author e-mail: xsqin@ntu.edu.sg).

Rodriguez-Iturbe et al. [1], [4], [5] described three models, including rectangular pulse Poisson model (RPPM),Bartlett-Lewis rectangular pulse model (BLRPM) and Neyman-Scott rectangular pulse model (NSRPM),to represent rainfall intensity wherethe clustering effect of rainfall process was accounted for. BLRPM has many modifications, such as the modified Bartlett-Lewis model (MBL) [6], Bartlett-Lewis model with 2 cell types (BL2C), and Bartlett-Lewis Rectangular with cell depth distribution dependent on duration (BLRD) [7].Extensions to NSRPM include specialization of raincell [8] anda spatial-temporal NSRPM [9] etc. NSRPM was alsousedinRainSim software [10]. However, there were limitedstudies for comparing multipleversions of BLRPM and NSRPM.

Manyof the above-mentioned models have been successful appliedin US, Canada andUK,at different temporal scales and ranges;whereas,very limited studies were reportedfor Southeast Asia.One attempt was made byHanaish et al. [11], where three versions of Bartlett-Lewis rectangular pulse models were applied in Malaysia. Singapore isa typicalregion withtropical climate. Characterized byheavy rainfallswithshortdurations, itsoverall climatic features is important to the global climate change studies.Moreover, for evaluating the suitability of current storm water management systemsunder changing climatic conditions,the rainfall patterns in finer time resolutions for future conditions will need to be investigated. In this study, five models that could disaggregaterainfall fromdaily to hourly time series were evaluated. The rainfall dataset of Changi meteorological station, which is located in the eastern part of Singapore, was chosen for demonstration. We also compared the fitted precision using differentweight schemes inthe process of parameter estimations.

II.Model Description

(1) *Rectangular pulse Poisson model (RPPM).*The RPPM was developed with the following assumption. The occurrence of rainfall events follows the Poisson process as follows[4].

$$P_i = \frac{\lambda^i e^{-\lambda}}{i!}, i = 0, 1, 2, \cdots \quad (1)$$

where e is the base of the natural logarithm, i is the number of occurrences of an event, λ is a rate parameter (a positive real number) which is equal to the expected number of occurrences during the given time interval. The mean and standard deviation of the single pulse intensity are denoted by parameters$\mu_x$and σ_x, respectively. The corresponding quantity for duration L is denoted as μ_L . The RPPM can be characterized by a set ofparameters: (λ, μ_x, σ_x, μ_L). Readers are referred to Rodriguez-Iturbe et al. [4] for details.

(2) *Bartlett-Lewis rectangular pulse model (BLRPM).* This cluster-based modelwasproposed by Rodriguez-Iturbe et al. [2]. For BLRPM, the intervals between successive cells are independent and identically distributed [4]. The rainfall arrivesin a Poisson process ata rate of λ. Each rain cell also follows a Poisson distributionat a rate of β. In BLRPM, the rainfall and cell durations are by defaulttaken as having exponentialdistributions, anddenoted by parametersγ and η, respectively [5]. The mean cell intensity distribution is characterized by μ_x. The BLRPM can therefore be described by a set of parameters (λ, β, μ_x, γ, η). The related theorywas fully describedby Rodriguez-Iturbe et al. [4]. The explanatory sketches of BLRPM were shown in Figure 1(a).

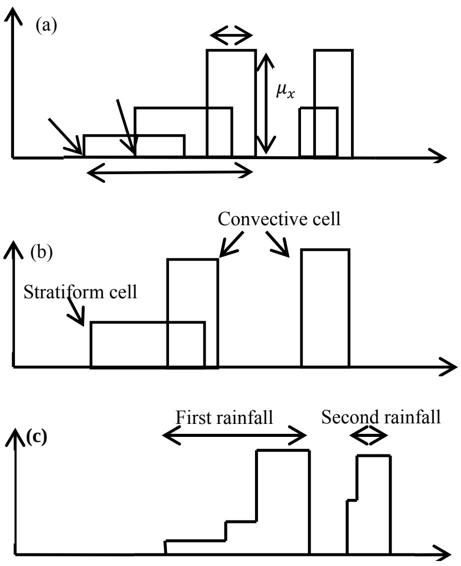

Fig. 1 Explanatory sketch of (a) BLRPM, (b) BL2C and (c) NSRPM [4] [5] [11]

