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Characteristics of Hydraulic Jump

Sumit Gandhi

Abstract**—**The effect of an abruptly expanding channel on the main characteristics of hydraulic jump is considered experimentally. The present study was made for supercritical flow of Froude number varying between 2 to 9 and approach to expanded channel width ratios 0.4, 0.5, 0.6 and 0.8. Physical explanations of the variation of these characteristics under varying flow conditions are discussed based on the observation drawn from experimental results. The analytical equation for the sequent depth ratio in an abruptly expanding channel as given by eminent hydraulic engineers are verified well with the experimental data for all expansion ratios, and the empirical relation was also verified with the present experimental data.

Keywords**—**Abruptly Expanding Channel, Hydraulic Jump, Efficiency, Sequent Depth Ratio.

I. Introduction

In an open channel when water at high velocity discharges into a zone of lower velocity, an abrupt rise on the surface and high turbulence at the bottom in the form of rollers occur. The rapidly flowing water is abruptly slowed and increases in height, converting kinetic energy into an increase in potential energy, with some energy irreversibly lost through turbulence. It is one of the important tasks for hydraulic engineers to design a safe and economical energy dissipater. Stilling basins are one of the possible solutions for hydraulic jump to dissipate kinetic energy to produce safe downstream flow, which causes no bed scour and bank erosion.

Stilling basin can be made gradually or abruptly expanding eventually extended by appurtenances. These not only dissipate additional energy and reduced the basin length, but also deflect the high velocity jets away from the basin bottom. Compared to a prismatic stilling basin, an abruptly expanding basin modifies not only the sequent depths but influences all other flow characteristics. Moreover, conditions were tail-water depth in a prismatic channel is so low that a classical jump does not form or it is impossible to depress the basin floor due to some design considerations or jump is no longer able to form even with the aid of appurtenances, a lateral expansion may be the only possibility for adequate energy dissipation.

As far as the rectangular channel is concerned, the different hydraulic jump characteristics can be determined easily, whereas for abruptly expanding rectangular channels, few results in terms of relative length of jump 'L_j/y_1' are available. Review on analytical and experimental studies by Rajaratnam and Subramanya [10], Herbrand [7], Hager [6], Smith [11], Bremen and Hager [3], [4], Ranga Raju [9], Agarwal [1] and Gandhi [5] are mainly devoted to the sequent depth ratio and the relative energy loss. Herbrand [7] investigated jumps in expanding channels and suggested

Sumit Gandhi, Assistant Professor, is with the Department of Civil Engineering, Jaypee University of Engineering and Technology, Guna - 473226, (M. P.), India. (e-mail: sumit_gandhi2001@yahoo.co.in)

$$\frac{y_1}{y_2} = \frac{Y_1}{Y_2}\sqrt{\frac{B_1}{B_2}} \tag{1}$$

Y_1/Y_2 is the depth ratio of jump in prismatic channels. Based on experimental results, a classical work was presented by Bremen and Hager [3], [4]. They proposed the sequent depth ratio

$$\frac{y_1}{y_2} = \frac{Y_1}{Y_2} - \left(\frac{Y_1}{Y_2} - 1\right)\left(1 - \sqrt{\frac{B_1}{B_2}}\right)\left[1 - \tanh\left(1.9\frac{x}{L_r}\right)\right] \tag{2}$$

where 'x' is the distance from the expansion section to the toe of jump and L_r is length of roller.

II. Theory

Hydraulic jump was first investigated experimentally by Bidone [2]. Thereafter, many studies were made and the results were quoted by many engineers. But, there is a lack of results on some of the jump characteristics, which may reasonably affect the hydraulic jump phenomenon. Therefore, their effects need to be explored for a better estimation of safe stilling basin design. Considerable amount of work has been done by eminent Hydraulicians on important parameters of hydraulic jump on abruptly expanding horizontal channel and the same are reproduced as follows:

Empirical relation is given by Herbrand [7] for an abruptly expanding channel valid for $3.1 < F_{r1} < 9$ and gives satisfactory estimation of y_2.

$$\frac{y_2}{y_{2R}} = \left(\frac{B_1}{B_2}\right)^{3/8} \tag{3}$$

however, $y_2/y_{2R} = (B_1/B_2)^{1/2}$ given in (4) below gives better result.

