International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:11, No:9, 2017

Cognitive SATP for Airborne Radar Based on Slow-Time Coding

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Abstract**—**Space-time adaptive processing (STAP) techniques have been motivated as a key enabling technology for advanced airborne radar applications. In this paper, the notion of cognitive radar is extended to STAP technique, and cognitive STAP is discussed. The principle for improving signal-to-clutter ratio (SCNR) based on slow-time coding is given, and the corresponding optimization algorithm based on cyclic and power-like algorithms is presented. Numerical examples show the effectiveness of the proposed method.

Keywords**—**Space-time adaptive processing (STAP), signal-to-clutter ratio, slow-time coding.

I. Introduction

TRADITIONAL space-time adaptive processing (STAP) involves multidimensional adaptive filtering which combines signals from several antenna elements and from multiple pulse repetitions to suppress clutter, interference and noise in both space and time. It is well known that STAP improves detection performance of targets in both mainlobe and sidelobe clutter and in jamming interference environments. A great deal of attention has been given to STAP algorithms and much of the work has been done in the past three decades [1], [2].

The signal design for radar performance improvement has been an active area of research in the last decades; however, the majority of previous works have considered pulse compression radar and clutter-free scenarios. For detecting a particular target in the presence of additive signal-dependent noise, waveform optimization theory, developed by Guerci [3]-[5], is evaluated in terms of the signal-to-interference-plus-noise ratio (SINR) under a particular model of the system, interference, clutter and targets. The suboptimal solution of the phase-coded waveform for detecting a particular target in additive noise was proposed in [6]. For designing code for MTD in pulsed-Doppler radar, [7] proposed to resort to average and worst case performance metrics and several algorithms to solve highly nonconvex design problems.

It should be pointed out that waveform design problem for STAP has been discussed from the viewpoint of fast-time coding. In [8], [9], waveform design and waveform scheduling problems in STAP for airborne radar are formulated with a cost function, and least-squared solutions for the designed waveform were obtained. In this paper, slow-time coding for STAP for improving SCNR of STAP output is discussed.

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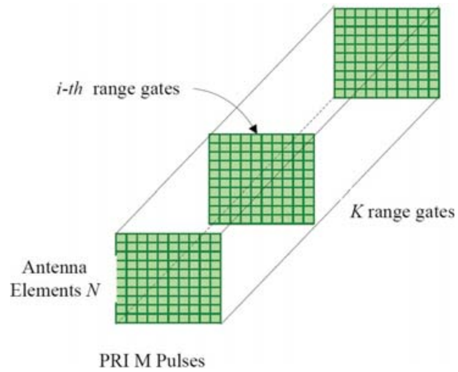

Fig. 1 Phased-array radar CPI datacube

II. STAP Model

The system under consideration is a pulsed Doppler radar residing on an airborne platform. A uniformly spaced linear array consisting of N elements is used as the radar antenna, and the echoes are obtained in a coherent processing interval (CPI). The 3-D radar datacube is shown in Fig. 1, and K denotes the number of samples collected to cover the range interval, and M is the pulse number in a CPI. Actually the data is processed at the interested range bin, which corresponds to a slice of the CPI datacube. This slice, which is a $M \times N$ matrix with $N \times 1$ spatial snapshots for M pulses at the range of interest, is usually transformed into a colum-wise form, i.e., $NM \times 1$ vector $\mathbf{r}_i$, and termed the ith range gate space-time snapshot, $1 \leq i \leq K$ [1].

The radar space-time snapshot is then expressed for each of the two hypotheses in the following form:

$$\begin{cases} \mathbf{H}_0 : \mathbf{r}_i = \mathbf{c}_i + \mathbf{n}_i \\ \mathbf{H}_1 : \mathbf{r}_i = \alpha\mathbf{s} + \mathbf{c}_i + \mathbf{n}_i \end{cases} \tag{1}$$

where α is a zero-mean complex Gaussian random variable with variance σ_s^2, $\mathbf{s}$ denotes the echo signal, and $\mathbf{c}_i$ and $\mathbf{n}_i$ are clutter and noise, respectively. These three components are assumed to be mutually uncoorrelated.

