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Classifications of Neuroscientific-Radiological Findings on "Practicing" in Mathematics Learning

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Abstract**—**Many people know 'Mathematics needs practice!' statement or similar ones from their mathematics lessons. It seems important to practice when learning mathematics. At the same time, it also seems important to practice how to learn mathematics. This paper places neuroscientific-radiological findings on "practicing" while learning mathematics in a context of mathematics education. To accomplish this, we use a literature-based discussion of our case study on practice. We want to describe neuroscientific-radiological findings in the context of mathematics education and point out stimulating connections between both perspectives. From a connective perspective we expect incentives that lead discussions in future research in the field of mathematics education.

Keywords**—**fMRI, education, mathematics learning, practicing.

I. Introduction

MATHEMATICS education depends on an interdisciplinary exchange. So far, a neuroscientific-radiological perspective has played a rather subordinate role for problems and questions of mathematics education. Yet, neuroscientific and radiological findings as well as the latest research results concerning the function and structure of our brain can be used as a potential discipline of reference to question (already existing) concepts, discuss consequences and to gain new incentives.

As neuroscientific research was able to yield numerous findings and diagnoses about functions and structures of our brain in recent years, it seems interesting to look at existing concepts of teaching and learning mathematics from an interdisciplinary perspective. In this sense, the aim of this article is to develop a classification of neuroscientific-radiological findings on practice when it comes to learning mathematics in mathematics education and it will become clear whether "neuroscience research today is setting the scene for future developments in mathematics education [1, p.3]".

This paper is designed to report on the current state and to embed it into the context of mathematics education in order to develop further connective research elements, concepts and questions on this basis.

In the field of mathematics education, one seeks to "develop a clear relation to practice [2, p.4]", while practice already has a long (research) tradition as a concept and principle of mathematics education. "We usually refer to 'practice' when a set of elements of knowledge or a skill is practiced in a great number of similar tasks [3, p.177]". With regard to school practice, the image seems to be fluctuating, perhaps because "the boundaries between meaningful practice and stubborn drill […] remain undefined [4, p. 5]". It may also be due to the fact that a meaningful design of exercises and phases of practice in mathematical teaching and learning processes is perceived as challenging [2].

An initial finding from a review of neuroscientific-radiological studies [13], [27], [28] is that drill training or items based on drill training often play a decisive role in the inquiry, analysis and gain in (scientific) insight. We want to include these studies in our article that deal with drill training in relation to mathematical tasks and we understand this as a form of practice (in the sense of mathematics education). This enables us to establish connections between neuroscientific-radiological studies and a perspective of mathematics education.

II. Theoretical Framework

Over the centuries, the individual sciences have evolved into their present differentiation and accumulated a high degree of knowledge. However, they have also become independent units and partly demarcated, or as Reusser puts it, they "formed disciplines that cross boundaries but are still independent [5, p.224]". The cross-over effects between individual scientific disciplines are pronounced to a different degree, depending on the ability of mutual or one-sided ways to use them. Yet different sciences, including mathematics education cannot be studied in isolation, but rather in discourse with other disciplines of reference and their findings and experiences [6]. As Wußing also describes in his work "6000 years of Mathematics" [7], one is confronted with "a number of topics, depending on one's scientific goals and tendencies [7, p.1]". Over the centuries, medicine has not only experienced and appreciated the professional exchange between different peoples and nations in a traditional way but has also upheld the good tradition of acquiring knowledge from different disciplines and putting it into new targeted contexts, also through unorthodox recombination of individual knowledge spades. Not only has medicine literally been ballooned by the expansion through the integration of knowledge from other scientific disciplines. It has also benefited from the further development of established contents of knowledge through research and science. Other sciences, which fortunately have the same interest in further development, are also likely to be subject to similar effects.

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The (continuously advancing) development of magnetic resonance imaging as a diagnostic method within the medical field of radiology and in particular the subspate of functional magnetic resonance imaging (short: fMRI), enables structural and functional representations of cognitive processes in the human brain, i.e., the direct scientific insight into the head. This makes the (neuro-)radiologist an expert for the analysis and interpretation of Real-Time Data collected and obtained by fMRI [8]. Based on this premise, it seemed particularly important to describe (new) connections between neuroscientific-radiological findings and areas of mathematics education – using the case example of practice.

