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Cold Analysis for Dispersion, Attenuation and RF Efficiency Characteristics of a Gyrotron Cavity

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***Abstract*—**In the present paper, a gyrotron cavity is analyzed in the absence of electron beam for dispersion, attenuation and RF efficiency. For all these characteristics, azimuthally symmetric TE_{0n} modes have been considered. The attenuation characteristics for TE_{0n} modes indicated decrease in attenuation constant as the frequency is increased. Interestingly, the lowest order TE_{01} mode resulted in lowest attenuation. Further, three different cavity wall materials have been selected for attenuation characteristics. The cavity made of material with higher conductivity resulted in lower attenuation. The effect of material electrical conductivity on the RF efficiency has also been observed and has been found that the RF efficiency rapidly decreases as the electrical conductivity of the cavity material decreases. The RF efficiency rapidly decreases with increasing diffractive quality factor. The ohmic loss variation as a function of frequency of operation for three different cavities made of copper, aluminum and nickel has been observed. The ohmic losses are lowest for the copper cavity and hence the highest RF efficiency.

***Keywords*—**Gyrotron, dispersion, attenuation, quality factor.

I. Introduction

THE gyrotrons, which operates essentially in high power, millimeter and sub-millimeter wave regime, have drawn tremendous attention all over the world due to their wide range of applications in the areas of RF plasma production, heating, non-inductive current drive, plasma stabilization and active plasma diagnostics for magnetic confinement, thermonuclear fusion research, such as lower hybrid current drive (LHCD) (8 GHz), electron cyclotron resonance heating (ECRH) (28-170 GHz), electron cyclotron current drive (ECCD), collective Thomson scattering (CTS), heat-wave propagation experiments, and many more [1], [2].

The gyrotron is a high power high frequency coherent radiation source in which the magnetron injection gun produces an electron beam with the desired beam parameters, the beam is transported to the interaction region where the interaction cavity converts a fraction of beam power to the RF power and the spent beam is collected at the output collector [2]. The interaction region is usually a three-section smooth walled cylindrical open resonator cavity (Fig. 1). The input taper is a cut-off section which prevents backward waves towards the gun. Interaction takes place between RF wave and electron beam mainly in the uniform middle section where RF fields reach peak values. The up-taper connects the cavity with the output waveguide and the launcher. Cavity design becomes more stringent if one wishes to operate the device at elevated frequencies and higher power levels from long-pulse to CW operation [2]. The problem of wall losses is important for long pulse to CW operation of high power gyrotrons [3]-[6]. Wall losses are related to ohmic quality factor as $Q_{ohm} P_L = \omega W$; where Q_{ohm} is ohmic quality factor, P_L is loss in the cavity, ω is angular frequency of operation and W is the total energy stored in the cavity resonator. Knowledge of ohmic quality factor is useful for efficiency estimations [2], [7]. For this purpose a gyrotron cavity (Fig. 1) is analyzed in the present paper.

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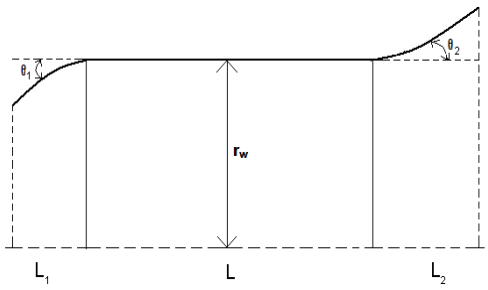

Fig. 1 Gyrotron cavity resonator

The organization of the paper is as follows: Firstly, the cold analysis (beam absent analysis) dispersion characteristic has been presented. The attenuation characteristics for different azimuthally symmetric modes have also been presented in order to observe the coupling of wave fields to the interaction structure metallic walls. Further, cavity resonator analysis for diffractive quality factor and efficiency has also been dealt with. Finally, the results and discussion have been presented.

II. Analytical Study

A. Dispersion Characteristic Equation

1. Beam Mode Dispersion Relation

The beam mode dispersion relation $\omega - \beta v_z - s\omega_c / \gamma = 0$ can be easily derived by noting that, over a cyclotron period, one or more RF cycles are completed. Here, ω is the wave frequency, β the propagation constant or wavenumber, s the beam mode (harmonic) number, γ the relativistic factor, and v_z the axial DC beam velocity. It may be mentioned that, in a gyro-device like gyrotron, the fast cyclotron wave couples to a forward wave of the interaction structure (circuit), unlike in a slow-wave device like TWT, in which the slow space-charge wave couples to the forward circuit wave [8].

