

# Comparison between Pushover Analysis Techniques and Validation of the Simplified Modal Pushover Analysis

N. F. Hanna, A. M. Haridy

**Abstract**—One of the main drawbacks of the Modal Pushover Analysis (MPA) is the need to perform nonlinear time-history analysis, which complicates the analysis method and time. A simplified version of the MPA has been proposed based on the concept of the inelastic deformation ratio. Furthermore, the effect of the higher modes of vibration is considered by assuming linearly-elastic responses, which enables the use of standard elastic response spectrum analysis. In this thesis, the simplified MPA (SMPA) method is applied to determine the target global drift and the inter-story drifts of steel frame building. The effect of the higher vibration modes is considered within the framework of the SMPA. A comprehensive survey about the inelastic deformation ratio is presented. After that, a suitable expression from literature is selected for the inelastic deformation ratio and then implemented in the SMPA. The estimated seismic demands using the SMPA, such as target drift, base shear, and the inter-story drifts, are compared with the seismic responses determined by applying the standard MPA. The accuracy of the estimated seismic demands is validated by comparing with the results obtained by the nonlinear time-history analysis using real earthquake records.

**Keywords**—Modal analysis, pushover analysis, seismic performance, target displacement.

## I. INTRODUCTION

STANDARD pushover analysis (POA) is a simple technique that has been effectively used to estimate the seismic demands of frame buildings. It is a static non-linear analysis method based on applying a selected lateral load pattern, which represents the equivalent lateral forces due to earthquake, along the height of the building frame, and the building is pushed laterally until a pre-defined failure state is reached. The expected damage in the main structural members is modeled using plastic hinges placed at strategic locations along the member. The main drawbacks of the standard POA are its inability to consider the effect of the higher modes of vibration and its inability to consider the changes in the lateral stiffness properties of the building as damage progresses during the pushover. The first drawback related to the higher vibration modes effect is deemed more important in practice.

Several advanced methods have been proposed in literature to account for the effect of the higher modes of vibration. The most popular method is known by the Modal Pushover Analysis (MPA). The lateral load patterns used in the MPA

are based on the mode shapes of vibration of the building frame. The main difficulty of the MPA is that it requires nonlinear time-history analysis of an equivalent single-degree-of-freedom system. This may not be practical in a design office, as typical engineers are not familiar with this advanced type of analysis. So, a simplified MPA (SMPA) method has been developed to overcome this difficulty. This simplified method utilizes the concept of the deformation ratio which relates the inelastic deformations of SDF system to the corresponding elastic deformations that can be directly estimated using the regular response spectrum analysis. Recent expressions for the deformation ratio will be presented and applied in the simplified POA.

POA can be either force-controlled or displacement-controlled. In force-controlled POA, the entire lateral load up to failure is known and hence applied along the height of the frame building. In force-controlled POA, some numerical problems that affect the accuracy of results may occur since the target drifts may be associated with a very small positive or even a negative lateral stiffness. On the other side, displacement-controlled POA is based on using a known lateral load pattern, but not the actual values of the loads till failure as mentioned earlier. This pre-selected lateral load pattern is using to push the building until a pre-defined failure state or target drift state has been reached. At this state, the actual lateral load can be determined from the static nonlinear analysis.

One of the main steps in the POA is the construction of the global capacity curve, which represents the nonlinear relation between the base shear and the global drift at the top of the building. After applying the SMPA method, the performance state of the building due to the given earthquake can be identified, and the associated seismic responses are determined: target drift, base shear, interstory drifts, and distribution of the plastic hinges. A good estimation of the building drifts is desirable for better judgment of the expected damage in the building due to the earthquake. So, the drift results obtained from the SMPA are compared with both the results determined by the MPA and by the non-linear time-history analysis using a set of real and artificial earthquake records. POA is becoming the preferred tool for seismic performance and evaluation of frame buildings, and several modern seismic design codes, e.g. Eurocode 8, have included provisions for performing static nonlinear analysis.

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## II. STANDARD PUSHOVER ANALYSIS

**Aspects of Pushover Analysis**—To appraise the seismic performance of existing building and newly designed buildings the static POA method is applied, which is a non-linear static analysis method. For performance based seismic design (PBSD) buildings are designed to perform according to a specific performance objective based on its function and type.

The POA method is considered to be a powerful tool, even though it still has no strict theoretical basis. As for the POA, there are three main assumptions that it relies on; the first mode of vibration is what controls the seismic response of the building, secondly through the elastic and inelastic response of the building the lateral load pattern remains constant, the third assumption is the relativity between the single degree of freedom system to the multi degree of freedom system.

What recent studies on POA have shown is that in case where the first mode of vibration dominates the responses, these assumptions can produce good evaluation of the seismic responses of building. In the other words, an approximate analysis method in which the building is subjected to monotonically expanding later forces with an invariant distribution over the height of the building till a pre-determined or target displacement is come, is what is known as the standard POA. For approximating the nonlinear relation between the base shear and the roof displacement of the building a nonlinear static analysis is required. A model for the building is built in the POA. First, gravity loads are implemented, afterwards it is applied the selected later load pattern onward the building height. To approximately perform the relative inertia actions developed throughout an earthquake, excitation is the main purpose and objective of this lateral load pattern. Till the displacement at the top of the building achieves a certain value or a breakdown mechanism evolves for the building, the later strengths are raised. Subsequently, the capacity of the building can be tested before and after yielding.

**Elastic Vibration Properties**—For demonstrating the reinforced concrete (RC) building frames, SAP200 is utilized as a standard structural analysis program. In order to represent the beams and columns of the RC frame, 2-D frame components are used and where two nodes situated at the ends of the element portray each frame elements. Every node has three degrees of freedom; one rotational degree of freedom for in-plane bending, and two translational degrees of freedom through the vertical and horizontal directions. For deciding the vibration properties of the RC building frames, these 2D frames are utilized like the time of vibration, the modal participation mass ratios, the modal participating factors, and the mode shapes of vibration.

The satisfactory number of vibration modes required for the examinations of the frames must guarantee that no less than 90% of the total seismic weight is taking in consideration in the analysis; this was suggested by numerous latest seismic regulations. In general, to acquire the dynamic qualities of seismic analysis which mode shapes, participation factor, mass participation ratio and time period are involved. Fig. 1 shows

the first three modes of vibration.

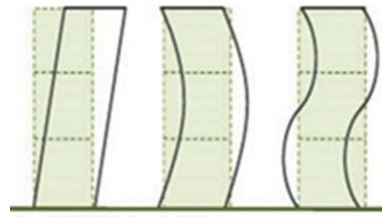


Fig. 1 First, second, and third modes of vibration

**Limitations**—Taking in consideration that POA is a nonlinear static analysis, it needs numerous features of its dynamic counterpart which might be basic in certain analysis cases. Nonetheless, it delivers the engineers with a practical option to the Non-Linear Time History Analysis (NLTHA) and demonstrates standard seismic codes of practice with basic tools for enhancing the seismic design of structures.