In the introduction of his work "The number sense", Dehaene states: „My hypothesis is that the answers to all these questions must be sought at a single source: the structure of our brain. Every single thought we entertain, every calculation we perform, results from the activation of specialized neuronal circuits implanted in our cerebral cortex. Our abstract mathematical constructions originate in the coherent activity of our cerebral circuits and of the millions of other brains preceding us that helped shape and select our current mathematical tools. Can we begin to understand the constraints that our neural architecture impose on our mathematical activities? [9, p.4]". In this context, arithmetic as a branch of mathematics often seems to be part of the exploratory instrument in the field of cognitive neuroscience. Delaezer gives two reasons for this: "first, simple arithmetic is an ideal field to study the acquisition of new expertise, since learning conditions and learning contents can be easily defined. Second, the acquisition of arithmetic facts is of crucial importance for young students, as well as for patients after acquired brain damage [10, p.839]".

In his paper "Neuroscience and subject-related educational research – discussed in the context of mathematics" Bauersfeld, a researcher in the field of mathematics education, discusses the relevance that neuroscience can have for subject-related educational research – using the example of mathematics [11]. As a picture of our brain, he draws a "heterogeneous group of speechless agents, each of whom is not capable of much on their own, but who are extremely powerful due to their division of labor, Marvin Minsky's 'society of mind' [11, p.4]" and "for some keywords of learning and teaching, he presents relevant results [...] and conclusions for teaching [11, p.4]".

Tall also identifies connections between the biological brain and a mathematical mind [12]. According to him, "the mathematical mind has all kinds of associations within the multi-processing brain [12, p.3]". The term "mathematical mind" is used to refer to the way "in which the processes and concepts of mathematics are conceived and shared between individuals [12, p.1]".

In the literature-based discussion of our case study on practice, we want to describe neuroscientific-radiological findings in the context of mathematics education and point out stimulating connections between both perspectives for future developments.

III. Material and Method

For the choice of publications used for our literature-based discussion, we used the platform „ScienceDirect" for certified scientific literature from Elsevier (connection to step 2), see below). ScienceDirect contains scientific research on current topics and new cognitions. In particular, it is characterized by a variety of scientific publications in health care. In addition, our literature research only included journal articles. Our literature-based discussion follows three steps:

1) Planning the research based on pre-defined categories,
2) Conducting research and choice,
3) Discussion of selected publications using a systematic scheme.

The following categories are applied to step 1: As the studies are supposed to be describable based on concept and principles of mathematics education, they need to contain mathematical items. Therefore, we investigated whether the selected studies used items with mathematical contents. The results should contain statements about mathematics (e.g. about methods of calculation such as multiplication, about methods such as solving equations or algorithms). At the same time, this entailed another category, the category of practice which is called "(drill) training" in the studies. In addition, this yields in a connection to the mathematical items, as they as well had to contribute to the question of the respective study through "training" or "drill training". The neuroscientific-radiological perspective was selected as a further category. It implies that the selected publications investigate the functioning of the brain and that they used fMRI as an imaging technique for the collection and presentation of data. These four categories were decisive for our step 2. In support of the literature-based discussion of our selection, we based the description on the following schematic presentation:

1. Question/Hypothesis
2. Setting
3. Output/Outcome
4. View on mathematical items within mathematics education (validity)
5. The perspectives that emerge for further research in mathematics education

In the following, the selected studies are discussed in the sense of a fact sheet and described and presented in accordance with our scheme.

IV. Results and Discussion

In the following, we want to describe neuroscientific-radiological findings on practice schematically and classify them within mathematics education for our paper. In this case, the studies are arranged in alphabetical order and not based on a hierarchy or the like.

A. Classification

1) Anderson et al.: A Central Circuit of the Mind [13]

a) Question/ Hypothesis

Data from complex tasks (e.g., solving linear equations) are to be understood by means of a developed model on cognitive architecture [13], in order to make predictions about which brain regions are activated.

b) Setting

The study examines children (aged 11-14).

- No information about previous knowledge of the children is provided
- Solving linear equations is practiced through drill training on 5 consecutive days:

$$\text{“}0-step\text{: }e.g., 1x+0=4$$
$$1-step\text{: }e.g., 3x+0=12 \text{ or } 1x+8=12$$
$$2-step\text{: }e.g., 7x+1=29 \text{ [13, p.5]”}$$

- 0. day: specific learning unit on solving equations
- 1.-5. day: 1h of practicing equations with an increasing level of difficulty (0-step to 2-step) on a computer.

In a former study by Anderson, we could find clues about solving equations on a computer [14]. As the study deals with problem-solving, it is not presented further in this article. An equation is displayed on the computer screen. Then, the participant can choose between different transformations, type in values, ask for clues and the evaluation of results or request a new equation.

