International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:4, No:8, 2010

Commuting Regular Γ-Semiring

S.Krishnamoorthy and R.Arul Doss

Abstract—We introduce the notion of commuting regular Γ-semiring and discuss some properties of commuting regular Γ-semiring. We also obtain a necessary and sufficient condition for Γ-semiring to possess commuting regularity.

Keywords—Commutative Γ-semiring, Idempotent Γ-semiring, Rectangular Γ-band, Commuting regular Γ-semiring, Clifford Γ-semiring.

I. Introduction

Commuting regular rings and semigroups were studied by H. Doostie, L.Pourfaraj in [4] and by Amir H Yamini, Sh.A.Safari Sabet in [1]. The notion of Γ-semiring was introduced by M.Muralikrishna Rao [7]. All definitions and fundamental concepts concerning Γ-semirings can be found in [5],[7]. In this paper we introduce the notion of commuting regular Γ-semiring. We also discuss some properties of commuting regular Γ-semiring and obtain a necessary and sufficient condition for Γ-semiring to possess commuting regularity.

Let S and Γ be two additive commutative semigroups. Then S is called Γ-semiring if there exists a mapping $S \times \Gamma \times S \to S$(image to be denoted by $a\alpha b$ for $a, b \in S, \alpha \in \Gamma$) satisfying the following conditions.

(i) $a\alpha(b + c) = a\alpha b + a\alpha c$
(ii) $(a + b)\alpha c = a\alpha c + b\alpha c$
(iii) $a(\alpha + \beta)b = a\alpha b + a\beta b$
(iv) $a\alpha(b\beta c) = (a\alpha b)\beta c$, for all $a, b, c \in S$ and for all $\alpha, \beta \in \Gamma$.

A non empty subset A of a Γ-semiring S is called a sub Γ-semiring of S if A is a sub semigroup of S and $A\Gamma A \subseteq A$. A Γ-semiring S is said to be commutative if $a\alpha b = b\alpha a$ for all $a, b \in S$ and for all $\alpha \in \Gamma$. An element e in a Γ-semiring S is said to be an idempotent in S if there exists an $\alpha \in \Gamma$ such that $e = e\alpha e$. In this case, we say that e is an α-idempotent. If every element of S is an idempotent, then S is called an idempotent Γ-semiring. For an element a in a Γ-semiring S, if there exists an element $b \in S$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha b\beta a$ and $b = b\beta a\alpha b$, then b is said to be an (α, β) inverse of a. In this case, we write $b \in V^{\beta}_{\alpha}(a)$. we also denote it by $a^{-1}_{\alpha,\beta}$ i.e., $a^{-1}_{\alpha,\beta} \in V^{\beta}_{\alpha}(a)$. An element s in a Γ-semiring S is said to be regular if $s \in s\Gamma S\Gamma s$, where $s\Gamma S\Gamma s = \{s\alpha x\beta s; x \in S; \alpha, \beta \in \Gamma\}$. A Γ-semiring S is said to be regular if every element of S is regular. A Γ-semiring S is called a rectangular Γ-band if $a\alpha b\beta a = a$ for all $a, b \in S$ and $\alpha, \beta \in \Gamma$. Two elements a and b of a Γ-semiring S are said to commute with each other if $a\alpha b = b\alpha a$ for all $\alpha \in \Gamma$. An element of a Γ-semiring S which commutes with every element of S, is called a central element of S. A regular Γ-semiring with the central idempotents is called clifford Γ-semiring.

S.Krishnamoorthy is with Ramanujan Research Centre, Department Of Mathematics, Government Arts College, Kumbakonam - 612 001, Tamilnadu, India.

R.Arul Doss is with Department Of Mathematics, Amrita School of Engineering, Amrita Vishwa Vidyapeetham University, Ettimadai, Coimbatore - 641 105, Tamilnadu, India. E-mail : aruldoss_ kmu@yahoo.co.in

II. Commuting regular Γ-semiring

Definition II.1. *A Γ-semiring S is called commuting regular if for each $x, y \in S$, there exists an element $s \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $x\alpha y = y\alpha x\beta s\gamma y\alpha x$.*

Theorem II.2. *Let S be a rectangular Γ-band. Then S is commutative if and only if S is commuting regular*