In the following, the major limitations of the standard pushover techniques are listed:

- It is simulated that the structural harm is a function of just the lateral deformation of the building. By this way, the earthquake duration impact neglecting duration impacts and the cumulative energy dissipation demand is neglected. Harm is a duty of both deformation and energy, and this was generally acknowledged. So, particularly for non-ductile structures which display pinched hysteretic behavior, the applicability of POA is slightly simplistic.
- Many researches recognized that the independent reactions between the structural capacity and the earthquake demand are suggested to be interconnected. In addition, the division between the loading input and structural reaction is not regularly sufficient because the nonlinear structural behavior is dependent on the load path.
- It was not accounted that progressive determination of the stiffness of the building because of the yield and harm evolving in the building prompts to period elongation. This is because of the invariant lateral load pattern utilized, which disregards the redistribution of the inertia forces as yielding rules the inelastic behavior of the building.
- The strain energy of the structures throughout a monotonic static loading is what is only concentrated on. Different sources of energy that are connected with the dynamic parts of forces are dismissed for example as the viscous damping energy and the kinetic energy.
- The impacts of the higher-modes on the seismic reactions and demands of the structures are not represented. If there should be an occurrence of mid-to high structures then the contribution of the higher-modes might be of great impact.

When accomplishing a POA, it was tried by [1] to recognize some potential pitfalls. Therefore, ten significant conditions that are supposed to be taken into consideration before the POA were outlined; the following are the most vital ones:

- The gravity loads and the shear failure mechanisms are not supposed to be neglected.
- Before the building is pushed, the performance purpose is supposed to be identified
- The underestimation of the loading shape function is not supposed to happen
- The P-Delta impacts should be included, because if not then the outcomes gained by the pushover could be non-conservative.
- As three-dimensional structures may require more than a one planar POA, then the pushover loading should not be mistaken for the actual earthquake.

A few progressed pushover techniques have been created to manage the limitations of the standard POA, due to the complexity of the nonlinear dynamic analysis contrasted with the simplicity and practicality of the standard POA. The theory that the seismic reaction of the building is monopolized by the primary mode of vibration is one of these limitations. In the case of low-rise structures, this presumption is real and rational, and great results are given by the standard pushover procedures. While modes of vibration higher than the fundamental mode have a significant impact that cannot be ignored in the case of mid-rise and high-rise structure.

The adequate number of modes that is supposed to be incorporated to accomplish acceptable exactness to be all modes with cumulative seismic mass more than 90% of the total seismic mass of the building is characterized by numerous updated seismic regulations like [2], [3]. Few pushover procedures have been created for the objective of incorporation the impact of the higher modes of vibrations.

### III. ADVANCED PUSHOVER TECHNIQUES

**Modal Pushover Analysis**— There are some limitations for the standards POA, which gave reasons for a more advanced pushover technique. This technique is considered to be more conservative when compared to the standard techniques either in its concept of the idea or in its application. What [4], [5] did is that, without losing the simplicity of the standard POA, the limitation of the unique vibration mode that commanded the behavior of the structure was considered. In order to reach this objective, the known and familiar modal analysis approach was joined with the POA for evolving the MPA which can generally be described as a more progressed pushover procedure. According to the limitations and shortcomings of the mentioned Standard POA, a MPA developed by [6]. MPA main upgrades are:

1. Taking the effect of higher modes into consideration.
2. Account progressive drop in stiffness as a result of yielding response of structure leading to period elongation.

The involvement of the higher modes will affect the seismic response especially in the mid-rise and high-rise buildings. Using nonlinear dynamic analysis is a procedure in the MPA to consider the effects of higher modes.

#### A. Steps of MPA

After the model is constructed the procedures of the MPA

as follow:

#### 1. Vibration Analysis

- Compute vibration properties of building using Sap2000 by choosing the number of modes required for the effective modal weight to vibrate, which is given to be more than 90% according to [7].
- Get natural frequencies  $\omega_n$  or natural periods  $T_n$

$$\omega = \frac{2\pi}{T} \quad (1)$$

- Get modes  $\Phi_n$ , for linearly elastic vibration of the building.

#### 2. Application of Gravity Load

- Assign Gravity load (Dead and Live). Using load combinations:

$$*U = 1.4 \text{ D.L} + 1.6 \text{ L.L} \quad (2)$$

$$*U = 1.12 \text{ D.L} + \Omega \text{ L.L} + \text{E} \quad (3)$$

$$*U = 0.8 (1.4 \text{ D.L} + 1.6 \text{ L.L} + 1.6 \text{ W.L}) \quad (4)$$

#### 3. Selection of Lateral Load Pattern

- Get mass (seismic weight/gravity) matrix using one of these methods:

##### I. Modal load in Sap2000

**OR**

##### II. Hand Calculations as follow for the rigid diaphragm

$$\begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & m_n \end{bmatrix} = [M] \text{ where } m_i = W_i/G, n = \text{floor}$$

- Get Mode shape vector using Sap2000

$$\{\Phi_j\} = \begin{bmatrix} \Phi_{1-1} \\ \Phi_{2-1} \\ \vdots \\ \Phi_{n-1} \end{bmatrix}$$

- Lateral Load Pattern  $\{f_j\} = [M] \{\Phi_j\}$  (5)

$$\{f_j\} = \begin{bmatrix} m_1 \Phi_{1-1} \\ m_2 \Phi_{2-1} \\ \vdots \\ m_n \Phi_{n-1} \end{bmatrix}$$

#### 4. Apply Selected Lateral Load Pattern in the Building

- Assign the values for the concentrated lateral load pattern forces per floor using the previous vector  $\{f_j\}$ .
- Develop the pushover curve for the whole building, Base Shear-Roof Displacement  $V_{by}-\Delta$  using nonlinear static analysis in Sap2000.

#### 5. Transform the Capacity Curve of SDOF System to Bilinear Using Standards by FEMA356

- Calculate the Area of the Capacity Curve “Actual Area” using Simpson’s rule  $A_a$ .

$$A_a = \int_0^{D_u} F(d) dd \approx \frac{\Delta D}{3} [F(D_0) + 4F(D_1) + 2F(D_2) + 4F(D_{n-1}) + 2F(D_{n-2}) \dots + F(D_n)] \quad (6)$$

Assuming yielding force “ $F_{by}$ ” that does not exceed the ultimate force.

- The slope of the first linear segment is taken as the same slope of the line connecting the origin point and 0.6 from the yielding point ( $0.6F_{by}$ ).