- Data collection with the help of fMRI
- 0. day: Checking, how long it takes all participants to solve the respective equations (0-step to 2-step).
- 1.-5. day: Controlling the respective equations (0-step to 2-step) after 1h of practice.

c) Output/ Outcome

- Fig. 1 shows the solving time throughout the days on which the study was conducted. Before carrying out the fMRI and capturing the solving times, the test subjects were asked to practice for an hour on every testing day.
- The exam day (day 5, Fig. 1) yielded the following results: All participants had clearly improved their solving time in comparison to the initial findings. Even with an increasing level of difficulty according to the classification into 0-step to 2-step.

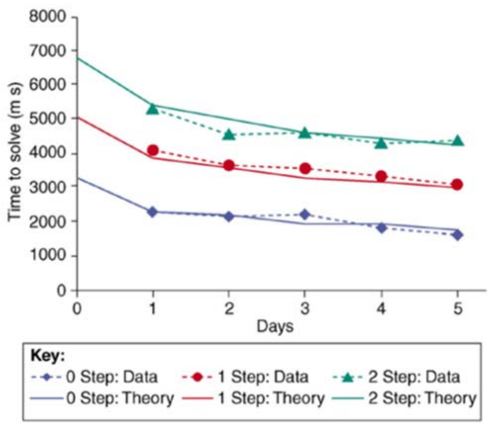

Fig. 1 Gathering the solving times on each day of testing for the respective equations (0-step to 2-step) [13, p.6]

d) View on Mathematical Items within Mathematics Education (Validity)

From the perspective of mathematics education, the scaling of the increasing levels of difficulty is to be criticized (0-step to 2-step). In this case, the cognitive level of requirements is considered to be correspondent to a number of steps that are required to solve simple equations through equivalent transformations – combined in one variable. In our opinion, this represents a clear condensation of models of difficulty in mathematics education [15]. Conceptual epistemological obstacles, for example, appear to be more important than the number of steps that is required for a solution: The occurrence of zero as a theoretical concept is a significant obstacle in school mathematics that is based on perception. Thus, the occurrence of zero as a theoretical term in conceptually mathematics derived from the view is an essential obstacle, i.e., solving an equation like $1x+0=4$ can be more difficult than solving $7x+1=29$.

e) The Perspectives that Emerge for Further Research in Mathematics Education

Based on a case study, this study provides neuroscientific evidence for the thesis that repetitive formats of practice (“drill training”) can lead to processes of automatization and that the brain can be relieved regarding necessary processes of activation. With regard to findings within mathematics education, this assumption does not seem surprising. Still, the neuroscientific-radiological perspective can contribute to the reasoning.

2) Delazer et al.: Learning by Strategies and Learning by Drill-Evidence from an fMRI Study [10].

a) Question/ Hypothesis

- Two main topics are considered in this study
  - First: The effects of training (comparison of new and trained items based on the same algorithm)
  - Second: Effects of different methods of training (comparison of items that have been learned through different methods).3
- Hypothesis: „numerical training leads to a shift of activation within parietal areas [10, p.845]”.

b) Setting

- Arithmetic was selected as the topic. This has two reasons: First of all, arithmetic is an ideal field to investigate the acquisition of new technical knowledge because learning conditions as well as learning content can be easily defined. Second, the acquisition of arithmetic facts is vitally important for university students on the one hand and for patients who suffered brain damages on the other.
- 16 test subjects (9 females), aged 26, participated in the study. 9 (4 females) stayed for the FMRI test. Participants were either university students or had other academic degrees.
- All participants have good arithmetic skills.
- The participants were trained in two new mathematical operations. One part of the mathematical operations was

practiced through drill training. The other through a given algorithm.

- There were 5 sessions of 45 min. (from Monday until Friday). Each daily session contained a drill training of a mathematical operation and a strategy training of another mathematical operation.
- 90 blocks of training were provided for each mathematical operation throughout the five sessions.
- The reaction time as well as the accuracy was registered.
- The problems were displayed on a screen and the participants typed in a two-digit number on the numeric keypad on the right of the keyboard. They immediately received positive and negative feedback. If there was a mistake, the problem was repeated until the answer was correct.

Strategy Learning

- In their first training session, the participants received an exemplary worksheet with a description of the algorithm as well as six exemplary problems. According to [16] to whom the study referred to in terms of the items, this is a 3-step arithmetic algorithm:

"#" stood for an operation/ a sign.