(3)*Bartlett-Lewis model with 2 cell types (BL2C).* The original Bartlett-Lewis model only shows one type of rain cell within the same rainfall event. However, observational studies on precipitation field have shown that multipletypes of cells may exit in one storm, such as heavy convective rain cell and light stratiform rain cell. They are featured by: (i) two random variables for the intensity distribution $\{x_i, i = 1,2\}$, with means $\{\mu_{x_i}, i = 1,2\}$ and mean square intensities $\{\mu_{x_i^2}, i = 1,2\}$; (ii) two duration distributions with exponential parameters $\{\eta_i, i = 1,2\}$; (iii) two probabilities $\{\Psi_i, i = 1,2\}$ of occurrences for cell type, $\sum_{i=1}^{2} \Psi_i = 1$. This model is defined by 9 parameters, i.e. (λ, μ_{x_1},μ_{x_2}, η_1, η_2, Ψ_1, Ψ_2, β, γ) [11]. The explanatory sketch of cell types are shown in Figure 1(b).

(4)*Bartlett-Lewis Rectangular with cell depth distribution dependent on duration (BLRD).* The dependent depth-duration model can introduce dependence between cell intensity and cell duration distribution to improve the wet-dry properties [12]. The parameter set of (λ, β, γ, η) of BLRPM are also available for this model, representing the rainfall arrival, cell arrival, cell duration and cell duration, respectively. But the cell intensities X are now specified through the distribution of X conditional upon the cell duration$L$. The conditional mean and standard deviation of cell intensity limit for 0 cell duration are denoted by $\mu_{x|0}$and $\sigma_{x|0}$. The mean durations of cell and rainfall event are distributed with parameters δ_c and δ_s, where$\delta_c = 1/\eta$ and $\delta_s = 1/\gamma$. The mean number of cells in each rainfall event is denoted by μ_c, and $\mu_c = 1 + \beta/\gamma$. Therefore, the mechanistic parameters for BLRD are (λ, μ_x, $\mu_{x|0}$, μ_c, σ_x, $\sigma_{x|0}$, δ_c, δ_s) [12].

(5)*Neyman-Scott rectangular pulse model (NSRPM).* In this model, the position of cells isdetermined by a set of independent and identically distributed random variables, where the location of this distribution is given by the rainfall origion [5]. The parameters of λ, β, γ, and ηalsorepresent the rainfall arrival, cell arrival, rainfall duration and cell duration, respectively. The number of cells per rainfall event is an independent random variable, where the correspondingmean is μ_c.The explanatory of NSRPM is depicted in Figure 1(c).

Among the above-mentioned models, RPPM is the fundamental form for all other versions that are characterized as rectangular pulses models. BL2C and BLRD are modified versions based on BLRPM. Both BLRPM-type models and NSRPM are empirical and it is normallydifficult to simply choosewhich one is superiorforspecific applications [4].

III.Study area and data Input

In this study, the hourly rainfall datasetbased on ground observation from Changimeteorological station was used.The station is located in the eastern part of Singapore andthe period of the dataset is from 1980 to 2005. Singapore's climate is classified as tropical rainforest climate (*Koppen climate classification Af*), without obvious seasonal variations. The annual rainfall normally exceeds 2300*mm*[13].

There are two monsoon seasons for Singapore. The Northeast Monsoon occurs from December to early March,and Southwest Monsoon season occurs from June to September [14]. Based on rainfall levels, the wet season is from November to January and the rest months are relativelydry. The December and February are the wettest and driest month per year, respectively. Therefore, these two months will be chosen as typical months to fit different weight schemes whenestimating parameters.Since both months are in the Northeast Monsoon,we also takeJune into consideration for comparison, as it falls in the Southwest monsoon which has relatively constant rainfall patterns.

IV.Parameter estimation and Model fitting

The parameters are estimated based on the method of moments. Specifically, let $T = (T_1, T_2, \cdots)$bethe vector of the observed rainfall summary statistics, and let $\tau(\theta) = [\tau_1(\theta), \tau_2(\theta), \cdots]$ be the expected value of T according to the model, θ is a parameter vector. The fundamentalmethod of moments is to choose the bestθthat can minimize a quadratic function [11]:

$$S(\theta) = [T - \tau(\theta)]' W [T - \tau(\theta)] \quad (2)$$

Where $W$is the weight matrix.