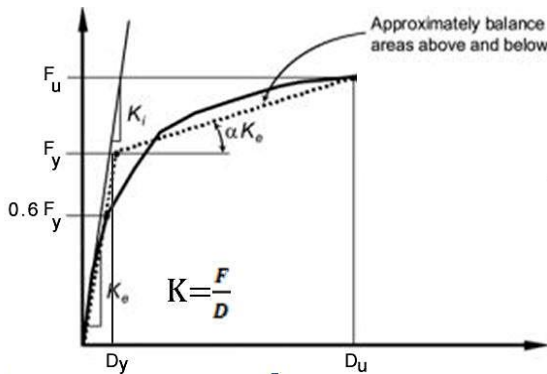


Fig. 2 Conversion of capacity curve to bilinear curve

- The slope of the second segment is determined by connecting the yielding force “ $F_y$ ” with the ultimate force “ $F_u$ ” from the actual curve.
- Calculate the area under the bilinear curve “Bilinear Area”

$$\frac{1}{2} A_{bi} = (F_u + F_y) / 2 \cdot \frac{1}{2} D_y F_y + (F_u + F_y) / 2 \cdot (D_u - D_y) \quad (7)$$

- Checking that  $(A_{bi}) = \text{Area of actual curve } (A_a)$ , if not change assumed  $F_y$ .

#### 6. Capacity Curve “F-D”

Transform the pushover curve to the force–deformation curve “capacity curve” of an equivalent SDOF system:

$$* D = \Delta / r \Phi_r \quad * F_{sy} / L_n = V_{by} / M^* = \omega^2 D_y$$

$\Delta$  = the roof drift of the building,  $\Phi_r$  = the ordinate of the first mode shape at roof of the building  $F_{sy}$  = the yield force of the equivalent SDOF system.

$$L_n = \Phi_r^* m l$$

$V_{by}$  = the base shear of the building at yield,  $M^*$  = the effective modal mass,  $\omega$  = the natural vibration frequency for the equivalent,

SDOF system;  $D_y$  = the yield displacement of the equivalent SDOF system,  $r_n$  = the modal participation factor and is calculated as follow:

$$r_n = (\{\Phi_n\}^T [M] \{1\}) / (\{\Phi_n\}^T [M] \{\Phi_n\}) \quad (8)$$

$$[1] \text{ is a vector of unit values } \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, T = 2\pi\sqrt{D/F}$$

#### 7. Compute the Peak Deformation $D_n$ for each mode

- Using the stiffness of spring “ $K_i$ ” from the bilinear curve.
- Apply Earthquake using Sap2000.
- Get the peak inelastic displacements or deformation of the equivalent SDOF “ $D_n$ ” using nonlinear dynamic analysis.

#### 8. Transform the Equivalent SDOF System Back

- This displacement of the SDOF system is transformed back to its corresponding target displacement of the building

$$* \Delta = D_r \Phi_r \quad (9)$$

$$* V_{by} = F_{sy} M^* \quad (10)$$

#### 9. Get the Desired Responses

- From the modal pushover steps (step 3), extract values of desired responses ( $r_n$ ) such as floor drifts, story drifts and plastic hinge rotations due to the combined effects of gravity and lateral loads at roof displacement equal to  $\Delta$

$$\text{Summation of all modes } r = \sum \sqrt{r_n^2} \quad (11)$$

- Repeat steps for as many modes as required for sufficient accuracy.

**Advantages and Disadvantages**— The advantages of the MPA can be summarized in the following points:

- The technique is based on a well-known structure dynamic theory giving it a reliable basis.
- It requires only one or two modes to be in a good accuracy which is a small number compared to all of the vibration modes of a building.
- Taking into consideration the effect of higher modes.
- Elastic-strain hardening instead of elastic-perfectly plastic providing better accuracy.
- The method can be used in both new as well as existing buildings.

For the MPA, there are also disadvantages to be mentioned, such as:

- Complexity of determining SDOF inelastic peak response using nonlinear dynamic analysis.
- Requiring of the NLDA for SDOF system which can be done without using the MPA from the beginning consuming more time.
- Neglecting the coupling happening between modes during inelastic response of structures.

The change in vibration shapes and stiffness is ignored by using invariant load patterns.

- The MPA requires as much standard POA as many the modes are involved.

- The MPA uses approximate rule to get the peak responses (the-square-root-sum-of-squares “SRSS”)

**SMPA**—After the development of the MPA, some limitations were found in the method. However, the main disadvantage of the MPA was the time consumption using the NLDA in the steps. In order to minimize the computational effort and time consumption, SMPA was firstly introduced by [8] assuming that the higher modes cause only elastic responses to the building.

**Concept of Deformation Ratio**—Deformation ratio concept has been under considerable investigations; in order to perform the NLDA for the determination of peak elastic displacement of the equivalent SDOF system. The concept depends on relating the inelastic displacement  $u_p$  with the elastic displacement  $u_{ep}$  with a deformation ratio factor as given by (12).

$$C = \frac{u_p}{u_{ep}} \quad (12)$$

where  $u_p$  = the peak inelastic displacement;  $u_{ep}$  = the peak elastic displacement and  $C$  = the deformation ratio.

Therefore, to find the value of the deformation ratio “ $C$ ”, an NLDA should be performed calculating both the elastic and inelastic peak displacements. The value of  $C$  is determined by applying earthquake acceleration-time history, in which the value can be affected by the earthquake characteristics. As a result, [9] studied the effect of some parameters such as rapture distance, site classes, earthquake magnitude, and near fault condition on the computed value of  $C$ .

Reference [9] introduced two empirical equations to compute the value of the deformation ratio based on SDOF system represented by elastic-strain hardening. The two equations depend on different factors, the first for systems with known ductility  $C_\mu$  as shown in (13), while the second for systems with known yield-strength reduction factor  $C_R$  as shown in (13). The median values of  $C$  are for any ground motion ignoring values for

$$C_\mu = 1 + [(L_\mu - 1)^{-1} + (\frac{a}{\mu^b} + c)(\frac{T}{T_c})^d]^{-1} \quad (13)$$

$a$ ,  $b$ ,  $c$ , and  $d$  are constants determined from regression analysis = 105, 2.3, 1.9 and 1.7.  $T$  = the elastic natural vibration period, while  $T_c$  = the period separating acceleration sensitive and velocity sensitive region.  $L_\mu$  = the deformation ratio  $C_\mu$  but for zero period systems and is given by

$$L_\mu = \frac{\mu}{1 + (\mu - 1)\alpha} \quad (14)$$

where  $\alpha$  is the ratio between post-yield stiffness to the elastic stiffness of the system, while the second relation is used for systems with known strength reduction factor as:

$$C_R = 1 + [(L_R - 1)^{-1} + (\frac{a}{R^b} + c)(\frac{T}{T_c})^d]^{-1} \quad (15)$$