For example 4 # 17 = ?

1. $step: 17 - 4 = 13$ (left number subtracted from number on the right)

2. $step: 13 + 1 = 14$ (add 1 to the result)

3. $step: 17 + 14 = 31$ (add the result from step 2 to the number on the right)

Then this worksheet was removed for the following training sessions. Afterwards a problem was displayed. It was to be solved when the problem was presented a second time and the participants typed in their solution with the keyboard. The problem was then visible until the test subjects typed in their answers (there was no time limit).

- In the third and fifth training sessions, the participants were examined and the strategies they used were recorded. „How did you solve the preceding operation? a = algorithm, g = retrieval, s = other [10, p.840]".

Drill Training

The participants were instructed to memorize the drill-problems, without using a Back-up-Strategy. In this case, the problems were presented twice as well. The first time, the problem as well as the correct solution was presented. The second time, the solution was supposed to be typed into the keyboard by the test subjects.

FMRI Test

Three conditions were used:

- „Drill" (items that have previously been learned by drill),
- "Strategy" (Items that have previously been learned through the application of strategies)İ
- „New" (new problems, which are solved with the same algorithm as the strategy items). The problems were displayed on the screen and the correct number of two had to be specified by pushing a button. The correct number and the wrong one were in a range of 18-35.

Known problems were used for "drill" and "strategy", whereas new problems e.g. with a similar level of difficulty were generated for the category "new".

c) Output/Outcome

- The acquisition of new arithmetic operations causes a change of cerebral activation (the brain changed its activity related to the brain area as well as the brain area itself) depending on the understanding and the repeated practice.
- Newly acquired arithmetic knowledge was implemented into already existing arithmetic processes and memories.
- Training leads to a change of cognitive processes in mental arithmetics.
- The authors were able to show that both, the brain activity as well as the participating brain areas in a mathematical network depend on the method of learning.

d) View on Mathematical Items within Mathematics Education (Validity)

Artificial algorithms were created for this study, which enabled the test subjects to "solve" predetermined tasks in a specific number of steps. At first, the selected items seemed to be strange from the perspective of mathematics education – but with regard to the design of the study the choice makes sense. As a new set of rules is preset, it is not possible to draw on existing, already automatized knowledge. Certainly, there can still be great differences regarding the implicitly available, procedural knowledge [17]. In this case as well, a pretest regarding the selected items would have been useful from the perspective of mathematics education.

e) The Perspectives that Emerge for Further Research in Mathematics Education

In the present study, the so-called scheme aspect [18], which is to be avoided in comprehension-oriented mathematics classes, is systematically brought into the foreground. The study focuses on the aspect that special methods and strategies lead to the fact that mathematical knowledge is processed and stored in fundamentally different ways. This indicates that people construct their (mathematical) knowledge autonomously, domain specific and individually in training situations as well [19], [20]. This provides a strong incentive to investigate heterogeneous approaches as well as training sequences within mathematics education.

3) Ischebeck et al.: Flexible Transfer of Knowledge in Mental Arithmetic – An fMRI Study [21].

a) Question/ Hypothesis

The aim of the present fMRI-study is to examine, whether and how newly acquired arithmetic knowledge of trained problems of multiplication can be transferred to problems of division.

b) Setting

- There were 17 participants (7 females), 25 years old and right-handed. The students came from the University of Innsbruck (without any known neurologic or psychiatric diseases).

- 50 problems of multiplication with a comparable level of difficulty were created (of the form; that two-digit numbers, multiplied by a number of one digit, results in a three-digit result). The division tasks were created in relation to the multiplication tasks. If 17x7(=119) then 119:7(=17).
- 10 out of 50 tasks are practiced with the participants. Each of the 10 tasks is solved 72 times (we could not find any clues, why the tasks were repeated 72 times. The authors claim that one wanted to make sure that the results could be retrieved from memory).
- The training was carried out in individual sessions on a computer and lasted at most 2h. After every process of solving a task, feedback was provided immediately. The task was repeated until the correct answer was given.
- The multiplication and the division were tested separately, with 20 tasks each.
- Pre- and post-tests were also carried out on a computer, but no feedback was provided.
- The fMRI scan was carried out one day after the training.