$$\frac{E_L}{E_1} = \frac{E_1 - E_2}{E_1} = \frac{\left(2 - 2\frac{y_2}{y_1}\right) + F_{r1}^2\left(1 - \frac{1}{\eta^2 \frac{y_2^2}{y_1^2}}\right)}{2 + F_{r1}^2} \tag{4}$$

Assuming hydrostatic pressure distribution and uniform velocity distribution Matin et al [8] developed the expression for sequent depth ratio as

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$$\frac{y_2}{y_1} = \frac{\left(\sqrt{1+8F_{r1*}^2}-1\right)}{2} \tag{5}$$

were, $F_{r1*}^2 = k_1 + \frac{2F_{r1}^2}{k_2}$, is the modified Froude number in which k_1 and k_2 depend on the jump position and the expansion ratio, defined as

$$k_1 = \frac{\left(\frac{y_2^2}{y_1^2} - 2F_{r1}^2\frac{B_1}{B_2}\right) - \left(1 - \frac{B_1}{B_2}\right)}{2\left(1 - \frac{y_1}{y_2}\right)} \quad \text{and} \quad k_2 = \frac{1 - \left(\frac{y_1}{y_2}\right)}{\frac{B_1}{B_2}\left(1 - \frac{B_1 y_1}{B_2 y_2}\right)} \tag{6}$$

with the known dimensionless quantity $\frac{x}{L_r}$ that describes the jump location, k_1 and k_2 can be expressed as function of F_{r1} and $\frac{B_1}{B_2}$ as

$$k_1 = -\frac{x}{2L_r}\left(\ln\frac{B_1}{B_2}\right)F_{r1}^{1.5\left(\frac{B_1}{B_2}+1\right)} \text{and}\; k_2 = 1 - 0.4\left(\ln\frac{B_1}{B_2}\right)\left(1 + \ln F_{r1}\right) \tag{7}$$

The analytical equation for the sequent depth ratio in an abruptly expanding basin given by Ranga Raju [9] and Agarwal [1], respectively, is

$$\left[\left(\frac{y_2}{y_1}\right)^2 - 1\right] = 2F_{r1}^2\left[\left\{\left(-\frac{B_1^2}{B_2^2}\right)\times\left(\frac{y_1}{y_2}\right)\right\} + \left(\frac{B_1}{B_2}\right)\right] \tag{8}$$

$$\frac{E_L}{E_1} = \frac{F_{r1}^2\left[1 - \left(\frac{B_1^2}{B_2^2}\right)\left(\frac{y_1}{y_2}\right)^2\right] - 2\left(\frac{y_2}{y_1} - 1\right)}{2 + F_{r1}^2} \tag{9}$$

above equations have been solved graphically by Agarwal [1] which can be used to determine accurately the values of y_2/y_1, E_L/E_1 and E_1/y_1 for various values of B_1/B_2 at particular value of F_{r1}.

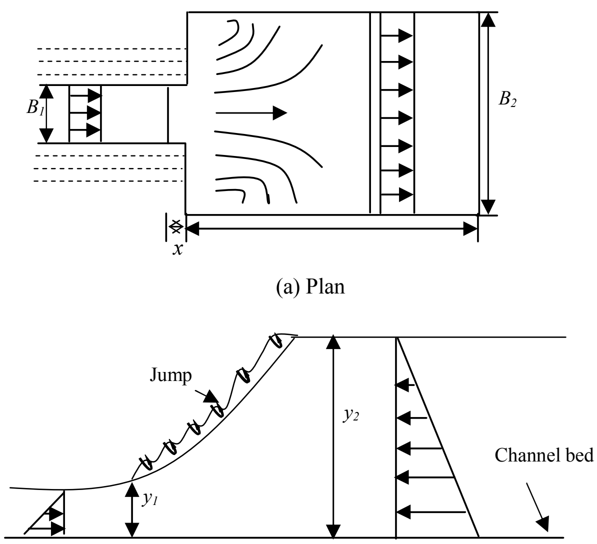
III. Experimental Set–Up and Methodology

A. Experimental Set–Up

In order to evaluate the various hydraulic jump characteristics sequent depth ratio 'y_2/y_1', efficiency 'E_2/E_1', relative height of jump 'h_j/E_1', relative energy loss 'E_L/E_1', relative post jump depth 'y_2/E_1', relative pre jump energy 'E_1/y_1', relative length of jump 'L_j/y_1', relative length of roller 'L_r/y_1' and relative energy loss 'E_L/y_1', the experiments were carried out on abruptly expanding channel and the setup was designed, fabricated and commissioned in the hydraulics engineering laboratory at MNNIT Allahabad, India.