The vector $\mathbf{s}$ is the $NM \times 1$ normalized space-time steering vector in the space-time look-direction and defined as

$$\mathbf{s} = \mathbf{b}_t \otimes \mathbf{a}_t \tag{2}$$

where $\otimes$ denotes the Kronecker product, $\mathbf{b}_t$ is the corresponding M-dimensional Doppler steering vector given by

$$\mathbf{b}_t = [1 \quad e^{j2\pi f_t} \quad \ldots \quad e^{j2\pi(M-1)f_t}]^T \tag{3}$$

and $\mathbf{a}_t$ is the N-dimensional spatial steering vector and given by

$$\mathbf{a}_t = [1 \quad e^{j2\pi\theta_t} \quad \ldots \quad e^{j2\pi(N-1)\theta_t}]^T \tag{4}$$

f_t and θ_t are Doppler frequency and direction angle of arrival of the target, and $(\cdot)^T$ indicates transpose operation of a matrix/vector.

The clutter $\mathbf{c}_i$ can be represented as

$$\mathbf{c}_i = \sum_{l=1}^{N_c} \xi_l^c \mathbf{b}(f_l^c) \otimes \mathbf{a}(\theta_l^c) \tag{5}$$

where N_c denotes the number of the clutter patches, ξ_l^c is the power of reflected signal by the lth clutter patch. $\mathbf{b}(f_l^c)$ and $\mathbf{a}(\theta_l^c)$, respectively, denote the spatial steering vector with the spatial frequency f_l^c and the temporal steering vector with the normalized Doppler frequency θ_l^c for the lth clutter patch. Thus, the clutter covariance matrix can be expressed as

$$\begin{aligned}\mathbf{C} &= \mathbb{E}\{\mathbf{c}_i\mathbf{c}_i^H\} \\ &= \sum_{l=1}^{N_c} \xi_l^c \left[\mathbf{b}(f_l^c)\mathbf{b}(f_l^c)^H\right] \otimes \left[\mathbf{a}(\theta_l^c)\mathbf{a}(\theta_l^c)^H\right]\end{aligned} \tag{6}$$

where $(\cdot)^H$ represents Hermitian transpose and $\mathbb{E}(\cdot)$ denotes expectation. The noise covariance matrix $\mathbf{M} = \mathbb{E}\{\mathbf{n}_i\mathbf{n}_i^H\}$ can be written as a scaled identity matrix $\sigma_n^2\mathbf{I}_{NM}$, where σ_n^2 is the noise power.

The optimum full-rank STAP filter obtained by an unconstrained optimization of the SCNR is given as follows:

$$\mathbf{w}_{\text{opt}} = \kappa(\mathbf{C} + \mathbf{M})^{-1}\mathbf{s} \tag{7}$$

The corresponding optimum SCNR is given by

$$\text{SCNR} = \mathbf{s}^H(\mathbf{C} + \mathbf{M})^{-1}\mathbf{s} \tag{8}$$

III. Problem Formulation

In this section, the radar system is considered to transmit a series of weighted pulse. Using (1), the target detection problem can also be cast as the following binary hypothesis test

$$\begin{cases} \mathbf{H}_0 : \mathbf{r}_i = \mathbf{c}_i \odot (\mathbf{p} \otimes \mathbf{1}) + \mathbf{n}_i \\ \mathbf{H}_1 : \mathbf{r}_i = \alpha\mathbf{s} \odot (\mathbf{p} \otimes \mathbf{1}) + \mathbf{c}_i \odot (\mathbf{p} \otimes \mathbf{1}) + \mathbf{n}_i \end{cases} \tag{9}$$

where $\mathbf{p} = [p_0 \quad p_1 \quad \ldots \quad p_{M-1}]^T$ are the transmit weights that are to be optimally designed, and $\mathbf{1} = [1 \quad 1 \quad \ldots \quad 1]^T$. The performance of the optimum detector depends on the following SCNR

$$\text{SCNR} = (\mathbf{s} \odot (\mathbf{p} \otimes \mathbf{1}))^H(\mathbf{PCP}^H + \mathbf{M})^{-1}(\mathbf{s} \odot (\mathbf{p} \otimes \mathbf{1})) \tag{10}$$

where $\mathbf{P} = \text{Diag}(\mathbf{p} \otimes \mathbf{1})$, $\text{Diag}(\cdot)$ denotes forming a diagonal matrix whose diagonal entries are formed by a vector.