Proof: If S is a commutative Γ-semiring, then for each $a, b \in S$, there exists an $\alpha \in \Gamma$ such that $a\alpha b = b\alpha a$. Since S is a rectangular Γ-band, there exists an element $c \in S$ and $\beta, \gamma \in \Gamma$ such that $a\alpha b = b\alpha a\beta c\gamma b\alpha a$. Hence S is a commuting regular Γ-semiring. Conversely if S is a commuting regular Γ-semiring, for each $x, y \in S$, there exists an element $z \in S$ and $\alpha, \beta, \gamma \in S$ such that $x\alpha y = y\alpha x\beta z\gamma y\alpha x$. Since S is a rectangular Γ-band, $x\alpha y = y\alpha x\beta z\gamma y\alpha x = y\alpha x$. Hence S is a commutative Γ-semiring. ∎

Theorem II.3. *If S is a commuting regular Γ-semiring with set E of the idempotents, then E is a regular sub Γ-semiring of S. Moreover for every element a of E, there exists an element $\alpha \in \Gamma$ such that $a \in V^{\alpha}_{\alpha}(a)$.*

Proof: If S is a commuting regular Γ-semiring, then for each $a \in S$, there exists an element $s \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a\alpha a = a\alpha a\beta s\gamma a\alpha a$. If $a\alpha a\beta s = b$, then $b\gamma b = (a\alpha a\beta s)\gamma(a\alpha a\beta s) = (a\alpha a\beta s\gamma a\alpha a)\beta s = a\alpha a\beta s = b$. Hence E is not empty. For elements $x\alpha x = x$ and $y\delta y = y$ in E, there exists an element $t \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $x\alpha y = y\alpha x\beta t\gamma y\alpha x$. Now $(x\alpha y)\delta(x\alpha y) = x\alpha(y\delta y)\alpha x\beta t\gamma y\alpha x = x\alpha(y\alpha x\beta t\gamma y\alpha x) = (x\alpha x)\alpha y = x\alpha y$. Consequently, E is a sub Γ-semiring of S and $x\alpha x\alpha x = x$ yields that E is a regular sub Γ-semiring of S. Hence $x \in V^{\alpha}_{\alpha}(x)$. ∎

Corollary II.4. *Let S be a commuting regular Γ-semiring with set E of the idempotents. Let $a \in S$ and let $\alpha, \beta, \gamma \in \Gamma$. If $b \in V^{\beta}_{\alpha}(a)$, then for any γ-idempotent e of S, (i) $a\alpha e\gamma b$ is β-idempotent (ii) $b\beta e\gamma a$ is α-idempotent.*

Proof: Let $b \in V^{\beta}_{\alpha}(a)$. Then $(a\alpha b)\beta(a\alpha b) = (a\alpha b\beta a)\alpha b = a\alpha b$ and $(b\beta a)\alpha(b\beta a) = (b\beta a\alpha b)\beta a = b\beta a$. Since $(e\gamma b\beta a)\alpha(e\gamma b\beta a) = e\gamma b\beta a$ and $(e\gamma a\alpha b)\beta(e\gamma a\alpha b) = e\gamma a\alpha b$, by theorem II.3, $e\gamma b\beta a, e\gamma a\alpha b \in E$. Now, $(a\alpha e\gamma b)\beta(a\alpha e\gamma b) = a\alpha(e\gamma b\beta a\alpha e\gamma b\beta a)\alpha b =$

$a\alpha e\gamma(b\beta a\alpha b) = a\alpha e\gamma b$. Moreover, $(b\beta e\gamma a)\alpha(b\beta e\gamma a) = b\beta(e\gamma a\alpha b\beta e\gamma a\alpha b)\beta a = b\beta e\gamma(a\alpha b\beta a) = b\beta e\gamma a$. ■

Theorem II.5. *Let S be a commuting regular Γ-semiring with set E of α-idempotents. Let $e, f \in E$ and $\alpha \in \Gamma$. Then the set $S^{\alpha}_{\alpha}(e, f) = \{g \in V^{\alpha}_{\alpha}(e\alpha f) \cap E;\ g\alpha e = f\alpha g = g\}$ is a regular sub Γ-semiring of S.*