The setup consists of a constant head tank of volume 3.6 × 3.6 × 3m^3, water reaches to the inlet tank of volume 0.46 × 0.45 × 0.61m^3 through the feeding pipe of diameter 10cm provided with regulating valve. The upstream face of inlet regulating gate is covered by stilling basin of length 3m and width 0.3m to prevent side wave reflection and surface undulation so that a stabilized flow is available at the inlet of main channel.

The experiments were performed in 2.1m × 0.445m × 1.2m rectangular channel made up of perspex sheet. Parallel rails were mounted at the top of the side walls for sliding of pointer gauge to measure the depth at different positions along the length and across the width of the main channel.



(a) Plan

(b) Elevation

Fig. 1 Showing the plan and elevation of jump formation in an abruptly expanding channel

TABLE I
RANGE OF DATA DETERMINED

B_1/B_2	F_{r1*}	y_2/y_1	E_2/E_1	h_j/E_1	E_L/E_1	y_2/E_1	E_1/y_1	L_j/y_1	L_r/y_1	E_L/y_1
0.4	2 - 9	3.1 - 6.7	0.13 - 0.45	0.08 - 0.29	0.58 - 0.85	0.09 - 0.46	2 - 36	13.23 - 20.65	08.80 - 14.90	15.45 - 31.23
0.5	2 - 9	3.3 - 8.0	0.14 - 0.47	0.10 - 0.33	0.53 - 0.75	0.12 - 0.49	3 - 40	14.10 - 21.50	11.34 - 17.32	20.87 - 35.48
0.6	2 - 9	3.0 - 8.8	0.19 - 0.52	0.15 - 0.35	0.48 - 0.72	0.15 - 0.50	5 - 48	18.14 - 25.65	12.67 - 19.20	24.76 - 38.67
0.8	2 - 9	4.1 - 10.2	0.20 - 0.55	0.17 - 0.44	0.42 - 0.70	0.20 - 0.57	7 - 49	19.43 - 25.16	14.87 - 20.06	30.56 - 42.34

B. Experimental Methodology and Analysis

A series of run for different values of incoming Froude number (varying between 2 to 9 using sharp edged regulating gates (both upstream and downstream) and feeding pipe with valve were performed with width ratio (approaching channel width to expanded channel width) varying in the proportion of 0.4, 0.5, 0.6 and 0.8. For each experiment pre and post jump depths, lengths of jump and length of roller were recorded. The discharge was computed by volumetric method. Figs. 1 (a) & (b) show the pattern of formation of hydraulic jump in abruptly expanding channel and the range of different hydraulic jump characteristics calculated are shown in Table I.

Experiment results obtained for sequent depth ratio at different F_{r1} for all the expansion ratios is also used to compare for verification with Ranga Raju's analytical equation in Fig. 11. Also, the equation given by Herbrand is verified with experimental data of the present study for expanding channel and it is found to be in good agreement between the two. Fig. 12 shows the comparison of sequent depths between experimental data & Herbrand's empirical relation for $B_1/B_2 = 0.4$, 0.5, 0.6 and 0.8.

IV. Results and Discussion

As such the present study is carried out in the form of graphical representation to study the variation of various characteristics against F_{r1*} for B_1/B_2 ratio = 0.4, 0.5, 0.6 and 0.8 considering the modified Froude number as given in (5), (6) and (7). All the characteristics of the jumps are then written as

