

# Classification of Earthquake Distribution in the Banda Sea Collision Zone with Point Process Approach

Henry J. Wattimanela, Udjianna S. Pasaribu, Nanang T. Puspito, Sapto W. Indratno

**Abstract**—Banda Sea Collision Zone (BSCZ) is the result of the interaction and convergence of Indo-Australian plate, Eurasian plate and Pacific plate. This location is located in eastern Indonesia. This zone has a very high seismic activity. In this research, we will calculate the rate ( $\lambda$ ) and Mean Square Error (MSE). By this result, we will classification earthquakes distribution in the BSCZ with the point process approach. Chi-square is used to determine the type of earthquakes distribution in the sub region of BSCZ. The data used in this research is data of earthquakes with a magnitude  $\geq 6$  SR for the period 1964-2013 and sourced from BMKG Jakarta. This research is expected to contribute to the Moluccas Province and surrounding local governments in performing spatial plan document related to disaster management.

**Keywords**—Banda sea collision zone, earthquakes, mean square error, Poisson distribution, chi-square test.

## I. INTRODUCTION

**T**ECTONIC earthquake is one of the natural disasters that are feared by humanity, because it causes great losses. Tectonic earthquakes usually occur at the interface between the two plates. The tectonic plates are always moving and mutually press each other. The movement of tectonic plates causes energy accumulation slowly. Later tectonic earthquakes occur due to the release of energy that has long been stored it. Tectonic earthquakes often found in Indonesia at subduction Banda (Banda Sea region), Banda Collision Zone and Papua [1], [2]. Research on the distribution of earthquakes with statistical science approach has been made by many researchers related magnitude aftershocks, earthquakes with great strength and observation time between earthquakes [3], [4]. On the other hand, the application of the concept of the point process, which is part of stochastic processes have been developed to observe and describe various aspects of the field, among others, forestry, geology, ecology and seismology [5]-[8]. Especially in the field of seismology, the application process has been done related point characteristics and patterns of occurrence of earthquakes. Application of the point

process to the case of earthquakes has been conducted by researchers associated earthquake aftershock and multi-dimensional residual analysis [9]-[11], and fractal characterization of the spatial distribution [8]. The application process has not been applied to the local point BSCZ particularly related to a distribution model that a large magnitude earthquakes. Due to the current research in this area is more emphasis on the case of seismology as discussing the structure of the area of the collision on BSCZ [11]-[14], related to the depth of the area BSCZ [15]-[18], and the tectonic activity of the arc Banda in BSCZ [19], [20].

In this research, we apply the theory of stochastic counting processes for classification earthquake distribution of earthquakes with magnitude  $\geq 6$  SR for period of 50 years (1964-2013) at each partition area (30 locations) of BSCZ region.

This article consists of an introduction, results, and discussion including the location and observation data, methods and distribution models and experimental results, which ended with the conclusion. This research is expected to contribute the development of statistical science and the Moluccas Province and surrounding local governments in performing spatial planning taking into account disaster management.

## II. RESULT AND DISCUSSION

### A. Location and Data Observations

In BSCZ represents an outstanding example of active arc-continent collision and one of the most tectonically complex and seismically active areas on earth and it is characterized by some peculiarities that make the region unique among other zones of convergence [26]. BSCZ region bounded by  $2^{\circ}$  LS -  $7^{\circ}$  LS and  $126^{\circ}$  BT -  $132^{\circ}$  BT. BSCZ region consist some islands: Buru, Ceram, and part of the Banda Sea (Fig. 1). The data used in this study was the incidence of earthquakes in 1964-2013 in BSCZ with magnitude  $\geq 6$  SR and it can be obtained on BMKG Jakarta-Indonesia (website). The number of earthquakes per year for the years 1964-2013 can be shown in Fig. 2.

H. J. Wattimanela is with Department of Mathematics, FMIPA, Pattimura University, Moluccas, Indonesia, PC: 97233; as Doctor student in Institute of Technology Bandung (corresponding author to provide e-mail: hwattimanela@yahoo.com).