Slow-time code design to improve the detection performance of the STAP system for a known target Doppler frequency shift f_t and arrival angle θ_t can be accomplished by maximizing the following performance metric:

$$\begin{aligned}\text{SCNR} &= (\mathbf{s} \odot (\mathbf{p} \otimes \mathbf{1}))^H(\mathbf{PCP}^H + \mathbf{M})^{-1}(\mathbf{s} \odot (\mathbf{p} \otimes \mathbf{1})) \\ &= \text{tr}\left(\mathbf{P}^H(\mathbf{PCP}^H + \mathbf{M})^{-1}\mathbf{Pss}^H\right) \\ &= \text{tr}\left\{((\mathbf{P}^H\mathbf{MP})^{-1} + \mathbf{C})^{-1}\mathbf{ss}^H\right\}\end{aligned} \tag{11}$$

where $\text{tr}(\cdot)$ represents the trace of a matrix.

In cases where Doppler frequency shift and arrival angle of the interested target lie in a certain area, we consider the following design metric:

$$\text{tr}\left\{((\mathbf{P}^H\mathbf{MP})^{-1} + \mathbf{C})^{-1}\mathbf{S}\right\} \tag{12}$$

were $\mathbf{S} = \mathbb{E}\{\mathbf{ss}^H\}$. To optimize the detection performance for a STAP radar, the metric above can be maximized under an energy constraint:

$$\begin{aligned}&\max_{\mathbf{P}} \quad \text{tr}\left\{((\mathbf{P}^H\mathbf{MP})^{-1} + \mathbf{C})^{-1}\mathbf{S}\right\} \\ &\text{s.t.} \quad \text{tr}(\mathbf{PP}^H) = e\end{aligned} \tag{13}$$

where e denotes the transmission energy.

IV. Code Design Algorithms

In this section, the algorithm is given based on cyclic algorithm and power-like iteration method.

We begin by note that as $\mathbf{S} \succeq 0$ there must exist a full column-rank matrix $\mathbf{V} \in \mathbb{C}^{NM\times\delta}$ such that $\mathbf{S} = \mathbf{VV}^H$. As a result,

$$\begin{aligned}&\text{tr}\left\{((\mathbf{P}^H\mathbf{MP})^{-1} + \mathbf{C})^{-1}\mathbf{S}\right\} \\ &\quad = \text{tr}\{\mathbf{V}^H\mathbf{P}^H(\mathbf{PCP}^H + \mathbf{M})^{-1}\mathbf{PV}\}\end{aligned} \tag{14}$$

Let $\Theta = \mathbf{V}^H\mathbf{P}^H(\mathbf{PCP}^H + \mathbf{M})^{-1}\mathbf{PV}$. As a consequence, the maximization problem in (13) can be transformed to

$$\begin{aligned}&\max_{\mathbf{P}} \quad \text{tr}(\Theta) \\ &\text{s.t.} \quad \text{tr}(\mathbf{PP}^H) = e\end{aligned} \tag{15}$$

For solving this problem, we give the following several lemmas.

Lemma 1: Θ^{-1} can be expressed as

$$\Theta^{-1} = \mathbf{T}^H\mathbf{R}^{-1}\mathbf{T} \tag{16}$$

where $\mathbf{T} = [\mathbf{I}_{\delta\times\delta} \quad \mathbf{0}_{NM\times\delta}]^T$, and

$$\mathbf{R} = \begin{bmatrix}\mathbf{0}_{\delta\times\delta} & j\mathbf{V}^H\mathbf{P}^H \\ j\mathbf{PV} & \mathbf{PCP}^H + \mathbf{M}\end{bmatrix} \tag{17}$$

The proof of Lemma 1 can be easily obtained, and the detailed verification process is omitted here.

Lemma 2: Let an auxiliary $(NM + \delta) \times \delta$ matrix $\mathbf{Y}$, $\Xi = \mathbf{Y}^H\mathbf{RY}$, and $\Theta^{-1}\Xi = \mathbf{I}_{\delta\times\delta}$. The solution of $\mathbf{Y}$ is given by

$$\mathbf{Y} = [\mathbf{Y}_1 \quad \mathbf{Y}_2]^T \tag{18}$$

International Journal of Information, Control and Computer Sciences
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TABLE I
THE PROPOSED ALGORITHM FOR OPTIMIZING SLOW-TIME CODE