$a$ ,  $b$ ,  $c$ , and  $d$  = 61, 2.4, 1.5, and 2.4 while  $L_R$  equals:

$$L_R = \frac{1}{R} (1 + \frac{R-1}{\alpha}) \quad (16)$$

**Steps of SMPA**—The SMPA is introduced as an upgrade or improved version for the MPA which is based on NLDA. These are the steps for the SMPA using SAP2000 as a modeling tool. From step 1 to step 6, it is the same steps as the steps of MPA discussed previously. Then, by applying earthquake response spectrum for the SDOF system, the peak elastic displacement  $U_{ep}$  is computed. For the inelastic response of the SDOF system, the empirical relation of the deformation ratio provided is used for this point, in (9) and (10.) For the MDOF system, the target drift at the roof of the building  $\Delta_p$  for each vibration mode can be calculated from the peak response of the equivalent SDOF system for each mode separately as:

$$\Delta_p = \Gamma \phi_r u_p \quad (17)$$

Moreover, all quantities of interest such as story drifts, plastic hinge rotations, and floor drifts can be extracted from the model at roof displacement ( $\Delta$ ). Therefore, the total peak response ( $r$ ) can be computed using SRSS rule for each  $j$ -vibration mode as shown in (18)

$$r = \sqrt{\sum_{j=1}^J r_j^2} \quad (18)$$

**Advantages and Disadvantages**—As the SMPA is thought-out as an advancement of the MPA (MPA) so the SMPA has many advantages compared to MPA, these advantages are:

- SMPA does not require performing any sort of pushover dynamic analysis, as it exposes the complication of determining the SDOF inelastic peak reaction by presenting the proximate idea of the inelastic deformation ratio.
- This estimated deformation ratio is resolved such that its lowest value is equivalent to unity; for long period building frames, this is constant with the conservative approximation of the equivalent displacement rule. Therefore, it is exacted that the drift outcomes are going to be on the conservative side, which for practical purposes is useful
- SMPA corresponds with the way of utilizing the design response spectrum suggested by the late worldwide and local seismic regulations.
- SMPA, utilizing its actual response spectrum and the design response spectrum also suggested by seismic regulations, can be utilized for each individual earthquake.

Nonetheless SMPA gains some limitations of the MPA some of these are:

- SMPA join distinctive modes contributions relying on the square root of sum squares (SRSS) combination rule, which rejects the algebraic sings.
- The plastic hinge rotations or other localized demands may not be assessed precisely. Therefore, it is suggested to utilize the proposed process in the estimation of the

probable plastic hinge area just lowering their numerical qualities.

- Because SMPA utilizes invariant lateral load pattern, accordingly it discards the collective harm happening in the structure. It is hard to fix this limitation without losing the common sense of the proposed technique, therefore this limitation stays unfixed.

#### IV. MODELING AND COMPARISON BETWEEN TECHNIQUES

**Model Description and Characteristics**—In order to compare between the previous different techniques a model is setup. This model is a 9-story steel structure; this building was designed by Brandow and Johnston Associates as a typical medium-rise building in Los Angeles, California.

**Building structure**— The elevation of the structure consists of moment-resisting frames as shown in Fig. 3. The plan of the evaluation model building is a symmetric square, 45.73 m each side as shown in Fig. 4; it is divided into five steel beams in each direction. The elevation of the structure consists of moment-resisting frames as shown in Fig. 3, whereas the dimensions of the building are described as the following: basement level height is 3.65 m, ground level height is 5.49 m, and from the 1<sup>st</sup> to the 8<sup>th</sup> floors are 3.96 m each. The connections of the columns are indicated using this sign  $\nabla$  in Fig. 4 and located at 1.83 m from the beam-column joint. Concrete foundation walls and surrounding soil are assumed to restrain the structure at the ground level from horizontal displacement [10].

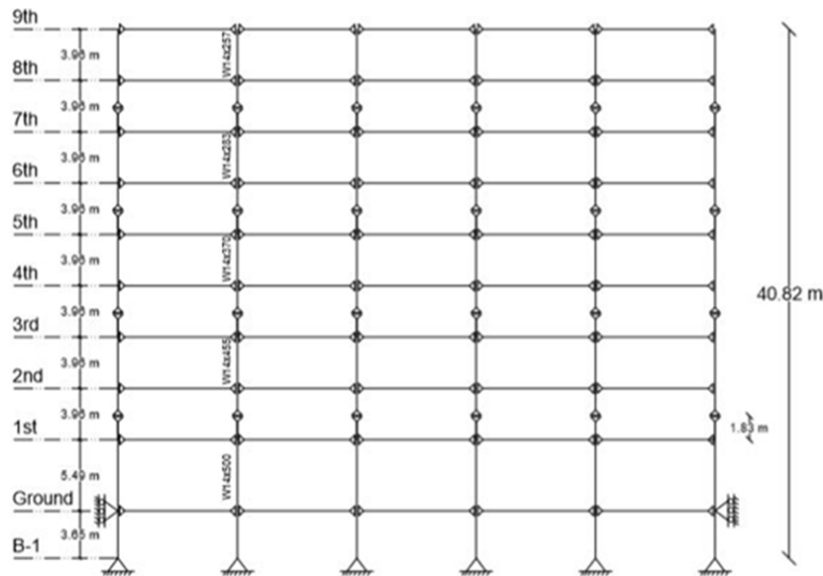


Fig. 3 Model building elevation

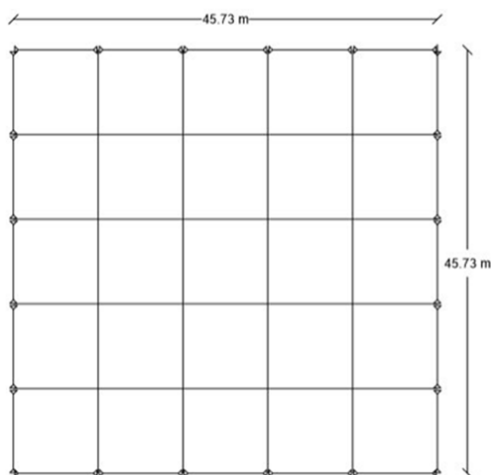


Fig. 4 Model building plan

**Sections Properties**—The section properties of the beams are as follow:

- From ground to the 2<sup>nd</sup> level: W36x160

- 3<sup>rd</sup> to 6<sup>th</sup>: W36x135
- 7<sup>th</sup> level: W30x99
- 8<sup>th</sup> level: W27x84
- 9<sup>th</sup> level: W24x68

While the maximum stress for all of the beams is 248 MPa. For the columns, the connections and the sections location are shown in Fig. 4. While the stress for all of the columns are the same, which is 345 MPa.

**Analysis Parameters**—The seismic masses of the building including all of its components were given to be  $9.65 \times 10^5$  kg for the ground level,  $1.01 \times 10^6$  kg for the 1<sup>st</sup> level,  $9.89 \times 10^5$  kg from the 2<sup>nd</sup> to the 8<sup>th</sup>,  $1.07 \times 10^6$  kg for the 9<sup>th</sup> level, and for the entire structure above ground is  $9.00 \times 10^6$  kg. It is mentioned that each frame resists only half of the seismic mass associated with the entire structure. EL Centro, Loma Prieta, and Northridge earthquakes were selected for the evaluation of the structure.

**Modeling using SPA**—To use the standard POA, the model is to be constructed on SAP2000 [11], then choose a load pattern to work on. According to the literature review, the

inverted triangular load pattern is the most accurate. Fig. 5 shows the chosen load pattern in the analysis. Then, the load pattern is used to push the building till reaching the maximum displacement observed from the nonlinear dynamic analysis.