U. S. Pasaribu, S. W. Indratno are with Statistics Research Group, FMIPA, Institute of Technology Bandung, Bandung, Indonesia, PC: 40132 (e-mail: udjianna@math.itb.ac.id, sapto@math.itb.ac.id).

N. T. Puspito is with Global Geophysics Research Group, FTTM, Institute of Technology Bandung, Bandung, Indonesia, PC: 40132 (e-mail: nanang@staff.itb.ac.id).

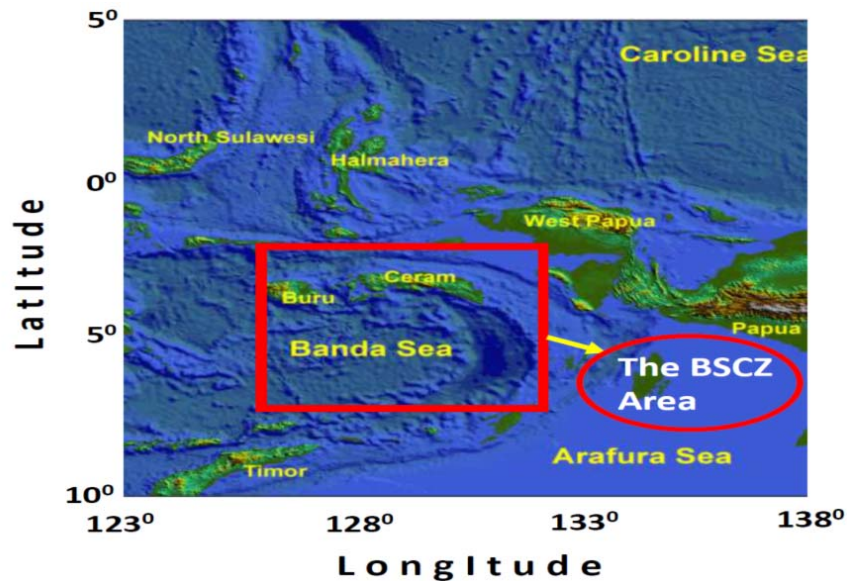


Fig. 1 Location of BSCZ Region in Moluccas Area

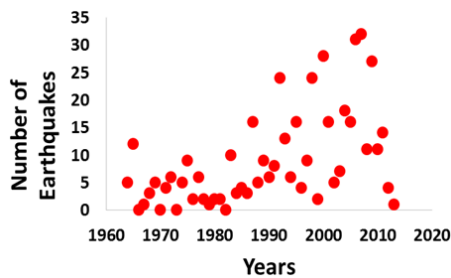


Fig. 2 Number of earthquakes BSCZ Region per years (1964-2013)

### B. Research Methods

In this study, the partitioning method used to form the set of models that is congruent with the approach of partitioning the set square. Partitioning is a representation of a set of disjoint regions. We suppose that  $A \subseteq R^d$ ,  $d = 1, 2, 3$  and finite partition of set  $A$  denoted by  $\vartheta$  is a collection of disjoint subset  $A_i$ ,  $i = 1, 2, \dots, n$ , or  $\vartheta = \{A_i\}$ , where  $A_i \subset A$  and  $A_i \cap A_j = \emptyset$  for  $i \neq j$ . Point processes  $\{N(A), |A| \geq 0\}$  is stochastic process with the realization as counting process if  $N(A)$  is stating the number of events that occur on any set  $A$  [9], [21]. Let  $N(A)$  denote the number of earthquakes in  $A$ . Then  $N(A)$  Poisson distribution with  $\lambda$  expressed rate earthquakes occurring per unit area location partition:

$$P(N(A) = y) = \frac{e^{-\lambda|A|}(\lambda|A|)^y}{y!}, y = 0, 1, 2, \dots, \quad (1)$$