$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8F_{r1*}^2} - 1\right) \tag{10}$$

$$\frac{E_L}{y_1} = \frac{\left(\frac{y_2}{y_1} - 1\right)^3}{4\left(\frac{y_2}{y_1}\right)} = \frac{\left(\sqrt{1+8F_{r1*}^2} - 4\right)^3}{16\left(\sqrt{1+8F_{r1*}^2} - 1\right)} \tag{11}$$

$$\frac{E_2}{E_1} = \frac{\left(8F_{r1*}^2 + 1\right)^{3/2} - 4F_{r1*}^2 + 1}{8F_{r1*}^2\left(2 + F_{r1*}^2\right)} \tag{12}$$

$$\frac{h_j}{E_1} = \frac{\left(\frac{y_2}{y_1} - 1\right)}{\left(1 + \frac{F_{r1}^2}{2}\right)} = \frac{\sqrt{1+8F_{r1*}^2} - 3}{F_{r1*}^2 + 2} \tag{13}$$

$$\frac{E_1}{y_1} = \frac{2 + F_{r1*}^2}{2} \tag{14}$$

$$\frac{y_2}{E_1} = \frac{y_2}{y_1}\left(\frac{2}{2 + F_{r1}^2}\right) = \frac{\left(\sqrt{1+8F_{r1*}^2} - 1\right)}{2 + F_{r1*}^2} \tag{15}$$

$$\frac{E_L}{E_1} = 1 - \frac{E_2}{E_1} = 1 - \frac{\left(8F_{r1*}^2 + 1\right)^{3/2} - 4F_{r1*}^2 + 1}{8F_{r1*}^2\left(2 + F_{r1*}^2\right)} \tag{16}$$

All the hydraulic jump characteristics considered in the present work have been computed from the measured data and are plotted against the Froude number as shown in Figs. 2 to 12. All these plots are developed using experimental results of Table I.