International Journal of Earth, Energy and Environmental Sciences
ISSN: 2517-942X
Vol:6, No:8, 2012

TABLE I

THE OBSERVED AND FITTED RAINFALL STATISTICAL PROPERTIES FOR FEBRUARY UNDER TWO WEIGHT SCHEMES

Properties	Timescales	Observed	RPPM		BLRPM		BL2C		BLRD		NSRPM	
			W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2
Mean	1 hour	0.163	0.163	0.163	0.163	0.187	0.162	0.163	0.162	0.163	0.163	0.151
Variance	1 hour	2.638	1.899	1.937	2.449	2.168	2.429	2.144	2.422	2.145	2.448	2.173
	6 hour	29.872	36.707	32.105	33.236	31.484	32.844	32.154	32.611	32.200	33.273	30.612
	24 hour	143.783	197.603	158.716	151.208	144.124	169.685	153.233	172.408	152.950	152.225	158.168
Lag-1 autocorrelation	1 hour	0.268	0.292	0.332	0.269	0.273	0.266	0.253	0.272	0.274	0.271	0.278
	24 hour	0.080	0.009	0.011	0.079	0.079	0.070	0.076	0.084	0.077	0.076	0.083
Probability of wet	1 hour	0.046	0.043	0.040	0.046	0.047	0.046	0.046	0.046	0.046	0.046	0.048
	24 hour	0.306	0.314	0.345	0.306	0.263	0.306	0.306	0.307	0.307	0.307	0.328

TABLE II

THE OBSERVED AND FITTED RAINFALL STATISTICAL PROPERTIES FOR DECEMBER UNDER TWO WEIGHT SCHEMES

Properties	Timescales	Observed	RPPM		BLRPM		BL2C		BLRD		NSRPM	
			W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2
Mean	1 hour	0.418	0.418	0.518	0.417	0.414	0.418	0.423	0.467	0.494	0.418	0.517
Variance	1 hour	6.185	3.650	2.127	5.214	4.631	5.330	4.588	7.019	6.951	5.220	2.127
	6 hour	67.084	78.127	55.207	75.414	75.709	75.901	78.296	61.087	52.195	77.298	55.207
	24 hour	390.982	454.415	470.450	403.761	408.910	377.871	387.395	343.920	359.788	365.269	470.450
Lag-1 autocorrelation	1 hour	0.280	0.328	0.331	0.283	0.286	0.285	0.283	0.285	0.282	0.278	0.286
	24 hour	0.182	0.011	0.011	0.180	0.181	0.183	0.181	0.188	0.178	0.177	0.182
Probability of wet	1 hour	0.131	0.127	0.145	0.131	0.130	0.131	0.131	0.137	0.132	0.131	0.145
	24 hour	0.624	0.637	0.472	0.624	0.638	0.624	0.622	0.535	0.501	0.624	0.472

TABLE III

THE OBSERVED AND FITTED RAINFALL STATISTICAL PROPERTIES FOR JUNE UNDER TWO WEIGHT SCHEMES

Properties	Timescales	Observed	RPPM		BLRPM		BL2C		BLRD		NSRPM	
			W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2	W_1	W_2
Mean	1 hour	0.174	0.175	0.175	0.174	0.202	0.174	0.175	0.175	0.188	0.174	0.197
Variance	1 hour	2.818	2.873	2.858	2.814	1.805	2.401	1.600	2.296	1.946	2.058	1.771
	6 hour	25.472	24.337	25.977	25.587	22.660	28.005	21.443	28.193	23.711	28.492	22.223
	24 hour	114.008	101.939	109.901	123.358	136.471	123.430	147.371	126.001	136.599	131.830	139.496
Lag-1 autocorrelation	1 hour	0.228	0.235	0.284	0.228	0.228	0.226	0.225	0.228	0.229	0.228	0.229
	24 hour	0.018	0.007	0.009	0.018	0.018	0.013	0.017	0.022	0.018	0.018	0.018
Probability of wet	1 hour	0.047	0.045	0.039	0.047	0.050	0.047	0.047	0.046	0.052	0.045	0.050
	24 hour	0.423	0.434	0.494	0.423	0.315	0.423	0.422	0.426	0.313	0.434	0.322

The Nelder-Mead optimization algorithm can be used to minimize the objective function [11]. The observed rainfall summary statistics applied in the study are: 1-hour mean (Mean1), 1-hour variance (Var1), 6-hours variance (Var6), 24-hours variance (Var24), 1-hour autocorrelation of lag 1 (Ac1), 24-hours autocorrelation of lag 1 (Ac24), 1-hour probability of wet (Pwet1), and 24-hours probability of wet (Pwet24).