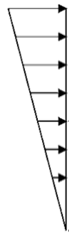


Fig. 5 Inverted triangle load pattern

**Modeling using MPA**—Firstly, start applying the steps mentioned above previously, after constructing the model on SAP2000. To get the load pattern for the analysis, a modal analysis is required as in [12]. While the force at each floor equals the mode shape component multiplied by the modal mass. Fig. 5 shows the mode shape components for the first three modes; however, the maximum component is unity for the multiplication simplicity. The distribution of the load patterns on the floors for each mode is according to the previous mode shapes and the modal masses which are given to be the seismic mass  $\times 0.5$  as it is said to be one half and converted from kilograms to Kips. $\text{sec}^2/\text{inch}$  as shown in Fig. 6. Then, the assigning of the plastic hinges for the beams as shown in Fig. 3 is required for the analysis. The next step from [13] is running the model for the three modes, while SAP2000 generates the pushover curves automatically as shown in Figs. 8-10.

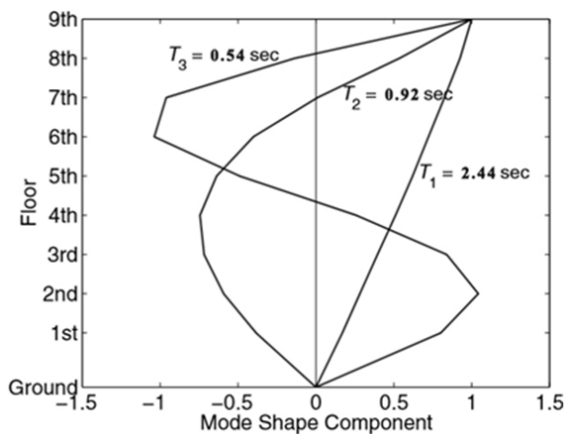


Fig. 6 Mode shape component per floor for each mode shape

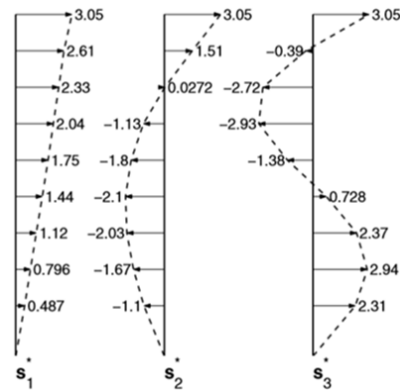


Fig. 7 Load pattern distributions for each mode

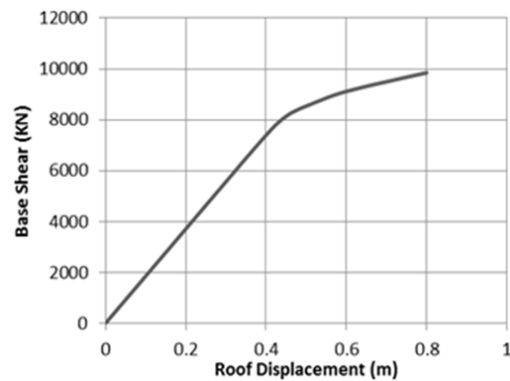


Fig. 8 Pushover curve for mode 1

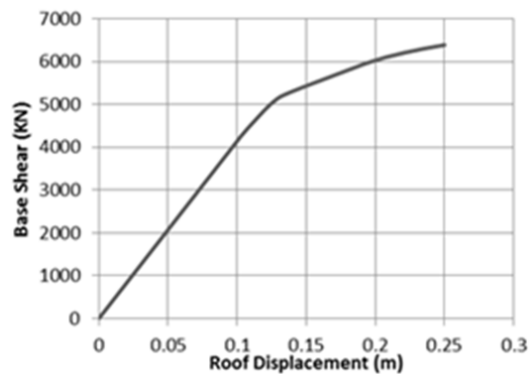


Fig. 9 Pushover curve for mode 2

The following idealized bilinear pushover curves are done to calculate the stiffness ( $K$ ) of the structure which is the slope of the line. For mode 1, the first segment slope is 18554.6 kN/m, and for the second segment is 5044.7 kN/m. For mode 2, the slope is equal to 41457.4 kN/m and 9755.7 kN/m, while mode 3 stiffnesses are 116642.9 kN/m and 27839 kN/m. After the idealization of the curve, comes step 6 of the MPA steps, which is converting from the MDOF system to the equivalent SDOF system by using the capacity curve method. Figs.11-13 demonstrate the capacity curves for the equivalent SDOF systems for the first three modes.

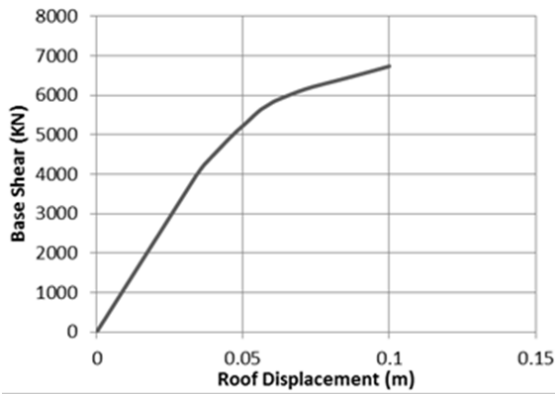


Fig. 10 Pushover curve for mode 3

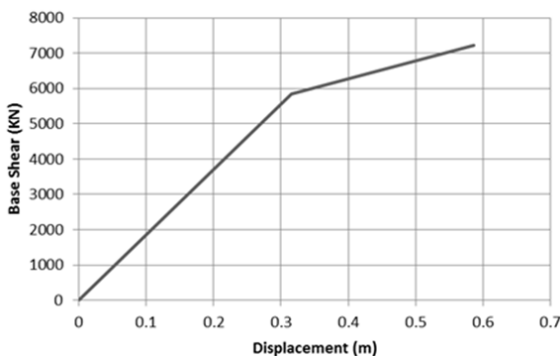


Fig. 11 Capacity curve for mode 1

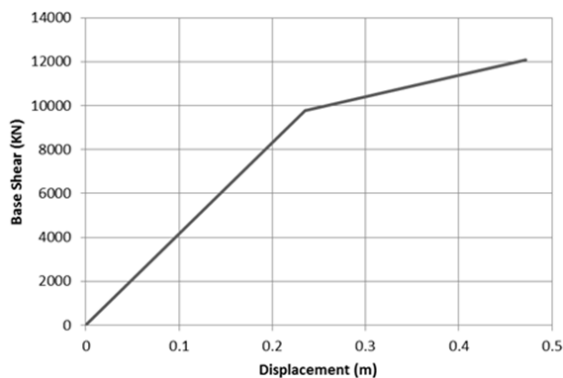


Fig. 12 Capacity curve for mode 2

Then, the stiffness can be calculated from the capacity curves which is the slope of the line segment  $K=V/D$ , and this stiffness is to be used in the SDOF using Sap2000 as a model of a vertical cantilever with unit length having the lumped mass attached to its vertical tip as shown in Fig. 14 and adjusted to provide the same natural period of free vibration with the stiffness calculated [14].