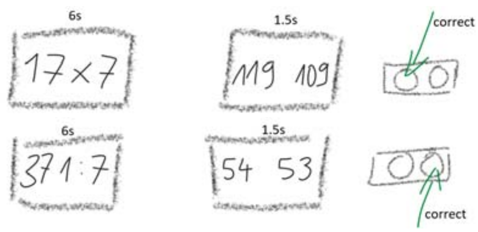

Fig. 2 Tasks as presented in the scanner

Fig. 2 shows how the tasks were presented to the participants in the scanner. A result had to be selected/chosen. The reaction time was measured.

c) Output/Outcome

The study was able to show that the left angular gyrus is not only involved in the retrieval of stored arithmetic facts to the highest degree, but that it is also decisive for the transfer between arithmetic operations (multiplication, e.g. 17·7 and division, e.g. 371:7).

While untrained multiplication problems activated several frontal and parietal brain areas more strongly than trained ones, it was discovered that trained multiplication problems almost exclusively activated the left angular gyrus and only discretely the rest of the fronto-parietal network. This means that practicing multiplication problems leads to the reduction of activity in the network and the focus on fewer centers, so that energy (activity) can be saved and that these centers can become “free” to some extent, for parallel activities regarding other cognitive functions. Keyword here is the inner calculative tendency of the brain.

The activation of the left angular gyrus was also higher for participants who showed a transfer effect in division tasks (i.e. those division tasks that were related to the previous problem of multiplication). Therefore, the study clearly shows that the transfer of knowledge between arithmetic operations (multiplication and division) is accompanied by changes regarding the activation in a mathematical network. Such a transfer is also accompanied by a changed involvement of brain areas in the network, in terms of an optimizing reduction regarding number, size and degree of activation.

d) View on Mathematical Items within Mathematics Education (Validity)

The items used in the study come from arithmetics and are related to the understanding of operations of primary school children [22], [23], which has been researched widely in mathematics education. These so-called inverse tasks offer references to the developmental-pychological term of reversibility by Jean Piaget. Even if the acquisition of an understanding of inverse tasks is characteristic of arithmetic classes, the cognition of the phenomenon of reversibility – which is referred to as “transfer” in the present study – is no longer described as a logical necessity. These connections rather have to be “learned” in a conventional sense. From the perspective of mathematics education, these items are indeed suitable to show learning progress; but with regard to elementary school children rather than adults who should be familiar with these connections under normal circumstances. In any case, the neuroscientific-radiological findings in the present study seem to indicate that a certain training effect leads to the shifting of networks of involved brain areas in terms of an optimizing reduction regarding the number, size and degree of activation – but there are no provable mathematical “skills centers” among non-specialized test subjects and those who are intellectually fixated on specific domains.

e) The Perspectives that Emerge for Further Research in Mathematics Education

The neuroscientific-radiological findings seem to indicate that in terms of the present study, a certain training effect leads to a permanent shift of the networks of the brain areas that are involved, in the sense of an optimizing reduction regarding number, size and degree of activation – the existence of “static” centers of mathematical skills cannot be proven.

The (left) angular gyrus which is linked to the visual and auditory centers as well as to higher sensory and motor cortical areas, as a higher area of association of the cerebral cortex, seems to be of central and functional importance to the study at the time when there is an explicit connection between the inverse tasks used in the study. As the neuroscientific-radiological findings suggest, a description of these networks that continuously reorganize themselves when it comes to learning mathematics should be highly relevant for mathematics education if it is discussed in interdisciplinary settings e.g. in comparison with findings from developmental psychologists such as Minsky [24].

According to the interpretation of the imaging techniques by the authors, several frontal and parietal brain areas are activated predominantly. After drill training sessions “only” the left angular gyrus is activated when it comes to learning arithmetic operations (multiplication and division – inverse tasks). In suitable settings, for instance, it could be investigated how structured, productive formats of practice [25], [2], [26] affect the activation of brain areas or neural networks and which conclusions can be drawn from the perspective of mathematics education and learning theory. On this basis, it would be particularly interesting to take a look at mathematical problem-solving processes.