Proof: Since S is a commuting regular Γ-semiring, there exists an element $s \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $e\alpha f = f\alpha e\beta s\gamma f\alpha e$. Then $(e\alpha f)\alpha(e\alpha f) = f\alpha e\beta s\gamma f\alpha(e\alpha e)\alpha f = (f\alpha e\beta s\gamma f\alpha e)\alpha f = e\alpha(f\alpha f) = e\alpha f$. Now, $(e\alpha f)\alpha(e\alpha f)\alpha(e\alpha f) = e\alpha f$, $(e\alpha f)\alpha e = f\alpha e\beta s\gamma f\alpha(e\alpha e) = e\alpha f$ and $f\alpha(e\alpha f) = (f\alpha f)\alpha e\beta s\gamma f\alpha e = e\alpha f$. This yields $(e\alpha f) \in S^{\alpha}_{\alpha}(e, f)$ and we can also prove that $f\alpha e \in S^{\alpha}_{\alpha}(e, f)$ which implies $S^{\alpha}_{\alpha}(e, f) \neq \phi$. Let $x, y \in S^{\alpha}_{\alpha}(e, f)$. Since S is a commuting regular Γ-semiring, there exists an element $t \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $x\alpha y = y\alpha x\beta t\gamma y\alpha x$. Then $(x\alpha y)\alpha(x\alpha y) = x\alpha(y\alpha y)\alpha x\beta t\gamma y\alpha x = x\alpha(y\alpha x\beta t\gamma y\alpha x) = (x\alpha x)\alpha y = x\alpha y$. $(e\alpha f)\alpha(x\alpha y)\alpha(e\alpha f) = (e\alpha f\alpha x\alpha e\alpha f)\alpha y\alpha e\alpha f = e\alpha f\alpha y\alpha e\alpha f = e\alpha f$. $(x\alpha y)\alpha(e\alpha f)\alpha(x\alpha y) = x\alpha(y\alpha e)\alpha(f\alpha x)\alpha y = (x\alpha y)\alpha(x\alpha y) = x\alpha y$. Hence $x\alpha y \in V^{\alpha}_{\alpha}(e\alpha f) \cap E$. Moreover, $(x\alpha y)\alpha e = x\alpha(y\alpha e) = x\alpha y$ and $f\alpha(x\alpha y) = (f\alpha x)\alpha y = x\alpha y$. Let $x \in S^{\alpha}_{\alpha}(e, f)$. Then $x\alpha(e\alpha f)\alpha x = (x\alpha e)\alpha(f\alpha x) = x\alpha x = x$. This shows that $s^{\alpha}_{\alpha}(e, f)$ is a regular sub Γ-semiring of S.

■

Remark II.6. *The set $S^{\alpha}_{\alpha}(e, f)$ is called the (α, α) sandwich set of e and f. It has an obvious alternative characterization $S^{\alpha}_{\alpha}(e, f) = \{g\alpha g = g \in S;\ g\alpha e = g = f\alpha g,\ e\alpha g\alpha f = e\alpha f\}$.*

Lemma II.7. *Let S be a commuting regular Γ-semiring. Let $a, b \in S$ and let $\alpha, \beta, \gamma \in \Gamma$. Suppose $a' \in V^{\beta}_{\alpha}(a)$ and $b' \in V^{\alpha}_{\beta}(b)$. Then for each $g \in S^{\alpha}_{\alpha}(a'\beta a, b\beta b')$, $b'\alpha g\alpha a' \in V^{\beta}_{\beta}(a\alpha b)$.*

Proof: $(a\alpha b)\beta(b'\alpha g\alpha a')\beta(a\alpha b) = a\alpha(b\beta b'\alpha g)\alpha a'\beta a\alpha b = a\alpha(g\alpha a'\beta a)\alpha b = a\alpha g\alpha b = a\alpha(a'\beta a\alpha g\alpha b\beta b')\alpha b = (a\alpha a'\beta a)\alpha(b\beta b'\alpha b) = a\alpha b$. Moreover $(b'\alpha g\alpha a')\beta(a\alpha b)\beta(b'\alpha g\alpha a') = b'\alpha(g\alpha a'\beta a)\alpha(b\beta b'\alpha g)\alpha a' = b'\alpha(g\alpha g)\alpha a' = b'\alpha g\alpha a'$. Hence $b'\alpha g\alpha a' \in V^{\beta}_{\beta}(a\alpha b)$. ■