$|A|$  is area size of  $A$ . Partitions that will be used in this study are congruent square models and process can be done by doing the arithmetic sum of all events within each subset. Since  $A_i \cap A_j = \emptyset$ ,  $i \neq j$ , and  $A = \bigcup_{i=1}^n A_i$ ,  $N(A) = N(\bigcup_{i=1}^n A_i) = \sum_{i=1}^n N(A_i)$ ; where  $N(A_1)$ ,  $N(A_2)$ , ...,  $N(A_n)$  are independent random variables [27]. The other side  $\lambda = \hat{\lambda} = \frac{\sum_{n=0}^k n \cdot f_n}{n}$ ,  $k = 0, 1, \dots, r$  with  $f_n$  is the location frequencies of  $n$

earthquakes. We applied MSE to see the degree of closeness between observed and expected value.  $MSE = \frac{\sum_{n=0}^k f_n (Y_n - \hat{Y}_n)^2}{\sum_{n=0}^k f_n}$ ,  $k = 0, 1, \dots, r$  where  $Y_n$  is result of observation  $n$ th and  $\hat{Y}_n$  is result of expectation  $n$ th. To test whether the data the number of earthquakes follow a Poisson distribution, we apply the Chi-Square test. In this test the null hypothesis  $H_0$  is data number of earthquakes follow a Poisson distribution.  $H_0$  is reject if chi-square calculation ( $\chi^2_{calc}$ ) > chi-square table ( $\chi^2_{table}$ ) [22].

### C. Case Study

BSCZ observation area is partitioned into 30 sub regions with each of the length and width of 1 degree ( $1^\circ$ ) as show in Fig. 3.

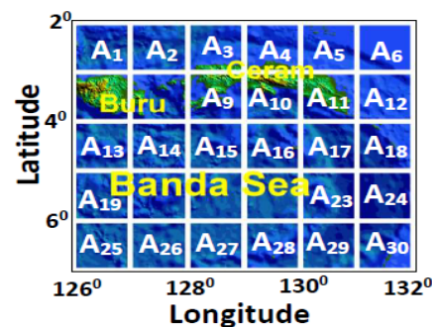


Fig. 3 Partitions Area of BSCZ Region (30 Locations)

The distribution of earthquakes from the year 1964-2013 can be seen in Fig. 4.

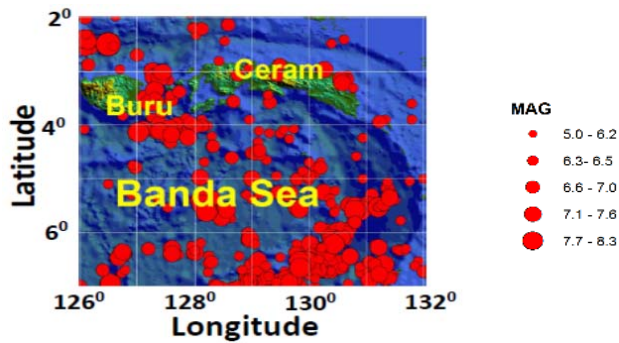


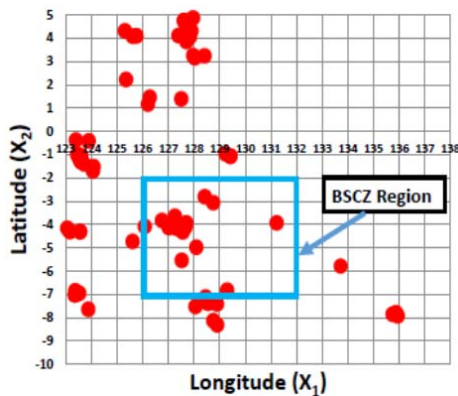
Fig. 4 Earthquakes Distribution with Mag  $\geq 6$  SR in BSCZ Region (1964-2013). Earthquakes (Mag  $\geq 6$  SR mostly occur in the area bounded  $3^0$  LS -  $5^0$  LS &  $127^0$  BT -  $128^0$  BT and  $5^0$  LS &  $128^0$  BT -  $132^0$  BT

Determination of the set of the partitions on the BSCZ regions is used to avoid repeating calculation of an event happening earthquake on a different partition regions. We