Step 0: initialize $\mathbf{p}^{(0)}$ using uniform code, $k=0$;
Step 1: calculate the matrices $\mathbf{S}$ and $\mathbf{V}$,
Step 2: calculate $\mathbf{Y}_2 = j(\mathbf{PCP}^H + \mathbf{M})^{-1}\mathbf{PV}$;
Step 3: calculate $\mathbf{K} = \mathbf{C} \odot (\mathbf{Y}_2\mathbf{Y}_2^H)$;
Step 4: calculate $\overline{\mathbf{p}}^{(k+1)} = \mathbf{K}\overline{\mathbf{p}}^{(k)}$;
Step 5: update $\mathbf{p}_{opt} = (\mathbf{HH}^H)^{-1}\mathbf{H}^H\overline{\mathbf{p}}_{opt}$;
Step 6: set $k=k+1$, repeat the steps (2)$\sim$(5) until a certain stop criterion

TABLE II
AIRBORNE PHASED-ARRAY RADAR PARAMETERS

Parameters	Value
Carrier frequency	10.0GHz
System bandwidth	5MHz
Pulse repetition frequency	5000Hz
Flight velocity	37.5m/s
Antenna array spacing	1.5cm
Elements of antenna array N	16
Number of pulses M	16
Clutter-to-noise ratio(CNR)	40dB

where $\mathbf{Y}_1 = \mathbf{I}_{\delta\times\delta}$, $\mathbf{Y}_2 = j(\mathbf{PCP}^H + \mathbf{M})^{-1}\mathbf{PV}$.

Proof: By substituting (18) into $\mathbf{Y}^H\mathbf{RY}$, we have

$$\begin{aligned}\mathbf{Y}^H\mathbf{RY} &= [\mathbf{Y}_1^H \quad \mathbf{Y}_2^H]\begin{bmatrix}\mathbf{0}_{\delta\times\delta} & j\mathbf{V}^H\mathbf{P}^H \\ j\mathbf{PV} & \mathbf{PCP}^H + \mathbf{M}\end{bmatrix}\begin{bmatrix}\mathbf{Y}_1 \\ \mathbf{Y}_2\end{bmatrix} \\ &= j\mathbf{Y}_2^H\mathbf{PVY}_1 + j\mathbf{Y}_1^H\mathbf{V}^H\mathbf{P}^H\mathbf{Y}_2 \\ &\quad + \mathbf{Y}_2^H(\mathbf{PCP}^H + \mathbf{M})\mathbf{Y}_2\end{aligned} \tag{19}$$

Note that when $\mathbf{Y}_1 = \mathbf{I}_{\delta\times\delta}$, $\mathbf{Y}_2 = j(\mathbf{PCP}^H + \mathbf{M})^{-1}\mathbf{PV}$,

$$\begin{aligned}\Xi &= \mathbf{Y}^H\mathbf{RY} \\ &= \mathbf{V}^H\mathbf{P}^H(\mathbf{PCP}^H + \mathbf{M})^{-1}\mathbf{PV} = \Theta\end{aligned} \tag{20}$$

With Lemma 2, we can conclude $\mathbf{Y}^H\mathbf{U} = \mathbf{I}_{\delta\times\delta}$. The optimization problem in (15) can be transformed to

$$\begin{aligned}\max_{\mathbf{P},\mathbf{Y}} \quad & \mathrm{tr}(\Xi) \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{PP}^H) = e \\ & \Theta^{-1}\Xi = \mathbf{I}_{\delta\times\delta}\end{aligned} \tag{21}$$

In the following optimization process, alternating direction method is considered to be used here. The auxiliary matrix $\mathbf{Y}$ is to guarantee the constraint condition $\Theta^{-1}\Xi = \mathbf{I}_{\delta\times\delta}$, and $\Xi = \Theta$. Therefore, for fixed $\mathbf{P}$, the solution of $\mathbf{Y}$ can be obtained by Lemma 2.

For fixed $\mathbf{Y}$, the optimization problem in (21) is simplified to

$$\begin{aligned}\max_{\mathbf{P}} \quad & \mathrm{tr}(\mathbf{Y}_2^H(\mathbf{PCP}^H + \mathbf{M})\mathbf{Y}_2) \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{PP}^H) = e\end{aligned} \tag{22}$$

(22) can also be described by

$$\begin{aligned}\max_{\overline{\mathbf{p}}} \quad & \overline{\mathbf{p}}^H(\mathbf{C} \odot (\mathbf{Y}_2\mathbf{Y}_2^H))\overline{\mathbf{p}} \\ \text{s.t.} \quad & \overline{\mathbf{p}}^H\overline{\mathbf{p}} = e\end{aligned} \tag{23}$$

where $\overline{\mathbf{p}} = \mathrm{diag}(\mathbf{P})$, $\mathrm{diag}(\cdot)$ means the diagonal entries of a matrix. Note that the positive semidefiniteness of $\mathbf{C}\odot(\mathbf{Y}_2\mathbf{Y}_2^H)$ guarantees the convexity of (23). The QCQP in (23) can be solved efficiently using the power-like iteration method.