4) Klein et al: White Matter Neuro-Plasticity in Mental Arithmetic: Changes in Hippocampal Connectivity Following Arithmetic Drill Training [27].

a) Question/ Hypothesis

Researchers expected that the structural connectivity of the hippocampus would increase through extensive drill training. Furthermore, the change of the hippocampus' connectivity was to be considerably greater than the change of the connectivity of the angular gyrus, which reflects its more important role in the retrieval of arithmetic facts.

b) Setting

- There were 32 participants (range 18-25 years) and no reports on or prehistories of problems with calculating.
- The participants had a short extensive drill training of 34 different multiplication tasks in the form of 36x8.
- Each participant of the study completed two fMRI-sessions as well as one intense multiplication training (one unit lasted about 30-60 min) with five sessions (consecutive).
- The 34 multiplication problems were trained until each of them was solved correctly a least once.
- fMRI- and DWI-weighted scans were carried out before and after the training, after the end of the multiplication training – always at the same time of day and with one week in between.
- In the fMRI, participants had to choose the correct of two results of a multiplication task as quickly as possible.

c) Output/Outcome

This study concludes that a significant increase in connectivity in the area of nerve fibers of the left hippocampal gyrus could be determined globally in the fMRI after a short and extensive drill training, whereas there was barely an increase in activity in the area of the angular gyrus. This means that a high neuro-plasticity (in simple terms: New formation and modification of nerve fibers and nerve cells and enhancement of their function) has developed perihippocampal in the area of the white matter, as a reaction to the drill training (five training sessions) on complex multiplication tasks.

The background of the study is particularly interesting concerning the evaluation of the role and the tasks of the angular gyrus. Klein et al. [27] were able to show that, through drill training, the angular gyrus is no longer directly involved in processes of calculating, but rather practices a mediative network function (it fulfils an intermediary function to other centers in the network)

d) View on Mathematical Items within Mathematics Education (Validity)

A drill training was developed for this study as well, i.e. an intense multiplication training (one unit lasted about 30-60 min) with five sessions (successive). 34 multiplication tasks in the form of 36x8 were practiced, until each of them was solved correctly once. From the perspective of mathematics education, it would be interesting to find out how the tasks were solved, e.g. whether they were exclusively calculated mentally or whether half-written calculation strategies could be applied as well (in this case, participants would note down intermediate steps and solutions). This way, it would be possible to receive information on different strategic approaches of the participants (e.g. place value strategy: Both numbers are split into their place values, calculated individually, and then combined with each other [23]).

With regard to its quality and validity from the perspective of mathematics education, the choice of the items is to be questioned critically with reference to the research question. At first – and this can more or less be applied to all the present studies except for [10] – it is to be criticized again that skills are trained, which have already been consolidated (there is even a high probability that they have already been automatized). With regard to validity, the question arises whether the items and the test design facilitate statements on remembering previous knowledge rather than acquiring new knowledge. Thus, the specific gain in (scientific) insight for mathematical teaching and learning processes are estimated as rather low. Instead, the availability of (factual) knowledge would be tested, which has already been consolidated. However, it seems interesting and certain that (from the perspective of mathematics education) this very basic remembering can actually be described measurably in a neuro-plasticity. It is questionable, whether this simply represents a reaction to any arbitrary cognitive activity (paired with a special aspect of attention in a setting which is very exceptional for the test subjects) or if it actually permits to draw specific conclusions on the neurologic dimension of arithmetic training processes.

e) The Perspectives that Emerge for Further Research in Mathematics Education

In principle, the study by Klein et al. [27] is interesting regarding the neurological results that were measured. This means that it can verify actual physical changes in the brain caused by drill training. From the perspective of mathematics education and with regard to the findings from similar studies as well, it would now be necessary to find out to what extent these can be interpreted specific to mathematical teaching and learning processes. With regard to the provable neurological processes of growth in comparatively less cognitive challenging settings, it seems profitable (as it has already been mentioned for Ischebeck et al. [21]), to think about what actually intelligent, productive mathematical formats of practice would lead to and which processes could then be represented neurologic-radiologically. In turn, these observations could provide insights and impulses on how productive formats of practice could be designed that also consider physiological aspects.

One may already claim that the hippocampus processes information from different sensory systems (non-subject-specific) and transfers it to the cortex under the premise of transferring theses contents to different types of memory (e.g. short-term, intermediate, long-term memory). It is self-evident that mathematics is being processed in this case as well and that an adequate drill training results in an increased neuro-plasticity in the area of the electric wires and in the area of memory and working cells of the hippocampus itself. Still, the hippocampus is not exclusively specialized in mathematical teaching and learning processes.

5) Popescu et al.: The Brain-Structural Correlates of Mathematical Expertise [28].

a) Question/ Hypothesis

The authors intend to investigate distinctive mathematical brain structures. For this purpose, they compared mathematicians to non-mathematicians.

b) Setting

- 19 mathematicians (5 females) and 19 non-mathematicians (14 males), PhD candidates or Post-Docs (all right-handed) were tested.
- There were mathematicians from the University of Oxford (Faculties such as algebra, logic und number theory) and non-mathematicians from the University of Oxford (departments of English and other languages, classics and history – liberal arts).