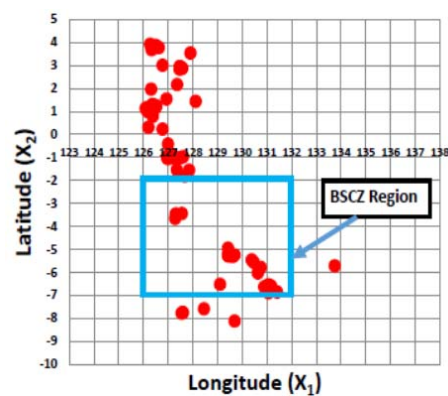
partition the BSCZ region into 30 sub regions in the following way:

$$\begin{aligned} A_{0+k} &= \{(x_1, x_2) | 125 + k \leq x_1 < 126 + k; -3 < x_2 \leq -2\} \\ A_{6+k} &= \{(x_1, x_2) | 125 + k \leq x_1 < 126 + k; -4 < x_2 \leq -3\} \\ A_{12+k} &= \{(x_1, x_2) | 125 + k \leq x_1 < 126 + k; -5 < x_2 \leq -4\} \\ A_{18+k} &= \{(x_1, x_2) | 125 + k \leq x_1 < 126 + k; -6 < x_2 \leq -5\} \\ A_{24+k} &= \{(x_1, x_2) | 125 + k \leq x_1 < 126 + k; -7 \leq x_2 \leq -6\} \end{aligned}$$

where  $k = 1, 2, 3, 4, 5, 6$ ;  $|A_1| = |A_2| = \dots = |A_{30}| = 1$  per unit area. The earthquakes distribution in BSCZ region for years 2000 and 2007 are given in Fig. 5. Based on Fig. 3,  $A = \bigcup_{i=1}^{30} A_i$ , and  $N(A) = N(\bigcup_{i=1}^{30} A_i) = \sum_{i=1}^{30} N(A_i)$ . In this research, we assume that the random variables of  $N(A_1), N(A_2), \dots, N(A_{30})$  are independent. The calculation will be done  $\hat{\lambda}$  dan MSE for earthquakes each 30 locations for 50 years period (1964-2013). Calculation of MSE preceded by calculating observation and expectations of data by using (1) for a lot of earthquakes per 30 locations to 50 years. Based on data, sub regions  $A_6$  and  $A_{18}$  have not earthquake with a Mag  $\geq 6$  SR.  $\hat{\lambda}$  and MSE of 30 locations are given in Table I.



(a) Scatter plot of earthquakes in year 2000. Earthquakes (Mag  $\geq 6$  SR) mostly occur in the area bounded  $2^0$  LS -  $6^0$  LS and  $126^0$  BT -  $129^0$  BT



(b) Scatter plot of earthquakes in year 2007. Earthquakes (Mag  $\geq 6$  SR) mostly occur in the area bounded  $4^0$  LS -  $7^0$  LS and  $128^0$  BT -  $132^0$  BT

Fig. 5 Earthquakes Distribution with Mag  $\geq 6$  SR in BSCZ Region

TABLE I  
THE RESULTS OF CALCULATIONS OF  $\hat{\lambda}$  AND MSE

Sub Region	$\hat{\lambda}$	MSE	Sub Region	$\hat{\lambda}$	MSE
A <sub>1</sub>	0.24	0.01252	A <sub>16</sub>	0.44	0.02871
A <sub>2</sub>	0.08	0.00137	A <sub>17</sub>	0.12	0.00283
A <sub>3</sub>	0.14	0.00111	A <sub>19</sub>	0.64	6.97E-07
A <sub>4</sub>	0.04	0.00036	A <sub>20</sub>	0.04	6.97E-07
A <sub>5</sub>	0.06	3.66E-06	A <sub>21</sub>	0.36	0.03905
A <sub>7</sub>	0.02	4.18E-08	A <sub>22</sub>	0.32	0.03549
A <sub>8</sub>	0.66	0.09006	A <sub>23</sub>	0.56	0.03207
A <sub>9</sub>	0.14	0.00099	A <sub>24</sub>	0.44	0.04314
A <sub>10</sub>	0.10	0.00127	A <sub>25</sub>	0.36	0.02750
A <sub>11</sub>	0.12	0.00514	A <sub>26</sub>	0.20	0.00387
A <sub>12</sub>	0.08	1.2E-05	A <sub>27</sub>	0.56	0.05493
A <sub>13</sub>	0.08	0.00032	A <sub>28</sub>	1.04	0.02339
A <sub>14</sub>	0.42	0.07659	A <sub>29</sub>	1.56	0.09884
A <sub>15</sub>	0.20	0.00651	A <sub>30</sub>	0.64	0.07574