These indicators were fully discussed by Chandler et al. [15], [16] and Hanaish et al. [11]

V. RESULTS AND DISCUSSIONS

Tables I to III provided the disaggregation analysis of February, December, and June; theyarerepresentative forthe dry, wet, and medium seasons, respectively.To compare the fitness of models, we employed two weight schemes to obtain different rainfall statistical indicators. The first weight scheme is based on the diagonal element, expressed as $w_i = 1/var(T_i)$, where $var(T_i)$ is the variance of the related monthly statistics across all years.The second weight scheme is given by $w_i = [1/\overline{T_i}]^2$, where $\overline{T_i}$ is the mean of the related monthly statistics across all years [11]. In the tables, $w_1$represents the weight from the inversely proportional variance and $w_2$represents the weight from inversely proportional mean squared.

From Table I, the value of variance in timescales of 6-hour and 24-hour are both over-estimationsfor February. BL2C and BLRD have a higher fitting resultin estimating variances of 6-hour and 24-hour underw_1 weight scheme compared with those under w_2. All models perform wellin fitting1-hour mean.From Table 2, all models show a lower accuracy under w_2 scheme forDecember.

Generally, the fitted values of 24-hour variance are much less accurate than those of other time resolutions.This may due to the fact that a larger value of time interval leads to smaller weight in the object function.All versions of Bartlett-Lewis model have better performance than Neyman-Scott model. This could be the reason that Bartlett-Lewis models assume independent and identically distributed intervals between consecutive rain cells, which may bemore suitable indescribinglocal rainfall patterns.Due to the Southwest Monsoon influence, the rainfall variation is smoother from June to September than those in other months. From Table 3, it appears that the performance of fitness is better than other two monthsin terms of Var6 and Var24; this probably because the rainfall distribution is more homogeneous in June.

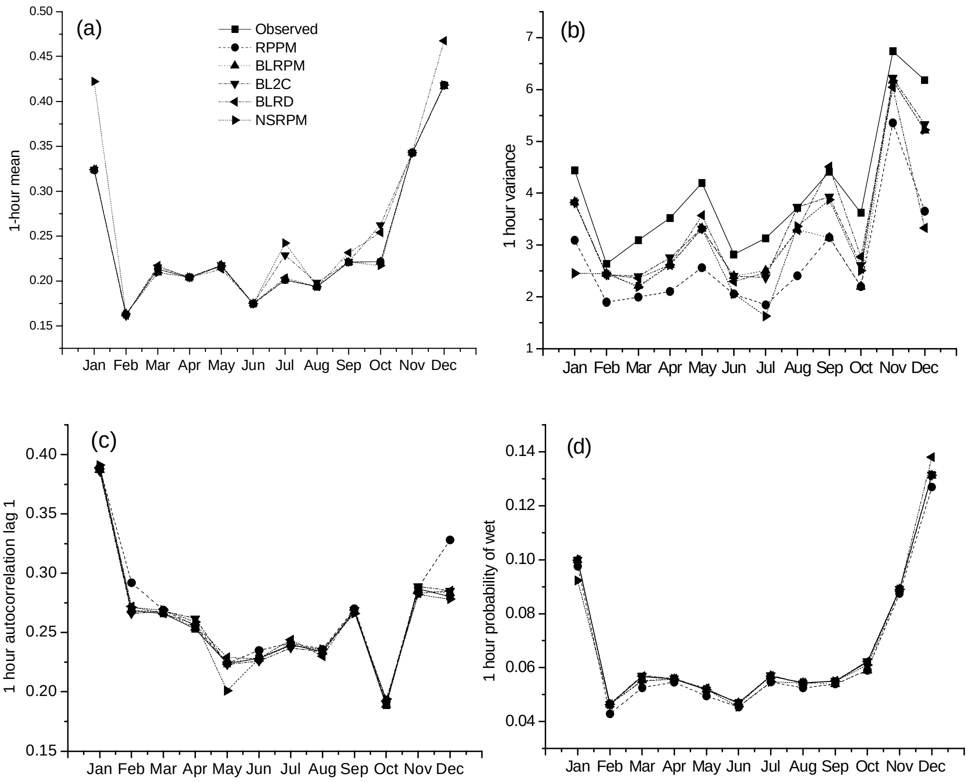

Fig. 2 Comparison between observed and fitted rainfall statistical properties under w_1 weight scheme.(a) Mean1, (b) Var1, (c) Ac1, (d) Pwet1.