The local optimized solution of $\overline{\mathbf{p}}$ is given by the power-like iterations:

$$\begin{aligned}\min \quad & \left\|\overline{\mathbf{p}}^{(k+1)} - \mathbf{K}\overline{\mathbf{p}}^{(k)}\right\|_2^2 \\ \text{s.t.} \quad & \overline{\mathbf{p}}^{(k+1)H}\overline{\mathbf{p}}^{(k+1)} = e\end{aligned} \tag{24}$$

where $\mathbf{K} = \mathbf{C} \odot (\mathbf{Y}_2\mathbf{Y}_2^H)$, $\overline{\mathbf{p}}^{(k+1)}$ and $\overline{\mathbf{p}}^{(k)}$ are the $(k+1)$th and kth iteration results. $\|\cdot\|_2^2$ denotes the ℓ_2 norm. In every iteration, $\overline{\mathbf{p}}^{(k+1)}$ is given by

$$\overline{\mathbf{p}}^{(k+1)} = \mathbf{K}\overline{\mathbf{p}}^{(k)} \tag{25}$$

The iterations stop when the difference between two consecutive iteration results is small enough, i.e., $\left\|\overline{\mathbf{p}}^{(k+1)} - \overline{\mathbf{p}}^{(k)}\right\|_2 \leq \epsilon$, where ϵ is a predefined value. Note that the optimal value $\overline{\mathbf{p}}_{opt}$ is actually the Kronecker product of $\mathbf{p}_{opt}$ and $\mathbf{1}$. Let $\mathbf{H} = \mathbf{I}_M \otimes \mathbf{I}_N$, the optimal value of $\mathbf{p}_{opt}$ can be obtained by least-square (LS) estimator:

$$\mathbf{p}_{opt} = (\mathbf{HH}^H)^{-1}\mathbf{H}^H\overline{\mathbf{p}}_{opt} \tag{26}$$

The whole optimization process is summarized as Table I.

V. NUMERICAL EXAMPLES

In this section, we asses the proposed optimization algorithm using simulated radar data. The parameters of the simulated radar platform are shown in Table II. The thermal noise is modeled as a Gaussian white noise with unity power. The jamming, clutter and target powers can be referred to the white noise power. For all simulations, the clutter-to-noise-ratio (CNR) is fixed at 40 dB. All presented results are averages over 1000 independent Monte Carlo runs.

To valuate SCNR improvement by incorporating slow-time coding, the metric of $\mathrm{IMP}_{\mathrm{SCNR}}$ is defined as

$$\mathrm{IMP}_{\mathrm{SCNR}} = 10\log_{10}\frac{\mathrm{SCNR}_{\mathrm{opt}}}{\mathrm{SCNR}}(dB) \tag{27}$$

where $\mathrm{SCNR}_{\mathrm{opt}}$ and SCNR denote calculated SCNR with optimized slow-time coding and without coding.

To optimize the slow-time code, the interested area in spatial-frequency and Doppler-frequency plane should be selected. Considering ground clutter appears on a ridge of spatial-frequency and Doppler-frequency plane in STAP output, we pick a certain area, which is parallel to the clutter ridge, for optimizing the slow-time code. The relationship of the spatial-frequency θ_s and the Doppler-frequency f_s in the selected area is given by

$$\theta_s = f_s \pm \frac{\Delta(s)}{N} \tag{28}$$

where $\Delta(s)$ indicates the difference between θ_s and f_s, s is the parallel line width of the interested area.

Fig. 2 shows the metric of $\mathrm{IMP}_{\mathrm{SCNR}}$ on the whole spatial-frequency and Doppler-frequency plane with $s=2$. About 2dB SCNR improvement can be obviously seen form the area, which is parallel to the clutter ridge, and no SCNR loss appears on the clutter ridge. Therefore, slow-time coding can effectively improve slow moving target detection performance for STAP radar system.