Based on Table I, it is seen that there is no calculation of  $\hat{\lambda}$  and MSE for the location of the  $A_6$  and  $A_{18}$  because there are no earthquakes with magnitudes  $\geq 6$  SR during the years 1964 to 2013. It shows that  $\hat{\lambda}$  for location  $A_{29}$  (part of the Banda Sea) is greater than other location because the highest number of earthquakes in the area BSCZ that occurred in the year that is equal to 78 earthquakes with strength magnitude  $\geq 6$  SR. Otherwise  $A_7$  has the smallest value of  $\hat{\lambda}$  and the location is located on Buru Island. The other sub regions such as  $A_2$ ,  $A_{12}$ , &  $A_{13}$ ;  $A_3$  &  $A_9$ ;  $A_4$  &  $A_{20}$ ;  $A_{11}$  &  $A_{17}$ ;  $A_{15}$  &  $A_{26}$ ;  $A_{16}$  &  $A_{24}$ ;  $A_{19}$  &  $A_{30}$ ;  $A_{21}$  &  $A_{25}$ ; and  $A_{23}$  &  $A_{27}$  have the same value of  $\hat{\lambda}$  (Fig. 6). Also based on Table I sub regions  $A_{29}$ , it has the largest MSE value and location  $A_7$  has the smallest MSE value. Fig. 7 shows that the graph of the expectations and observation data for some locations having Poisson distribution. Therefore, we divide on two classes based on proximity graphs observations and expectations that are close

and not close. Based on Fig. 7 (a), it is seen that the expectation graphs in  $A_2$ ,  $A_3$ ,  $A_9$ ,  $A_{10}$ ,  $A_{13}$ , and  $A_{15}$  are close with the observation graphs. On the other side, the expectation and observation graph in  $A_8$ ,  $A_{16}$ ,  $A_{23}$ ,  $A_{24}$ ,  $A_{27}$  and  $A_{29}$  are not close as show in Fig. 7 (b).

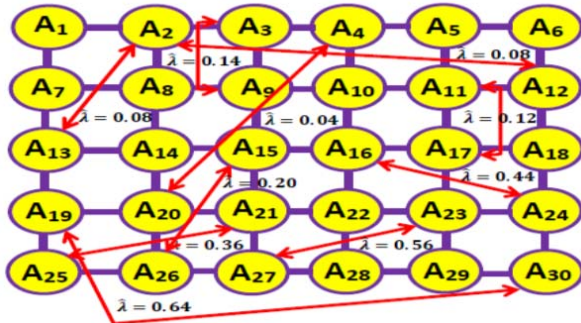
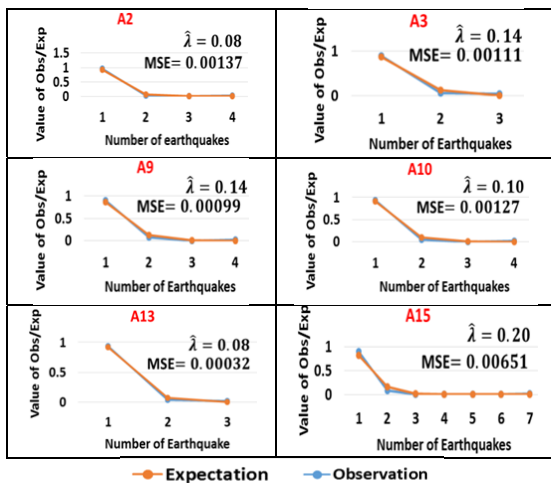
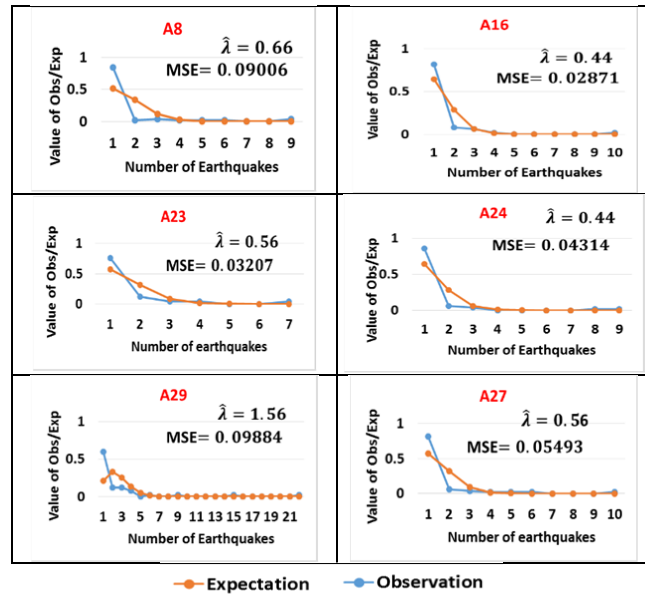