Consider a beam completing its one gyration in time T_c.

RF phase of the electric field component is given as $\omega t - \beta z$. After one gyration time period, T_c, the initial phase will change to $\omega(t+T_c) - \beta(z + v_z T_c)$.

Now putting $s(2\pi) = (\omega - \beta v_z)T_c$, the initial phase change will be $\omega t - \beta z + s(2\pi)$.

Considering,

$$s(2\pi) = (\omega - \beta v_z)T_c \tag{1}$$

The cyclotron frequency is given by $\omega_c = 2\pi/T_c$, so (1) can be written as

$$\omega - \beta v_z - s\,\omega_c = 0\,. \tag{2}$$

where, $\omega_c = eB_0/m_e$, e/m_e is electron charge to mass ratio and B_0 is axial DC magnetic flux density.

But, gyro-devices are relativistic devices, so considering relativistic mass factor, γ, (2) can be written as

$$\omega - \beta v_z - s\,\omega_c/\gamma = 0\,. \tag{3}$$

This is the *beam-mode dispersion relation*.

2. Waveguide Mode Dispersion Relation

The magnetic field intensity in a cylindrical waveguide excited in TE_{mn} mode is given by [8], [9]

$$H_z = C_{mn} J_m(\gamma_n r)\exp\{j(\omega t - \beta z - m\theta)\} \tag{4}$$

where, r, θ, z are cylindrical polar coordinates, m and n are the azimuthal and radial index of wave mode, γ_n is the radial phase propagation constant, J_m is the mth order ordinary Bessel function of first kind, and the field constant is given by

$$C_{mn} = \frac{1}{J_m(v_{mn})\sqrt{\pi\left(v_{mn}^2 - m^2\right)}} \tag{5}$$

and

$$\gamma_n = \left(k_0^2 - \beta^2\right)^{1/2} = \left(\frac{\omega^2}{c^2} - \beta^2\right)^{1/2} = \frac{v_{mn}}{r_w} \tag{6}$$

where $k_0 = \omega(\mu_0\,\varepsilon_0)^{1/2} = \omega/c$ is the free-space propagation constant; c being velocity of light, μ_0 and ε_0 are the permeability and permittivity of vacuum, respectively and $v_{mn} = \gamma_n r_w$.

Now considering the axial magnetic field intensity H_{z0}, at the axis of the guide, i.e., at $r = 0$. The non-azimuthally varying mode $m = 0$ will yield $J_0(0) = 1$. Hence, $H_z\big|_{r=0} = C_{mn} = H_{zo}$. Thus, (4) can be expressed as

$$H_z = H_{z0} J_m(\gamma r)\exp\{j(\omega t - \beta z - m\theta)\}\,. \tag{7}$$

The azimuthal components of electrical field intensity may be written as

$$E_\theta = \left(\frac{j\omega\mu_0}{\gamma_t}\right) H_{z0} J_m'(\gamma_n r)\exp j(\omega t - \beta z - m\theta)\,. \tag{8}$$

Consideration of appropriate boundary conditions at the wall interface of the waveguide, i.e., $r = r_w$ will make azimuthal component of electric field intensity zero, i.e. $E_\theta(r = r_w) = 0$, which implies that Bessel derivative must be zero. So,

$$J_m'(\gamma_n r) = 0\,. \tag{9}$$

Equation (9) has an infinite number of roots. Let us put n^{th} root $\gamma_n = v_{mn}/r_w$ in (6)

$$\frac{v_{mn}^2}{r_w^2} = k_0^2 - \beta^2 = \gamma_n^2 \tag{10}$$

Thus,

$$\omega^2 - \beta^2 c^2 - \omega_{cut}^2 = 0 \tag{11}$$

where, $\omega_{cut} = \gamma_n c$.

This is the *waveguide-mode dispersion relation* and the $\omega - \beta$ dispersion plot is a hyperbola.