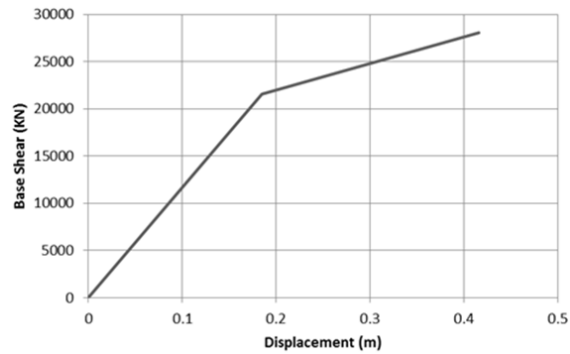


Fig. 13 Capacity curve for mode 3

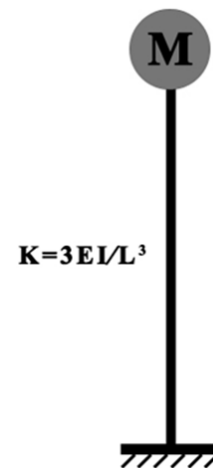


Fig. 14 Equivalent SDOF system

After applying the ground motion for nonlinear dynamic analysis,  $D$  which is the peak displacement of the equivalent SDOF system is then transformed back to its MDOF system, thus seismic responses can be obtained for the MPA.

**Modeling using SMPA**—The Simplified POA is introduced as a modification technique for the MPA as when coming to step 7 of the MPA steps, there is no need for the NLDA as the analysis to be linear, and by the concept of deformation ratio, the inelastic displacement is to be calculated. In this model, the deformation ratio equation that will be used is the one for the known strength reduction factor  $CR$  equation 4.17, while the four constants  $a$ ,  $b$ ,  $c$ , and  $d$  are equal to 61, 2.4, 1.5, and 2.4. Then, after calculating the deformation ratio for each mode, the inelastic displacement can be easily calculated from the elastic displacement for the SDOF system

**Comparison of Results**—For the comparison between the methods, each technique is done separately, while the validation of any of the method is due to its relativity to the NLTHA. The following tables show the results of the analysis using ElCentro ground motion scaled to 0.25 and 1.5, Loma Prieta and Northridge earthquakes exported from university of Berkeley database.



TABLE I  
PEAK FLOORS DISPLACEMENT FOR EL CENTRO X 0.25

Floor	Displacement /building Height (%)						Error %		
	SPA Mode 1	1 Mode	MPA 2 Modes	3 Modes	SMPA All Modes	NLDA All Modes			
1st	0.04	0.04	0.04	0.04	0.04	0.05	-29.69	-17.41	-16.50
2nd	0.06	0.07	0.07	0.07	0.07	0.09	-29.54	-18.30	-18.67
3rd	0.09	0.09	0.10	0.10	0.10	0.12	-26.98	-18.01	-17.52
4th	0.11	0.12	0.12	0.12	0.12	0.15	-25.18	-15.89	-15.54
5th	0.14	0.15	0.15	0.15	0.15	0.17	-19.69	-13.63	-12.69
6th	0.16	0.17	0.17	0.17	0.17	0.19	-14.31	-9.15	-8.44
7th	0.19	0.19	0.19	0.19	0.20	0.20	-6.76	-2.66	-1.41
8th	0.21	0.21	0.22	0.22	0.22	0.21	-0.26	2.81	5.08
9th	0.23	0.23	0.24	0.24	0.24	0.23	0.00	2.06	4.65

TABLE II  
MAXIMUM INTER-STORY DRIFTS FOR EL CENTRO X 0.25

Floor	Inter-story drift /Floor Height (%)						Error %		
	SPA Mode 1	1 Mode	MPA 2 Modes	3 Modes	SMPA All Modes	NLDA All Modes			
1st	0.25	0.28	0.29	0.33	0.29	0.22	12.50	49.12	33.61
2nd	0.24	0.27	0.28	0.29	0.26	0.22	6.82	28.35	18.19
3rd	0.23	0.26	0.26	0.23	0.25	0.22	8.14	8.20	15.56
4th	0.22	0.26	0.26	0.24	0.25	0.23	-4.35	2.33	7.74
5th	0.25	0.25	0.24	0.25	0.23	0.23	8.70	7.60	-0.48
6th	0.22	0.24	0.22	0.25	0.22	0.23	-4.30	5.59	-7.92
7th	0.23	0.23	0.22	0.22	0.23	0.25	-8.08	-13.18	-9.54
8th	0.22	0.21	0.22	0.20	0.23	0.26	-12.87	-22.37	-10.87
9th	0.20	0.17	0.19	0.18	0.19	0.21	-8.24	-15.51	-10.37

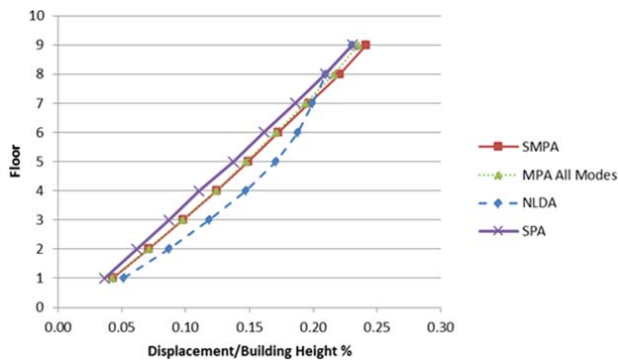


Fig. 15 Displacement/Building height % comparison for El Centro x 0.25

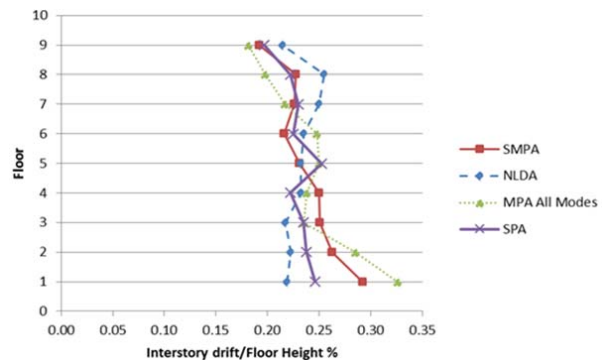


Fig. 16 Inter-story drift/Floor height % comparison for El Centro x 0.25

TABLE III  
PEAK FLOORS DISPLACEMENT FOR EL CENTRO X 1.5

Floor	Displacement /building Height (%)						Error %		
	SPA Mode 1	1 Mode	MPA 2 Modes	3 Modes	SMPA All Modes	NLDA All Modes			
1st	0.18	0.24	0.26	0.26	0.26	0.32	-42.37	-16.66	-18.61
2nd	0.33	0.42	0.44	0.45	0.45	0.52	-36.47	-13.21	-14.25
3rd	0.49	0.61	0.63	0.63	0.63	0.71	-31.38	-10.58	-10.95
4th	0.64	0.79	0.81	0.81	0.81	0.88	-26.80	-8.32	-8.24
5th	0.79	0.94	0.95	0.96	0.95	1.02	-22.35	-6.51	-6.60
6th	0.93	1.07	1.08	1.08	1.08	1.12	-17.20	-3.58	-3.94
7th	1.08	1.21	1.21	1.21	1.21	1.19	-9.71	1.54	1.23
8th	1.23	1.33	1.34	1.34	1.34	1.28	-3.93	4.29	4.31
9th	1.37	1.43	1.45	1.45	1.45	1.37	0.00	5.41	5.39