Fig. 6 Graph of linkages between locations that have the same  $\hat{\lambda}$  value

The results expectations and observations graph is as show in Fig. 7, it can be presumed that the data follow a Poisson distribution. We test whether the data of 28 locations (partitions) follow a Poisson distribution by the chi-square test. Based on calculation result,  $\chi^2_{calc}$  for data of earthquakes  $A_1$ - $A_{30}$  for 50 years of BSCZ region and its result that  $\chi^2_{table} = 40.113$  ( $n = 28$ ,  $\alpha = 0.05$ ), show that 13 locations (43%) is  $\chi^2_{calc} < \chi^2_{table}$  of 28 locations. This means that  $H_0$  is accepted for 13 locations ( $A_1$ - $A_5$ ,  $A_7$ ,  $A_9$ ,  $A_{10}$ ,  $A_{12}$ ,  $A_{13}$ ,  $A_{19}$ ,  $A_{20}$ , and  $A_{26}$ ) of data earthquakes, so we can be said that the earthquakes data to follow a Poisson distribution (Table II).



(a) Graphs of Observation and Expectation data locations are close



(b) Graphs of Observation and Expectation data locations are not close

Fig. 7 Graphs of Observation and Expectation data locations are close and not close

TABLE II  
THE RESULTS OF CHI-SQUARE CALCULATION

Sub Region	$\chi^2_{calc}$	Sub Region	$\chi^2_{calc}$	Sub Region	$\chi^2_{calc}$
$A_1$	6.217557	$A_{12}$	0.0025330	$A_{23}$	133.86560
$A_2$	5.147991	$A_{13}$	0.1439085	$A_{24}$	18809.782
$A_3$	0.375105	$A_{14}$	3.566E+16	$A_{25}$	34672047
$A_4$	0.499995	$A_{15}$	5496.8068	$A_{26}$	7.6623401
$A_5$	0.000804	$A_{16}$	364620.87	$A_{27}$	46924.300
$A_7$	9.973E-68	$A_{17}$	52.391166	$A_{28}$	2782.0113
$A_8$	6943.0002	$A_{19}$	0.0001592	$A_{29}$	8.556E+14
$A_9$	1.0722449	$A_{20}$	0.0001592	$A_{30}$	410379998
$A_{10}$	2.7331167	$A_{21}$	17387383		
$A_{11}$	52.476994	$A_{22}$	17754052		

Poisson distributed locations (13 locations) can be mapped on BSCZ region as shown in Fig 8.