TABLE I
COGNITIVE CATEGORY AND TEST FOR THE STUDY

Cognitive category	Test
Intelligence	IQ test (PIQ section)
	IQ test (VIQ section)
Working memory	Digit span (forward)
	Digit span (backward)
	Letter span (forward)
Attention	ANT:alerting
	ANT:orienting
	ANT:executive
Mental imagery	Mental rotation task (MRT)
Numerical skills	Number acuity (w)
	Number line (positive numbers)
	Number line (negative numbers)
	Numerical Stroop
	Numerical agility
	Numerical strategies
	Arithmetic task
Logic	Wason logic task
Verbal reasoning	Verbal reasoning task
Social skills	Emotional recognition task
	Gaze task
	Face recognition task
	Autism spectrum quotient (ASQ)
Arithmetic strategies questionnaire	Visuo-spatial
	Inner verbalisation
	Outer verbalisation
	Kinaesthetic

- The cognitive working memory, the attention, the IQ, as well as numerical and social qualities were tested.
- The cognitive category as well as the test is listed in Table I).

c) Output/Outcome

The main claim is that the density analysis regarding the behavior of the grey matter in the left frontal gyrus was denser in mathematicians than among non-mathematicians. This leads to the conclusion that a targeted “occupation” with a topic leads to an increase and augmentation of the so-called memory unit in the responsible area of the cortex of the cerebrum. This means that a thematic specialization leads to selective neuro-plasticity and thus to a selective development of skills and possibilities in the particular field.

Increased densities and activities were found in mathematicians than among non-mathematicians at the following anatomical sites: right superior parietal lobule, right intraparietal sulcus, left inferior frontal gyrus.

Learning effects as well as effects of practice functionally and structurally change neurological structures, concerning both the grey and the white matter (mainly involving the cortex in the present example). This means that professionalization in a specific area can lead to a shift of brain function to entirely different brain areas, and therefore in the areas of other networks and centers of association as well. This seems to indicate a very high functional dynamic and flexibility of the brain and its centers as well as enormous expandability through learning and practicing effects. It is to be assumed that the shifting of tasks to other centers is carried out by the brain, because there is a higher storage capacity, networking, possibility of demands that do not depend on other influences.

d) View on Mathematical Items within Mathematics Education (Validity)

In comparison to the other present studies, the approach of this study is clearly more holistic – many different aspects about known cognitive psychology instruments such as IQ-tests are considered. With regard to the state of the discussion in mathematics education, further specific testing procedures e.g. on predicative and functional thinking [29] could be considered. In addition, the designed research setting uses a statistically well-founded approach. The choice of the test subjects within a comparative design of control groups seems to be constructive with regard to the research questions described in 1. From this perspective, the study offers a good possibility to classify the selective results of the studies described above against the background of a broader and more holistic frame. Still, the same question arises to what extent the results of adults can be transferred to mathematical teaching and learning processes of children.

e) The Perspectives that Emerge for Further Research in Mathematics Education

The results of the study suggest that in comparison to control groups a continuous exposure to (challenging) mathematics leads to special changes and adaptations in the

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brain. The continuous exposure to this subject (within a professional environment) brings about exceptional, neoplastic adaptations. However, the question of specificity remains, or to ask in a different way: Which mathematical processes of knowledge development (e.g. of a problem-solving nature) are responsible for these adaptations and to what extent and which conclusions can be drawn for “non-professional” mathematicians? To what extent can results then be transferred and to what extent are they related specifically to mathematics, or from another perspective, how can mathematics education contribute to the development of such “mathematical centers”?

Based on the state-of-the-art selection of the items, as well as the design and implementation, the present study offers great potential for connections and specific research projects in mathematics education with holistic perspectives in learning through (productive) practice and even problem-solving.

V. Concluding Discussion

By means of processes that become visible using fMRI through “drill training” (practice) of mathematical tasks in the fields of arithmetic and algebra, the studies mentioned above draw conclusions on functional and structural processes in the brain. It is notable that the studies in question mainly focus on adults (except for [13]). In this case, there seems to be a certain inversion with regard to mathematics education and neurosciences. While mathematics education often investigates the developmental processes of children and makes certain claims to that effect, neuroscience seems to work with adults more frequently in order to answer their scientific research questions and draw conclusions to that effect.