Theorem II.8. *Let S be a commuting regular Γ-semiring. Let $a, b \in S$ and $\alpha, \beta \in \Gamma$. Then $V^{\alpha}_{\beta}(b)\Gamma V^{\beta}_{\alpha}(a) \subseteq V^{\beta}_{\beta}(a\alpha b)$.*

Proof: Let $a, b \in S$ and let $\alpha, \beta \in \Gamma$. Suppose $a' \in V^{\beta}_{\alpha}(a)$ and $b' \in V^{\alpha}_{\beta}(b)$. Then by lemma II.7, $b'\alpha g\alpha a' \in V^{\beta}_{\beta}(a\alpha b)$ for all g in $S^{\alpha}_{\alpha}(a'\beta a, b\beta b')$. Now, $(a'\beta a)\alpha(a'\beta a) = (a'\beta a\alpha a')\beta a = a'\beta a$ and $(b\beta b')\alpha(b\beta b') = (b\beta b'\alpha b)\beta b' = b\beta b'$. Then by theorem II.5, $b\beta b'\alpha a'\beta a \in S^{\alpha}_{\alpha}(a'\beta a, b\beta b')$. By lemma II.7, $b'\alpha(b\beta b'\alpha a'\beta a)\alpha a' \in V^{\beta}_{\beta}(a\alpha b)$. Hence $b'\alpha a' \in V^{\beta}_{\beta}(a\alpha b)$. ■

Theorem II.9. *Let S be a commuting regular Γ-semiring with set E of idempotents. Let $\alpha, \beta \in \Gamma$. Then $V^{\beta}_{\alpha}(e) \subseteq E$ for every e in E.*

Proof: Suppose $x \in V^{\beta}_{\alpha}(e)$. Then $x\beta e\alpha x = x$ and $e\alpha x\beta e = e$. Since $x\beta e$ is α-idempotent and $e\alpha x$ is β-idempotent, $x\beta e \in V^{\alpha}_{\alpha}(x\beta e)$ and $e\alpha x \in V^{\beta}_{\beta}(e\alpha x)$. Then by theorem II.8, $(e\alpha x)\beta(x\beta e) \in V^{\alpha}_{\beta}(x\beta e\alpha e\alpha x)$ which implies that $e\alpha x\beta x\beta e \in V^{\alpha}_{\beta}(x)$. Now $x = x\beta(e\alpha x\beta x\beta e)\alpha x = (x\beta e\alpha x)\beta(x\beta e\alpha x) = x\beta x$ which implies $x \in E$. ■

Theorem II.10. *Let S be a Γ-semiring. Then S is a commuting regular if and only if $S\Gamma S$ is a clifford Γ-semiring*