Mathematically, synchronism means that the intersection between the beam mode line and waveguide mode curve described by (3) and (11) occurs when the phase velocity of the beam mode and the RF wave are equal, namely where the beam-wave interaction occurs. In a gyrotron, a grazing situation where the two curves graze each other is usually desired for operation. To achieve the grazing situation, the DC axial magnetic field is tuned to the value of grazing magnetic field. At grazing condition, the group velocity of the beam mode and RF wave are same. The gyrotron is operated close to the waveguide cut-off frequency at near-zero group velocity that makes the energy velocity slow enough to keep electromagnetic energy in the waveguide resonator cavity for interaction with the electron beam in the device [2], [8]-[11].

B. Attenuation Characteristic Equation

Power propagating through the structure can be found by integrating the real part of the z-component of the complex Poynting vector over the waveguide cross-section as [8].

$$P_t = \frac{1}{2}\mathrm{Re}\int_0^{2\pi}\int_{r=0}^{r_W}\left(\mathrm{E}\times\mathrm{H}^*\right)_z r\,dr\,d\theta$$

$$P_t = \pi\int_{r=0}^{r_W}\left(E_r H_\theta^* - E_\theta H_r^*\right) r\,dr\,. \tag{12}$$

If the thickness of the waveguide conductor is greater than the depth of penetration of the electromagnetic fields in conductor, there cannot be any electric field inside the conductor. Hence, it is assumed that there exist only surface current on the surface of the conductor [12], [13]. Now, for metallic conductors of high conductivity, we can use Ampere's circuital law to calculate surface current density J_S and express it as, $J_S = n_\perp \times H$ where $n_\perp$ is the normal to the surface of the conductor. Then, the time-averaged power-loss per unit length of the guide can be given by [8]

$$P_L = \frac{1}{2} \int_{surface\,area} |J_S|^2 R_S \, ds \tag{13}$$

where the integration is taken over the wall-surface of a unit area of the guide and R_S is the resistance of the conductor for a unit length and unit width and is called surface resistivity.

Further, it is worth noting that the field equations used here are the loss-free field equations and are only valid for low losses, such as ohmic losses. One may obtain the power loss per unit area at the surface of the smooth wall cylindrical waveguide of radius r_w by using the following equation:

$$P_L = \frac{R_S}{2} \int_0^{2\pi} \left\{ |H_\theta|^2 + |H_z|^2 \right\}_{r=r_w} r_w d\theta . \tag{14}$$

The propagation constant γ is a complex quantity and is expressed as $\gamma = \alpha + j\beta$, where α and β are the real and imaginary parts of γ and α is known as attenuation constant and is associated with the power losses in the structure. These losses may be due to lossy dielectric and/or finite conductivity of structure walls. If the operating frequency is way below the cut-off frequency, the attenuation constant becomes very large and non-propagation occurs. For the frequencies larger than the cut-off frequency of the structure, the propagation constant γ is pure imaginary and is equal to $j\beta$ if the medium is lossless inside the structure. Following the principle of conservation of energy, the attenuation constant per unit length accounting for the power loss along the structure can be expressed as,

$$\alpha = \frac{P_L}{2P_t} = \frac{R_S \left\{ (\omega_c/\omega)^2 + \left(n^2/\nu_{mn}^2 - n^2 \right)^2 \right\}}{r_w \eta_0 \sqrt{1-(\omega_c/\omega)^2}} \tag{15}$$

where, η_0 is intrinsic impedance.

C. Diffractive Quality Factor

The time τ for energy to travel out of the cavity determines diffractive quality factor and is related as [9]

$$Q_{d,\min} = \omega\tau = \omega \frac{L}{v_g} = \omega \frac{L}{c^2/v_p} = \frac{\omega L}{c^2} \frac{\omega}{\beta} = 4\pi \frac{L^2}{\lambda^2} \tag{16}$$

where Q_d, L, v_g, v_p, and λ are diffractive quality factor, effective length of the cavity, group velocity, phase velocity, and wavelength at operating frequency, respectively and $\beta L = \pi$.