International Journal of Information, Control and Computer Sciences
ISSN: 2517-9942
Vol:11, No:9, 2017

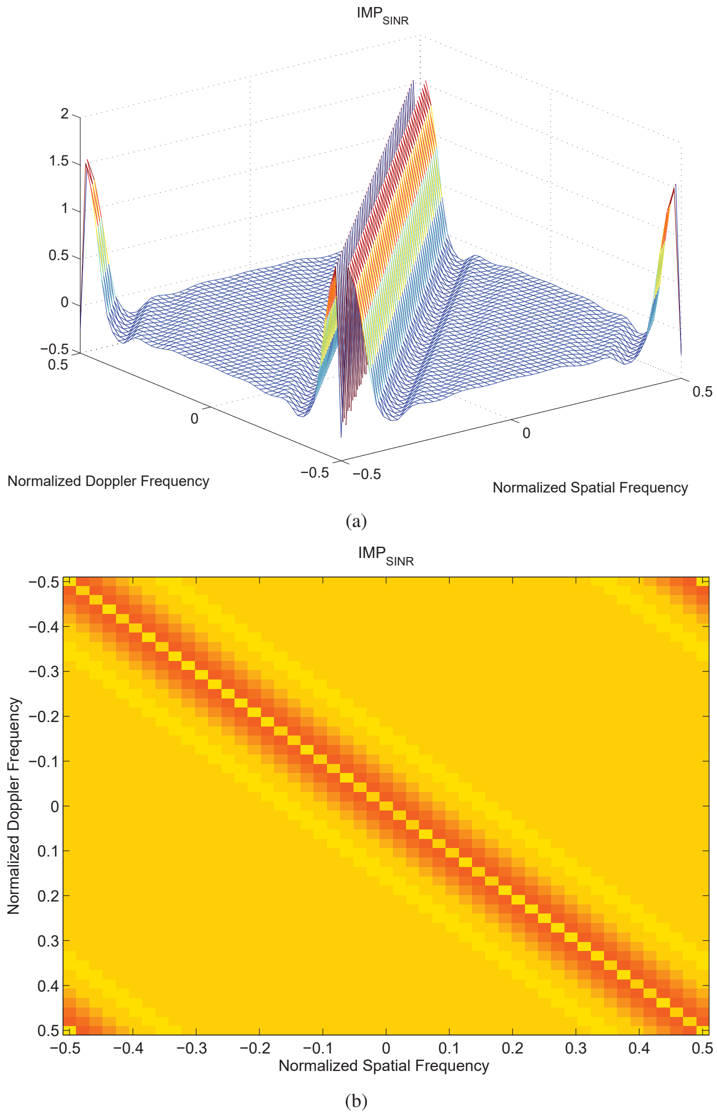

Fig. 2 The metric of $\mathrm{IMP}_{\mathrm{SCNR}}$, (a) view from side look, (b) view from top look

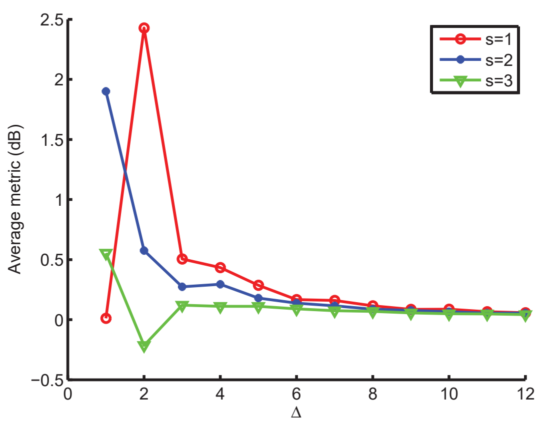

Fig. 3 Average metric in dB scale vs. Δ

Fig. 3 demonstrates the average metric over the interested area. As Δ increases, $\mathrm{IMP}_{\mathrm{SCNR}}$ decreased fast. Therefore, slow-time coding can only improving slow moving target detection. At the meanwhile, s should be carefully chosen. $s = 2$ indicates a better performance.

VI. Conclusions

The principle for STAP radar for improving SCNR based on slow-time coding is described in this paper. When slow-time code is optimized according to the selected area on spatial-frequency and Doppler frequency plane, SCNR metric can be improved accordingly. The optimization algorithm based on cyclic and power-like iteration algorithms is proposed. Numerical examples show the effectiveness of SCNR improvement around the clutter ridge. Therefore, slow-time coding can optimize slow moving target detection performance.

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