TABLE IV  
MAXIMUM INTER-STORY DRIFTS FOR EL CENTRO X 1.5

Floor	Inter-story drift /Floor Height (%)						Error %		
	SPA Mode 1	1 Mode	MPA 2 Modes	3 Modes	SMPA All Modes	NLDA All Modes	SPA	MPA	SMPA
1st	1.24	1.64	1.74	1.79	1.75	1.35	-8.36	32.53	29.43
2nd	1.39	1.71	1.76	1.76	1.77	1.28	8.70	37.73	38.31
3rd	1.45	1.73	1.74	1.70	1.73	1.31	11.20	29.98	31.95
4th	1.49	1.69	1.66	1.63	1.66	1.38	7.48	17.92	20.17
5th	1.39	1.43	1.37	1.38	1.37	1.40	-0.36	-1.09	-2.09
6th	1.29	1.24	1.17	1.21	1.18	1.41	-8.93	-14.65	-16.79
7th	1.37	1.23	1.20	1.19	1.19	1.49	-8.45	-20.39	-20.14
8th	1.45	1.17	1.22	1.18	1.22	1.53	-5.28	-22.94	-20.50
9th	1.33	0.91	1.03	1.04	1.03	1.29	3.13	-19.39	-19.88

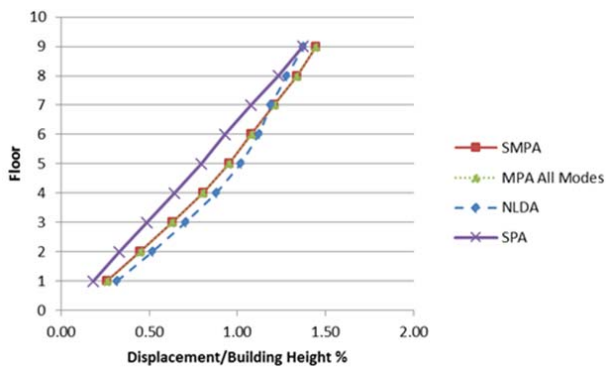


Fig. 17 Displacement/Building height % comparison for El Centro x 1.5

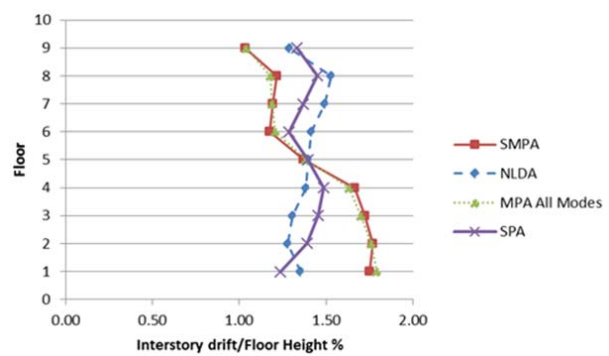


Fig. 18 Inter-story drift/Floor height % comparison for El Centro x 1.5

TABLE V  
PEAK FLOORS DISPLACEMENT FOR LOMA PRIETA

Floor	Displacement /building Height (%)						Error %		
	SPA 1 Mode	1 Mode	MPA 2 Modes	3 Modes	SMPA All Modes	NLDA All Modes	SPA	MPA	SMPA
1st	0.18	0.13	0.13	0.14	0.13	0.10	75.74	36.30	34.14
2nd	0.29	0.21	0.22	0.22	0.22	0.17	72.76	35.71	34.48
3rd	0.38	0.30	0.31	0.31	0.31	0.23	68.79	35.02	34.69
4th	0.48	0.38	0.39	0.39	0.39	0.32	51.19	23.29	23.38
5th	0.56	0.46	0.47	0.47	0.47	0.42	35.34	12.82	12.71
6th	0.64	0.54	0.54	0.54	0.54	0.53	21.52	3.64	3.40
7th	0.71	0.62	0.62	0.62	0.62	0.64	9.86	-3.75	-3.92
8th	0.77	0.69	0.70	0.70	0.70	0.75	2.84	-7.30	-7.28
9th	0.82	0.75	0.76	0.76	0.76	0.82	-0.65	-7.82	-7.82

TABLE VI  
MAXIMUM INTER-STORY DRIFTS FOR LOMA PRIETA

Floor	Inter-story drift /Floor Height (%)						Error %		
	SPA 1 Mode	1 Mode	MPA 2 Modes	3 Modes	SMPA All Modes	NLDA All Modes	SPA	MPA	SMPA
1st	1.19	0.86	0.90	0.92	0.91	0.68	75.74	36.30	34.14
2nd	1.03	0.80	0.82	0.82	0.83	0.61	68.18	34.81	35.00
3rd	0.93	0.79	0.80	0.78	0.80	0.59	58.37	33.22	35.26
4th	0.89	0.81	0.79	0.78	0.79	0.84	6.33	-6.61	-5.43
5th	0.79	0.77	0.74	0.74	0.74	0.93	-15.14	-20.51	-21.29
6th	0.70	0.72	0.69	0.70	0.69	1.02	-31.44	-31.52	-32.25
7th	0.65	0.73	0.71	0.71	0.71	1.12	-41.63	-36.39	-36.26
8th	0.59	0.69	0.72	0.70	0.71	0.99	-40.05	-28.98	-27.81
9th	0.43	0.54	0.59	0.59	0.59	0.68	-36.53	-13.18	-13.33