In conclusion, we can formulate the thesis that practice leads to relief processes within the brain. This is indicated by findings which reveal that repetitive exercise mechanisms within the same topic lead to a continuous temporal increase in speed of solving times [13]. Furthermore, they show that when it comes to learning arithmetic operations for example (multiplication and division – inverse tasks), several frontal and parietal brain areas are activated more strongly whereas after a drill training, the left angular gyrus is mainly activated [21]. Practice relieves the brain – the inner calculative tendency of the brain. Automatized knowledge can be relieving, e.g. when we think of algorithms that are automatized to a certain extent and are then available for further, advanced problems. “As a task to be learned is practiced, its performance becomes more and more automatic; as this occurs, it fades from consciousness, the number of brain regions involved in the task becomes smaller [30, p.51]”. Winter [4, p.8] also states that practice fulfils a “function of relief for mental work”. At the same time, he emphasizes that practicing mathematical skills “in a way that does not lead to the loss of understanding of meaning but rather improves it […] [can again as well] serve as instruments of problem-solving [4, p.8]”. Therefore, they can also be used as a basis for further knowledge development to a certain extent. According to Winter and Wittmann [4], [3], this can be referred to as productive practice, a concept of mathematics education. Wittmann and Müller understand productive practice as a form of practice “in which content-related and general aims of learning (mathematizing, exploring, reasoning, formulating) are trained in an integrated way. On the one hand, relationships between tasks play an important role even when it comes to basic and automizing practice. On the other hand, they are clearly revealed during productive practice in the form of concise mathematical patterns [25, p.114]”. Furthermore, productive practice involves a great number of tasks that are related to each other, in a superordinate context. In this regard, Krauthausen [31] mentions three possibilities of structuring in the style of Wittmann that create such a relationship. In problem-structured practice, the relationship between the tasks results from a superordinate problem or question. In an operationally structured practice, the results have a legitimate, operational connection and the relationship between the tasks results from systematic variation. In a factually structured practice, the relationship between the tasks is determined by a superordinate, factual context. Thus, from the perspective of mathematics education, it seems reasonable to consider concepts such as productive practice. With regard to the meaning of practice, Winter claims that “essentially, it [is a] resumption of an (explorative) learning process, the reproduction and reconstruction of learning situations. Students are intendedly and actively involved in the increasing (not immediately conveyed) mechanization of procedures, the interrelation of knowledge as well as the more common use of strategies. At the same time, it becomes clear that practice must have an integrated character in a sense that many and various relations to prior knowledge are created [4, p.10]”. Furthermore, Radatz and Schipper [32] distinguish between different (ideal-typical and classic) forms of practice for mathematics classes (Table II). Käpnick states that “mathematics can thus be considered as a subject that requires a high amount of practice […]. Still, from an educational perspective, it is very important to know the specific function of a particular form of practice, its particular demands, advantages, risks or limits [33, p.131]”.

TABLE II
FORMS OF PRACTICE FOR MATHEMATICS CLASSES ACCORDING TO [32]

Forms of practice for mathematics classes		
Form of practice	Aim	Theoretical Background
Automatized practice	Practicing basic knowledge and elementary techniques until they are mastered	Principle of algorithmic learning
Staged practice	Gradual development of skills through exercises that have an incremental increase in the degree of difficulty	Principle of isolating problems
operative practice	Expansion of the flexibility of thought by establishing diverse relations and connections	Operational principle
Practicing by applying	Transfer of what has been learned to new questions and situations	Principle of orientation towards application
ten-minute calculating	- warming-up - repetitive practice - preparative practice	Principle of stabilization

Concepts of practice play a decisive role in learning mathematics and they are frequently discussed, as the person doing mathematics practices and the person practicing also does mathematics [3].

If discoveries on neuro-plasticity were even made after a (mathematical) drill training, it would be interesting to observe the possible effects of productive practice according to Winter [4] and Wittmann and Müller [34], for example, or of other formats that follow the forms of practice by Radatz and Schipper [32]. Certainly, this requires appropriate research settings that take such an educational principle as that of productive practice seriously.

Interesting research questions would then be whether productive practice can lead to functional and structural processes of relief in the brain as well, and whether positive effects on the competence of problem-solving, according to Winter [4], can be determined with regard to the development of new, neuro-radiological network connections or the reduction to a new center of problem-solving. Another interesting question would be, in what way productive practice can lead to a higher neuro-plasticity than drill training, for instance. At the moment, statements on this matter can exclusively be made through the neuro-radiological procedures of magnetic resonance imaging and functional magnetic resonance imaging.

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