Proof: Suppose S is a commuting regular Γ-semiring. Then for $x \in S$ and an α-idempotent $e \in S$, there exists $s \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $x\alpha e = e\alpha x\beta s\gamma e\alpha x$. Now, $e\alpha x\alpha e = (e\alpha e)\alpha x\beta s\gamma e\alpha x = e\alpha x\beta s\gamma e\alpha x = x\alpha e$. Symmetrically, $e\alpha x\alpha e = e\alpha x$. Hence $e\alpha x = x\alpha e$ which follows that the idempotents in S are central. For any $x, y \in S$, there exists $z \in S$ and $\alpha_1, \beta_1, \gamma_1 \in \Gamma$ such that $x\alpha_1 y = y\alpha_1 x\beta_1 z\gamma_1 y\alpha_1 x$. Moreover, there exists $u \in S$ and $\alpha_2, \beta_2 \in \Gamma$ such that $z\gamma_1 y\alpha_1 x = y\alpha_1 x\gamma_1 z\alpha_2 u\beta_2 y\alpha_1 x\gamma_1 z$. Hence $x\alpha_1 y = (y\alpha_1 x\beta_1 y\alpha_1 x)\gamma_1 z\alpha_2 u\beta_2 y\alpha_1 x\gamma_1 z$. Since $y\alpha_1 x\beta_1 y\alpha_1 x$ is regular in commuting regular Γ-semiring S, $y\alpha_1 x\beta_1 y\alpha_1 x = y\alpha_1 x\beta_1 y\alpha_1 x\alpha_3 v\beta_3 y\alpha_1 x\beta_1 y\alpha_1 x$ for some $v \in S$ and $\alpha_3, \beta_3 \in \Gamma$. Let $e = y\alpha_1 x\beta_1 y\alpha_1 x\alpha_3 v$. Then e is a β_3-idempotent in S such that $y\alpha_1 x\beta_1 y\alpha_1 x = e\beta_3 y\alpha_1 x\beta_1 y\alpha_1 x$ and so $x\alpha_1 y = e\beta_3(y\alpha_1 x\beta_1 y\alpha_1 x\gamma_1 z\alpha_2 u\beta_2 y\alpha_1 x\gamma_1 z) = e\beta_3 x\alpha_1 y$. Now, there exists $w \in S$ and $\alpha_4, \beta_4 \in \Gamma$ such that $e\beta_3 x\alpha_1 y = x\alpha_1 y\beta_3 e\alpha_4 w\beta_4 x\alpha_1 y\beta_3 e$. Then $x\alpha_1 y = x\alpha_1 y\beta_3 e\alpha_4 w\beta_4(x\alpha_1 y\beta_3 e) = (x\alpha_1 y)\beta_3(e\alpha_4 w\beta_4 e\beta_3)(x\alpha_1 y)$ which implies that $x\alpha_1 y$ is regular. Hence $S\Gamma S$ is regular.
Conversely suppose $S\Gamma S$ is a clifford Γ-semiring. Since the idempotents of S are central, for any $x, y \in S$ and $\alpha, \beta \in \Gamma$, there exists an idempotent e such that $x\alpha y = e\beta(x\alpha y)$. Let $u = e\alpha x$ and $v = e\alpha y$. As $u, v \in S\Gamma S$, there exist $u', v' \in S$ and $\beta, \gamma, \delta \in \Gamma$ such that $u\beta u'\delta u = u$ and $v\delta v'\gamma v = v$. Since $u\beta u'$ and $v'\gamma v$ are central idempotents, $(y\alpha x)\beta(u'\delta v'\beta e\gamma x\alpha y\gamma u'\delta v')\gamma(y\alpha x) = v\delta(u\beta u'\delta v'\beta u\gamma v\gamma u'\delta v'\gamma v)\delta u = (v\delta v'\gamma v)\delta v'\beta u\gamma v\gamma u'\delta(u\beta u'\delta u) = ((v\delta v')\beta u)\gamma v\gamma u'\delta u = u\beta(v\delta v'\gamma v)\gamma(u'\delta u) = (u\beta u'\delta u)\gamma(v\delta v'\gamma v) = u\gamma v = e\gamma(x\alpha y) = x\alpha y$. Hence S is a commuting regular. ■

III. Green's Equivalences in Commuting Regular Γ-semiring

Definition III.1. *If a is an element of a Γ-semiring S, the smallest left ideal of S containing a is $S\Gamma a \cup \{a\}$. An equivalence $\mathfrak{L}$ on S is defined by the rule that $a\mathfrak{L}b$ if and only if $S\Gamma a \cup \{a\} = S\Gamma b \cup \{b\}$. Similarly we define the equivalence $\mathfrak{R}$ by the rule that $a\mathfrak{R}b$ if and only if $a\Gamma S \cup \{a\} = b\Gamma S \cup \{b\}$.*

Lemma III.2. *Let a and b be elements of a Γ-semiring S. Then $a\mathfrak{L}b$ if and only if there exist x and y in S and $\alpha, \beta \in \Gamma$ such that $x\alpha a = b$ and $y\beta b = a$. Also, $a\mathfrak{R}b$ if and only if there exist u and v in S and $\gamma, \delta \in \Gamma$ such that $a\gamma u = b$ and $b\delta v = a$*

Lemma III.3. *The relations $\mathfrak{L}$ and $\mathfrak{R}$ commute*

Theorem III.4. *Let S be a commuting regular Γ-semiring and $a, b \in S$. Then $a\mathfrak{L}b$ if and only if $a\mathfrak{R}b$.*

Proof: Suppose that $a\mathfrak{L}b$. By lemma III.2, there are x and y in S and $\alpha, \beta \in \Gamma$ such that $x\alpha a = b$ and $y\beta b = a$. So, there are t_1, t_2 in S and $\gamma_1, \gamma_2, \delta_1, \delta_2$ in Γ such that $b = x\alpha a = a\alpha x\gamma_1 t_1\gamma_2 a\alpha x$ and $a = y\beta b = b\beta y\delta_1 t_2\delta_2 y\beta b$ where $u = x\gamma_1 t_1\gamma_2 a\alpha x, v = y\delta_1 t_2\delta_2 y\beta b$. This implies $a\mathfrak{R}b$. Proof of the converse is similar. ■