Taking the reflection ρ at the end of the cavity [2], the actual quality factor can be represented by

$$Q = \omega \frac{W}{P} = \frac{Q_{d,mim}}{\tau_d} \frac{W}{P} = \frac{Q_{d,\min}}{P\tau_d/W} \tag{17}$$

Thus we have,

$$Q = \frac{Q_{d,\min}}{1-\rho} = 4\pi \frac{L^2}{\lambda^2} \left(\frac{1}{1-\rho} \right) \tag{18}$$

In a long wavelength gyrotron, the total Q is approximately equivalent to the diffractive quality factor (Q_d), resulting in negligible ohmic losses. However, at short wavelengths, the ohmic losses can severely reduce the output power of a gyrotron [4].

D. Gyrotron Efficiency Calculations

Total efficiency of a gyrotron is the ratio of output RF power to the input beam power. It is related to electronic-efficiency and efficiency-reduction due to ohmic losses and is given as $\eta_{out} = P_{out}/P_{in} = \eta_{el} \times \eta_Q$. The electronic efficiency, η_{el}, accounts for the fraction of beam power in the perpendicular direction which is given by [11]

$$\eta_{el} = \frac{\beta_0^2}{2\left(1-\gamma_0^{-1}\right)} \eta_\perp \tag{19}$$

where γ_0 is relativistic mass factor and $\eta_\perp$ is efficiency due to beam power in perpendicular direction and η_Q is the reduction in efficiency due to ohmic losses given by

$$\eta_Q = 1 - \frac{Q}{Q_{ohm}} . \tag{20}$$

The total Q of a resonant cavity is given by [2]

$$\frac{1}{Q} = \frac{1}{Q_{diff}} + \frac{1}{Q_{ohm}} . \tag{21}$$

The ohmic quality factor of a cavity excited in $TE_{m,n}$ mode is given by [2]

$$Q_{ohm} = \frac{r_w}{\delta}\left(1 - \frac{m^2}{v_{mn}^2}\right) \tag{22}$$

where $\delta = (\pi f \mu_0 \sigma)^{-1/2}$ is the skin depth at frequency f for the cavity having radius r_w, conductivity σ; and v_{mn} is the eigenvalue where m and n denote azimuthal and radial index of the $TE_{m,p}$ mode.

The RF efficiency is expressed as

$$\eta_{RF} = \frac{Q_{ohm}}{Q_{diff} + Q_{ohm}} \tag{23}$$

III. Results and Discussion

Dispersion characteristic for the uniform mid-section of the gyrotron-cavity (Fig. 1), where actual interaction between RF wave and electron beam takes place, has been obtained analytically for first eight azimuthally symmetric modes TE_{01}, TE_{02}, TE_{03}, TE_{04}, TE_{05}, TE_{06}, TE_{07}, and TE_{08} and is shown in Fig. 2. The standard routine *fsolve* of MATLAB software has been used, for obtaining the zeros of the dispersion relation (11), which uses the trust-region dogleg method ─ a variant algorithm of Powell dogleg method [14]. The operating point, i.e., desired frequency of operation of the gyrotron oscillator for a particular waveguide mode can be chosen with the help of the dispersion diagram.

The attenuation characteristics of the structure have also been numerically obtained from (15). The variation of attenuation characteristics has been observed for different azimuthally symmetric modes typically TE_{01}, TE_{02}, TE_{03}, TE_{04}, TE_{05}, TE_{06}, TE_{07}, and TE_{08}. From Fig. 3, it is evident that the attenuation for the azimuthally symmetric modes decreases to a very small value with increasing frequency. But, there exists more attenuation in the case of higher order azimuthally symmetric modes TE_{02}, TE_{03}, TE_{04}, TE_{05}, TE_{06}, TE_{07}, and TE_{08} in comparison of the TE_{01} mode because tangential magnetic field components in the case of TE_{02}, TE_{03}, TE_{04}, TE_{05}, TE_{06}, TE_{07}, and TE_{08} modes are more tightly coupled with the cavity surfaces.