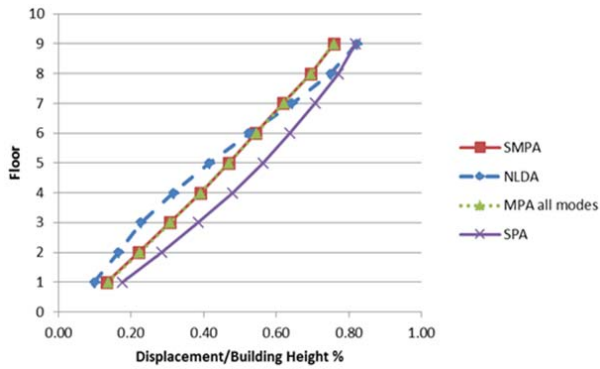


Fig. 19 Displacement/Building height % comparison for Loma Prieta

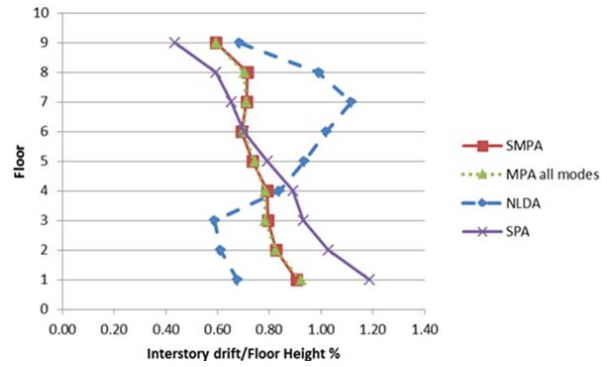


Fig. 20 Inter-story drift/Floor height % comparison for Loma Prieta

TABLE VII  
PEAK FLOORS DISPLACEMENT FOR NORTHRIDGE

Floor	Displacement /building Height (%)						Error %		
	SPA	1 Mode	MPA	2 Modes	3 Modes	SMPA	SPA	MPA	SMPA
1st	0.25	0.18	0.19	0.20	0.20	0.20	14.94	-8.19	-7.98
2nd	0.42	0.31	0.32	0.33	0.33	0.33	22.95	-3.46	-3.23
3rd	0.56	0.43	0.44	0.45	0.45	0.43	29.91	4.06	3.89
4th	0.69	0.56	0.57	0.57	0.57	0.56	22.53	0.89	0.95
5th	0.80	0.67	0.68	0.68	0.68	0.70	14.24	-2.32	-2.26
6th	0.89	0.78	0.79	0.79	0.79	0.82	8.74	-3.59	-3.52
7th	0.98	0.90	0.90	0.90	0.90	0.92	6.52	-2.38	-2.33
8th	1.07	1.00	1.01	1.01	1.01	1.04	2.27	-3.48	-3.45
9th	1.13	1.09	1.09	1.10	1.10	1.13	0.00	-2.71	-2.67

TABLE VIII  
MAXIMUM INTER-STORY DRIFTS FOR NORTHRIDGE

Floor	Inter-story drift /Floor Height (%)						Error %		
	SPA	1 Mode	MPA	2 Modes	3 Modes	SMPA	SPA	MPA	SMPA
1st	1.70	1.25	1.30	1.35	1.36	1.48	14.94	-8.19	-7.98
2nd	1.56	1.16	1.19	1.19	1.19	1.13	37.42	5.06	5.35
3rd	1.32	1.16	1.17	1.12	1.11	0.85	55.95	32.27	30.54
4th	1.21	1.16	1.15	1.11	1.12	1.23	-1.64	-9.50	-8.65
5th	1.04	1.10	1.07	1.09	1.09	1.29	-19.61	-15.42	-15.38
6th	0.91	1.04	1.00	1.04	1.04	1.17	-22.08	-10.76	-10.61
7th	0.85	1.05	1.03	1.03	1.03	0.96	-11.35	7.37	7.32
8th	0.77	1.00	1.02	0.98	0.98	1.11	-30.84	-12.03	-12.22
9th	0.56	0.78	0.83	0.84	0.84	0.79	-28.30	6.91	7.05

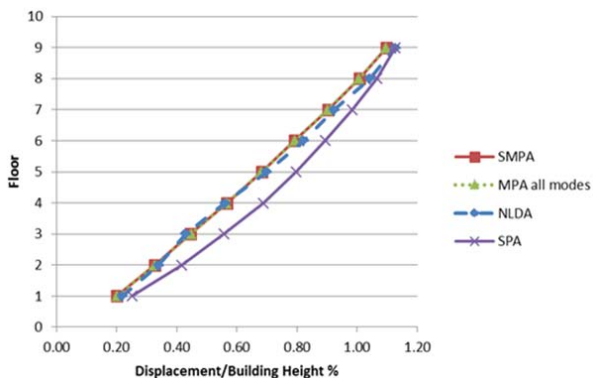


Fig. 21 Displacement/Building height% comparison for Northridge

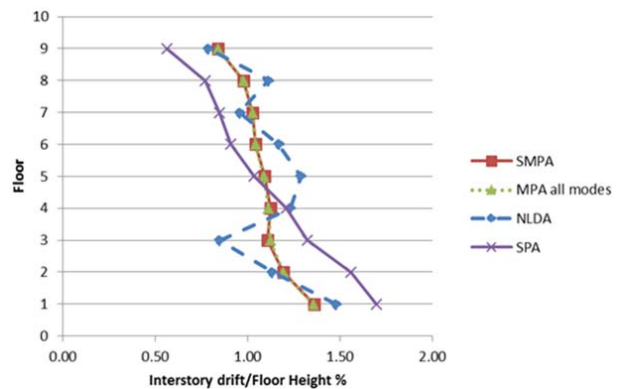


Fig. 22 Inter-story drift/Floor Height % comparison for Loma Prieta

## V. CONCLUSION

There are new techniques developed often in the seismology of structures, but the nonlinear dynamic analysis remains the most accurate and the used one. However, the NLDA cannot be used for everyday analysis because it is time consuming. Accordingly, the nonlinear static analysis is introduced as the meaning of POA. Nowadays, the POA has been important issue for its effectiveness in the seismic analysis. Due to the limitations of the POA, the method passed through many upgrades till reaching the MPA technique, which gains a great attention.

The MPA which is the improved version of the Standard POA had the same problem as the NLDA, for that reason the SMPA was created to overcome the shortcomings of the MPA which is the use of NLDA. In this thesis, the SMPA is presented and used in the analysis. To validate the SMPA method, a comparison between the techniques has been done and related to the most accurate technique which is the NLDA. The methods are introduced and used to compute the values of the floor displacements, base shear, and inter-story drifts.

**Case Study Conclusion**—The Results show the effectiveness of the proposed SMPA in estimating the seismic demand of the steel frame building. The results differ from the SPA, MPA, and SMPA. SPA values are far away from those computed using NLDA as SPA relying on the load pattern without any concern about the ground motion and don't consider any of the higher modes. Nevertheless, MPA and SMPA values are very efficient compared to the NLDA.

According to the three ground motions [15] used in the analysis, the outcomes illustrate that resonance may occur in a few stories, only if frequency at which ground shakes is steady at or near any of the natural frequencies of building and applied over an extended period of time. However, most of the results show that there is an underestimation (does not exceed 20%) in some floors according to the peak floor displacements and inter-story drifts in the weak ground motion, while the opposite happens in the strong ground motion. However, the number of modes used in the MPA shows that the higher modes effect is a must to be considered. As a result, the MPA for the three modes and the SMPA for all modes appear to be equal in most of the values, but the MPA took more analysis time.

Thus, SMPA simplifies the use of POA for everyday seismic design. As the SMPA does not need the nonlinear dynamic analysis, the method became an effective and time saving technique. In general, the SMPA results are conservative and better than those from the MPA.

**Future Work**—According to the presented study, some investigations are listed as follow for the future researches and study:

1. Application of the proposed SMPA for irregular frame buildings and shear walls.
2. Investigation of the SMPA under more real and artificial earthquakes time-history.
3. Study the accuracy of the following SMPA for the buildings designed using different codes of design including the Egyptian code of Practice.

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