Remark III.5. *The equivalence $\mathfrak{D}$ is a two sided analogue of $\mathfrak{L}$ and $\mathfrak{R}$. Also, we define the equivalence $\mathfrak{J}$ by the rule $a\mathfrak{J}b$ if and only if $S\Gamma a \cup a\Gamma S \cup S\Gamma a\Gamma S \cup \{a\} = S\Gamma b \cup b\Gamma S \cup S\Gamma b\Gamma S \cup \{b\}$ if and only if there exist $x, y, u, v \in S$ and $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \Gamma$ such that $x\alpha_1 a\alpha_2 y = b$ and $u\beta_1 b\beta_2 v = a$. It is immediate that $\mathfrak{L} \subseteq \mathfrak{J}$ and $\mathfrak{R} \subseteq \mathfrak{J}$. Hence since $\mathfrak{D}$ is the smallest equivalence containing $\mathfrak{L}$ and $\mathfrak{R}$, we get $\mathfrak{D} \subseteq \mathfrak{J}$.*

Theorem III.6. *If S is a commuting regular Γ-semiring, then $\mathfrak{D} = \mathfrak{J}$.*

Proof: By remark III.5, it is enough to show that $\mathfrak{J} \subseteq \mathfrak{D}$. For elements a and b in S, let $a\mathfrak{J}b$. Then there are $x, y, u, v \in S$ and $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \Gamma$ such that $x\alpha_1 a\alpha_2 y = b; u\beta_1 b\beta_2 v = a$. So there exists an element t_1 in S and $\gamma_1, \gamma_2 \in \Gamma$ such that $a = u\beta_1 b\beta_2 v = (u\beta_1 x\alpha_1 a)\alpha_2(y\beta_2 v) = (y\beta_2 v\alpha_2 u\beta_1 x\alpha_1 a\gamma_1 t_1\gamma_2 y\beta_2 v\alpha_2 u)\beta_1(x\alpha_1 a) = w_1\beta_1 c$ where $w_1 = y\beta_2 v\alpha_2 u\beta_1 x\alpha_1 a\gamma_1 t_1\gamma_2 y\beta_2 v\alpha_2 u$ and $c = x\alpha_1 a$ and so $a\mathfrak{L}c$. Combining the relations $x\alpha_1 a\alpha_2 y = b$ and $c = x\alpha_1 a$, we get $c\alpha_2 y = b$. Then there exists an element $t_2 \in S$ and $\delta_1, \delta_2 \in \Gamma$ such that $c = x\alpha_1 a = (x\alpha_1 u)\beta_1(b\beta_2 v) = b\beta_2(v\beta_1 x\alpha_1 u\delta_1 t_2\delta_2 b\beta_2 v\beta_1 x\alpha_1 u) = b\beta_2 w_2$, where $w_2 = v\beta_1 x\alpha_1 u\delta_1 t_2\delta_2 b\beta_2 v\beta_1 x\alpha_1 u$. This shows that $c\mathfrak{R}b$. Hence $\mathfrak{J} \subseteq \mathfrak{D}$. ■

References

[1] Amir H Yamini, Sh.A.Safari Sabet, *Commuting regular rings,* Int. J. Appl. Math., 14, No.4, 3357-3364(2003).
[2] Clifford A.H. and Preston G.B., *Algebraic theory of semigroups*, Volume I, American Mathematical Society, Providence(1961).
[3] Clifford A.H. and Preston G.B., *Algebraic theory of semigroups*, Volume II, American Mathematical Society, Providence(1967).
[4] Doostie H., Pourfaraj L., *On the minimal ideals of commuting regular rings and semigroups,* Int. J. Appl. Math., 19, No.2, 201-216(2006).
[5] Dutta T.K. and Sardar S.K., *On the operator semirings of a Γ-semiring*, Southeast Asian Bulletin of Mathematics, 26(2)(2002), 215-225.
[6] Howie J.M., *Fundamentals of semigroup theory,* Clarendon press, Oxford, Newyork(1995).
[7] Murali Krishna Rao M., *Γ-semirings-I,* Southeast Asian Bulletin of Mathematics, 19(1)(1995), 49-54.