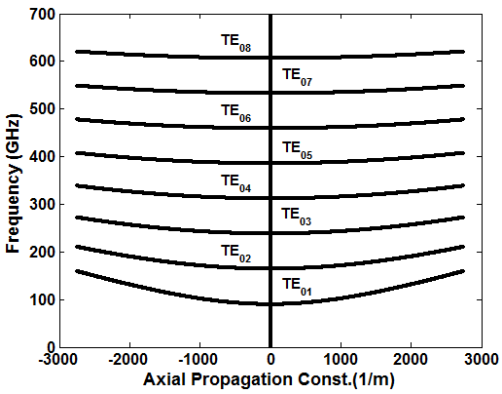

Fig. 2 Dispersion characteristics for a gyrotron interaction structure excited in different azimuthally symmetric modes TE_{0n}

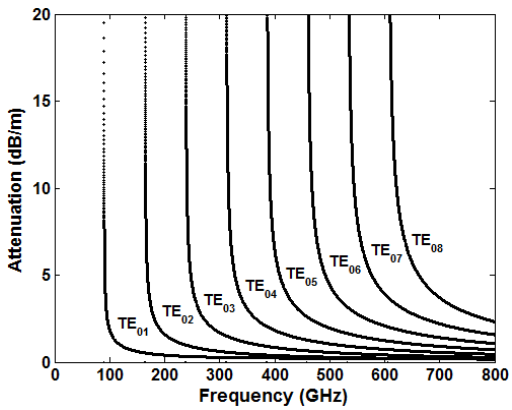

Fig. 3 Attenuation characteristics for different azimuthally symmetric modes TE_{0n} in a gyrotron cavity having electrical conductivity 5.8×10^7 S/m

The value of attenuation constant is very high near the cut-off frequency of the structure but decreases to a quite low value at frequencies above the cut-off. It has been observed that the attenuation constant for the azimuthally symmetric TE_{0n} waves would continue to decrease with increasing frequency. It can be understood from the fact that for TE_{01} wave, the magnetic field components tangential to the cavity surfaces decrease for constant value of the transmitted power as frequency increases. So, the current in the structure walls reduces and hence the losses involved also get reduced. Thus, it has been observed that azimuthally symmetric modes resulted in expected/desired low-loss characteristics at millimeter and sub-millimeter frequency range.

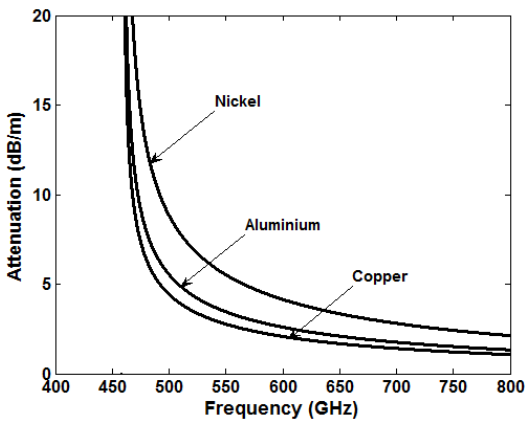

Fig. 4 Attenuation characteristic dependence on the electrical conductivity of a gyrotron cavity excited in azimuthally symmetric mode TE_{06}

Further, attenuation characteristics have been observed for the gyrotron interaction structure made of material of different electrical conductivity and excited in TE_{06} mode. Typically; copper, aluminum, and nickel materials (electrical conductivity 5.8×10^7 S/m, 3.72×10^7 S/m and 1.46×10^7 S/m, respectively) have been chosen for study. As expected, it can be quite clearly observed from Fig. 4 that the material with

higher conductivity results in lower attenuation. Even though aluminum is not used in the practical devices, because of its extensive use in cold testing, this material is also evaluated. Based on the attenuation characteristic results for different cavity materials (Fig. 4), the cavity made of copper can be used in practical devices due to low loss characteristics.

The effect of material electrical conductivity on the RF efficiency, for TE_{06} mode of operation, has also been observed and has been found that the RF efficiency rapidly decreases as the electrical conductivity of the cavity material decreases (Fig. 5). Further, RF efficiency variation with respect to diffractive quality factor variations has also been observed for three different cavity wall materials (Fig. 6). It has been observed that the RF efficiency rapidly decreases with increasing diffractive quality factor, Q_{diff}, (Fig. 6). Again, efficiency for copper cavity is highest among all the three considered cavities.