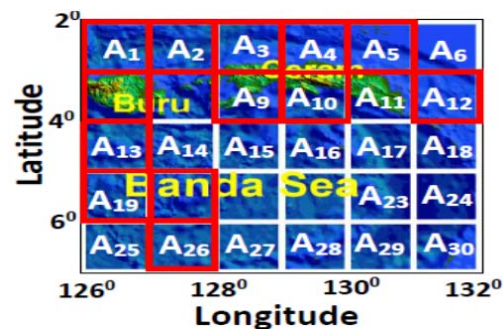


Fig. 8 The Mapping Poisson Distribution in Partition Area of BSCZ region



Fig. 8 shows that the location of the earthquake ( $\text{Mag} \geq 6$  SR) Poisson distributed in the area  $2^0 \text{ LS} - 4^0 \text{ LS} \& 126^0 \text{ BT} - 129^0 \text{ BT}$  and  $4^0 \text{ LS} - 7^0 \text{ LS} \& 126^0 \text{ BT} - 128^0 \text{ BT}$ . This area consist of the islands: Buru, Ceram, also included are some locations in the Banda Sea. We will be calculated  $\lambda$  and MSE value from the combination of several locations in the region BSCZ which has the same  $\hat{\lambda}$  of earthquake occurrence based on Table I. Suppose location  $B_1$  is result the combined of locations  $A_2, A_{12},$  and  $A_{13}$ . The same way for location  $A_3 \& A_9 = B_2$ ;  $A_4 \& A_{20} = B_3$ ;  $A_{11} \& A_{17} = B_4$ ;  $A_{15} \& A_{26} = B_5$ ;  $A_{16} \& A_{24} = B_6$ ;  $A_{19} \& A_{30} = B_7$ ;  $A_{21} \& A_{25} = B_8$ ; and  $A_{23} \& A_{27} = B_9$ . The results calculation  $\lambda$  and MSE in detail can be shown in Table III.

TABLE III  
THE RESULTS OF CALCULATIONS  $\hat{\lambda}$  AND MSE

Sub Region	$\hat{\lambda}$	MSE	Sub Region	$\hat{\lambda}$	MSE
$B_1$	0.24	0.00571	$B_6$	0.88	0.66459
$B_2$	0.28	0.00469	$B_7$	0.68	0.07486
$B_3$	0.08	0.00032	$B_8$	0.62	0.08179
$B_4$	0.24	0.01695	$B_9$	1.12	0.06177
$B_5$	0.40	0.01129			

We test whether the data of 9 locations follow a Poisson distribution using the chi-square test (Table IV).

TABLE IV  
THE RESULTS OF CHI-SQUARE CALCULATION

Sub Region	$\chi^2_{calc}$	Sub Region	$\chi^2_{calc}$
$B_1$	0.925009	$B_6$	8865.462612
$B_2$	0.718871	$B_7$	2192460.614
$B_3$	0.143905	$B_8$	5723704.621
$B_4$	81.422225	$B_9$	181.605093
$B_5$	106.442538		

Based on calculation result,  $\chi^2_{calc}$  for data of earthquakes  $B_1$ - $B_9$  (includes locations of  $A_1, A_5, A_7, A_8, A_{10}, A_{14}, A_{22}, A_{28},$  and  $A_{29}$ ) for 50 years of BSCZ region and its result that  $\chi^2_{table} = 27.587$  ( $n = 18, \alpha = 0.05$ ), show that 7 locations (38.9%) is  $\chi^2_{calc} < \chi^2_{table}$  of 18 locations. This means that  $H_0$  is accepted for 7 locations ( $B_1$ - $B_3, A_1, A_5, A_7,$  and  $A_{10}$ ) of data earthquakes, so we can be said that the earthquakes data to follow a Poisson distribution.

### III. CONCLUSION

There are two sub regions of BSCZ that do not have earthquakes with  $\text{Mag} \geq 6$  SR during periods 1964-2013, namely  $A_6$  (near Ceram) and  $A_{18}$  (part of Banda Sea). In addition, there are some sub regions of the BSCZ region which have the same value of  $\lambda$ , and there are two sub region which have the smallest and largest value of  $\lambda$ , namely  $A_7$  (part of Buru) and  $A_{29}$  (part of Banda Sea). The other sides, the data number of earthquakes in the BSCZ Poisson distributed as much as 43% in some location such as Buru, Ceram, and also some location of the Banda Sea. The location of known types of this distribution will facilitate the prediction of earthquakes in advanced research. The other side, this results could be used by the Moluccas Province government as

one of the references in preparing spatial Moluccas Province taking into account the risks related to disaster management and mitigation, including the development of infrastructure related to the strength of the building.

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