From Tables 1 to 3, w_1 weight scheme is considered relatively superior than $w_2$one for most models in terms of 1-hr statistics.Figure 2 shows the performance of five models under w_1 weight scheme for 1-hr statistics over all months. The values of Mean, Ac1 and Pwet1 all fit very well (mean relatively error < 5%).The 1-hr variance has more significant variations, as its value is generally bigger than other indicators anda lower weight would be assigned tothe objective function. Rainfall in Singapore is characterized by convective type cell, which demonstratingheavy rainfalls in short durations. Therefore, BLRPM which only allow for one type of rectangular pulse is more suitable for application in Singapore than BL2C,which represents 2 types of cell.

Another finding is that RPPM is relatively more accurate than others in estimating the 1-hour mean values, especially under w_1 weight scheme.

VI.CONCLUSION

The temporal characteristics of hourly rainfall in Changi meteorological station of Singapore were investigated. The studyresults indicated that thecharacteristics of rainfall in Singaporewere featured bylarge amounts in short duration, which belongs toconvective rain. The performances of different disaggregation models were also evaluated.In terms ofthe short timescales properties, all Bertlett-Lewis models have good performances.

Generally,comparing the resultsfrom different models under two weight schemes, BLRPM has a higher accuracy than other modelsunder both weight schemes; the scheme with weight of inversely proportional variance performs better than that from inversely proportional mean squared.Infurther studies, the description of extremes values from different models needs to be improved.

Acknowledgment

This research was supported by Earth Observatory of Singapore Project (M4080707.B50), and Singapore's Ministry of Education (MOM) AcRF Tier 1 Project (M4010973.030).

References

[1] C. G. Kilsby,A. Butron,S. J. Birkinshaw,A. M. Hashemi, P. E. O'Connell,"Extreme rainfall and flood frequency distribution modeling for present and future climates,"Proceedings of the British Hydrological Society Seventh National Hydrology Symposium, 2000, pp. 3.51-3.56.

[2] I. Rodriguez-Iturbe, V. K. Gupta, E. Waymire,. "Scale consideration in the modeling of temporal rainfall,"Water Resources Research 20 (11), 1984, 1611-1619.

[3] C. Onof,R. E. Chandler, A. Kakou,P. Northrop,H. S. Wheater,V. Isham, "Rainfall modeling using Poisson-Cluster processes: a review of developments,"Stochastic Environmental Research and Risk Assessment 14(6), 2000, 384-411.

[4] I. Rodriguez-Iturbe,D. R. Cox,V. Isham,. "Some models for rainfall based on stochastic point processes,"Proceedings of the Royal Society of London, Series A 410, 1987a, 269-288.

[5] I. Rodriguez-Iturbe,B. Febres de Power,J. B. Valdes,"Rectangular pulses point process models for rainfall: analysis of empirical data,"Journal of Geophysical Research 92, 1987b, 9645-9656.

[6] A. N. Engida, M. Esteves,"Characterization and disaggregation of daily rainfall in the Upper Blue Nile Basin in Ethiopia,"Journal of Hydrology 399, 2011, 226-234.

[7] C. Onof,"DEFRA Project: Improved methods for national spatial-temporal rainfall and evaporation modeling for BSM," 2003,unpublished.

[8] P. S. P. Cowpertwait, "A generalized point process model for rainfall,"Proceedings of Royal Society of London 447, 1994, 23-27.

[9] P. S. P. Cowpertwait,"A generalized spatial-temporal model of rainfall based on a clustered point process,"Proceedings of Royal Society of London 450, 1995, 163-175.

[10] A. Burton,C. G. Kilsby,H. J. Fowler, P. S. P. Cowpertwait, P. E. O'Connell,"RainSim: A spatial-temporal stochastic rainfall modeling system,"Environmental Modelling & Software 23, 2008, 1356-1369.

[11] I. S. Hanaish, K. Ibrahim,A. A. Jemain,"Stochastic modeling of rainfall in Peninsular Malaysia using Bartlett-Lewis rectangular pulse models,"Modelling and Simulation in Engineering, 2011.

[12] A. Kakou, "Point process based models for rainfall," Ph.D. thesis, University of London, 1997, unpublished.

[13] Meteorological Services Division, National Environmental Agency, http://www.nea.gov.sg

[14] Geography of Singapore, WIKIPEDIA, http://en.wikipedia.org/wiki/Geography_of_Singapore

[15] R. Chandler,G. Lourmas,J. Jesus, "Software for moment-based on fitting of single-site stochastic rainfall models,"2010, unpublished.

[16] J. Jesus and R. Chandler, "Estimating functions and the generalized method of moments," Interface Focus, 1(6), 2011, 871-885.