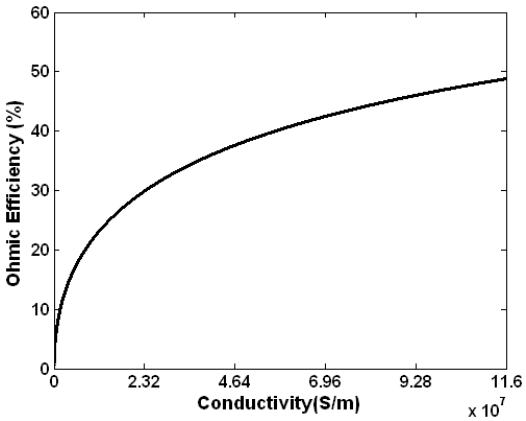

Fig. 5 Ohmic efficiency variations as a function of electrical conductivity of a 460 GHz gyrotron cavity wall excited in azimuthally symmetric mode TE_{06}

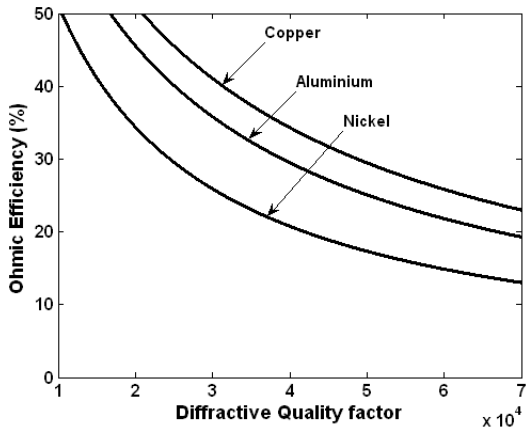

Fig. 6 Ohmic efficiency as a function of diffractive Q of a 460 GHz gyrotron cavity excited in azimuthally symmetric mode TE_{06}

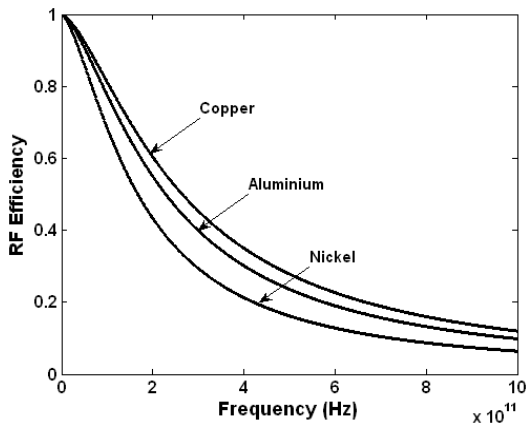

Fig. 7 RF efficiency as a function of frequency for a gyrotron cavity

The effect of material electrical conductivity on the RF efficiency, for TE_{06} mode of operation, has also been observed and has been found that the RF efficiency rapidly decreases as the electrical conductivity of the cavity material decreases (Fig. 5). Further, RF efficiency variation with respect to diffractive quality factor variations has also been observed for three different cavity wall materials (Fig. 6). It has been observed that the RF efficiency rapidly decreases with increasing diffractive quality factor, Q_{diff}, (Fig. 6). Again, efficiency for copper cavity is highest among all the three considered cavities.

The cavity design is a very important step in design process and depends on several factors. The design method, material and the process of fabrication of cavity highly affect the ohmic losses in the cavity. Fig. 7 shows ohmic losses, more specifically RF efficiency, as a function of frequency for the gyrotron cavity. Here again, the ohmic loss variation as a function of frequency of operation for three different cavities made of copper, aluminum and nickel has been observed. The ohmic losses are lowest for the copper cavity and hence the highest RF efficiency. For the gyrotron cavity made of copper material, the extracted power is reduced to 32% of the generated power due to ohmic losses in the walls of the cavity at the 460 GHz operating frequency. It indicates that a significant portion of the power generated in the cavity is not extracted and is instead deposited in the cavity walls in the form of ohmic heating. This can be explained with the help of (18) and (22). It is clear from (18) that quality factor Q is proportional to the square of the frequency, $Q \propto f^2$, and the ohmic quality factor is proportional to the square root of the frequency, as per (22), $Q_{ohm} \propto \sqrt{f}$. That is why ohmic loss increases rapidly with frequency and imposes a limit for output power at high frequencies.

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