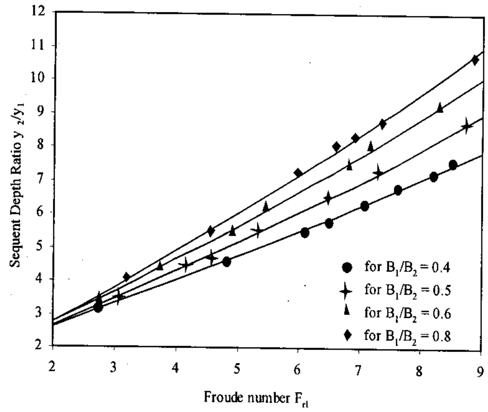


Fig. 2 Variation of Sequent Depth Ratio y_2/y_1 vs F_{r1}

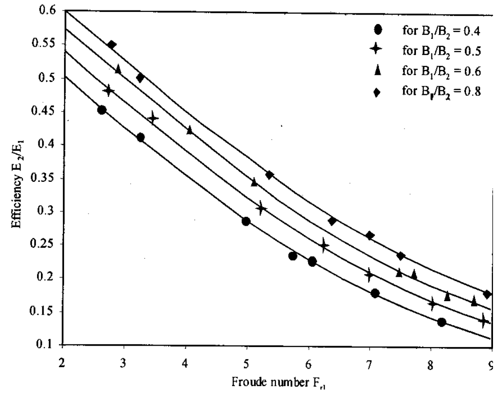


Fig. 3 Variation of Efficiency E_2/E_1 vs F_{r1}

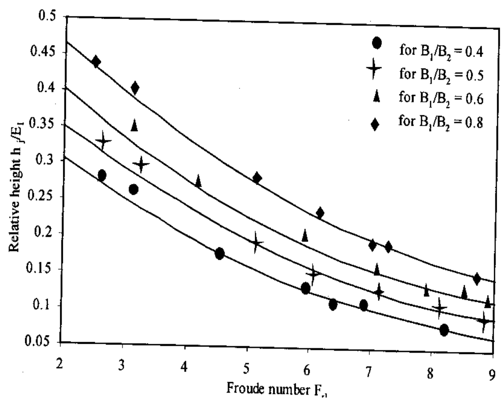


Fig. 4 Variation of Relative Height of Jump h_j/E_1 vs F_{r1}

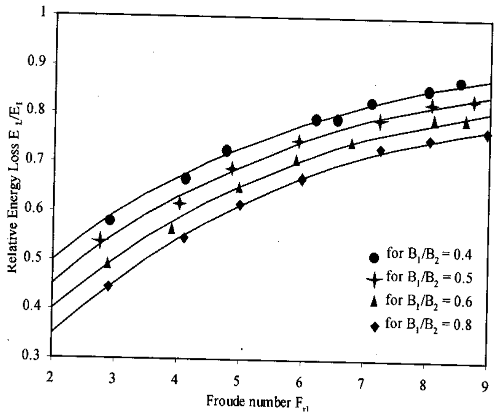


Fig. 5 Variation of Relative Energy Loss E_L/E_1 vs F_{r1}

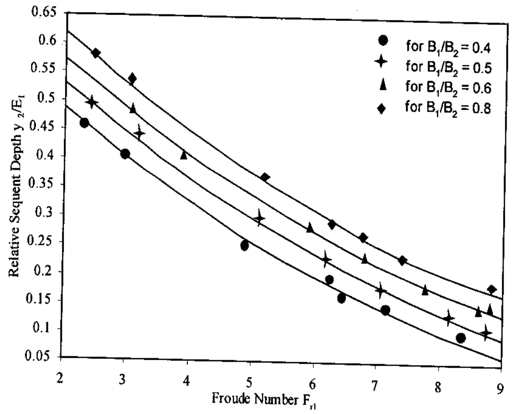


Fig. 6 Variation of Relative Postjump Depth y_2/E_1 vs F_{r1}

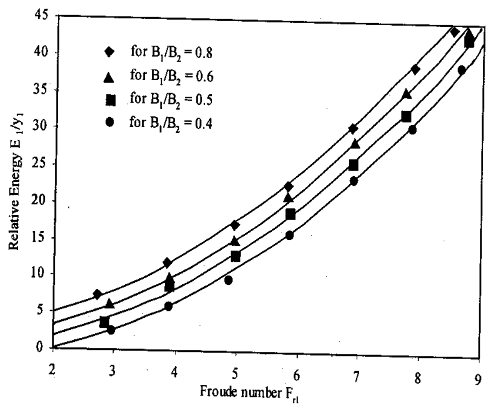


Fig. 7 Variation of Relative Prejump Energy E_1/y_1 vs F_{r1}

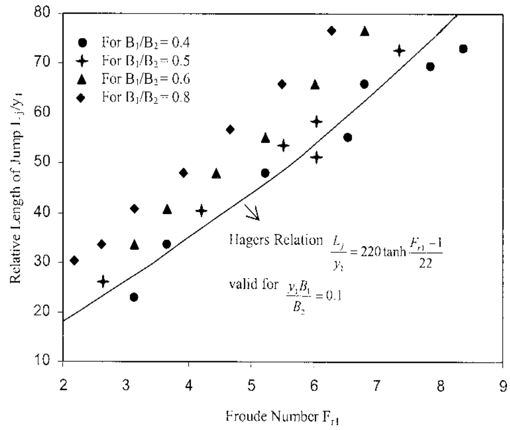


Fig. 8 Variation of Relative Length of Jump L_j/y_1 vs F_{r1}

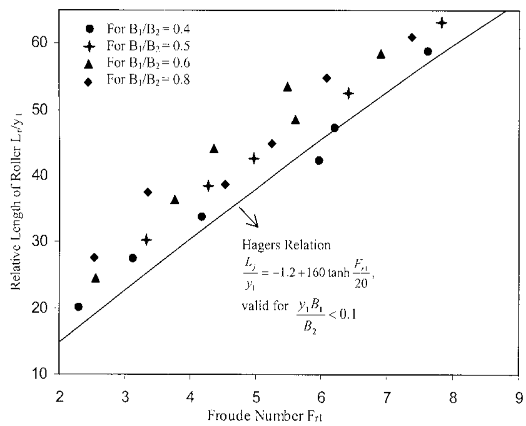


Fig. 9 Variation of Relative Length of Roller L_r/y_1 vs F_{r1}

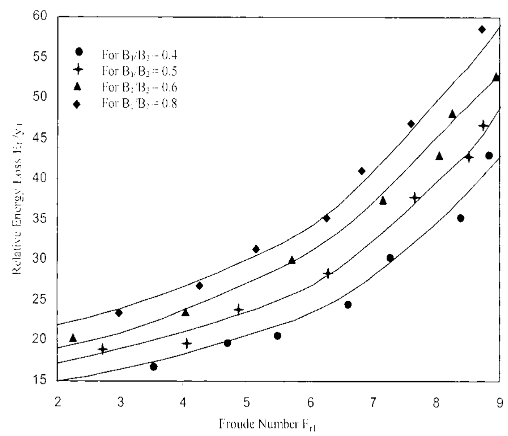


Fig. 10 Variation of Relative Energy Loss E_L/y_1 vs F_{r1}

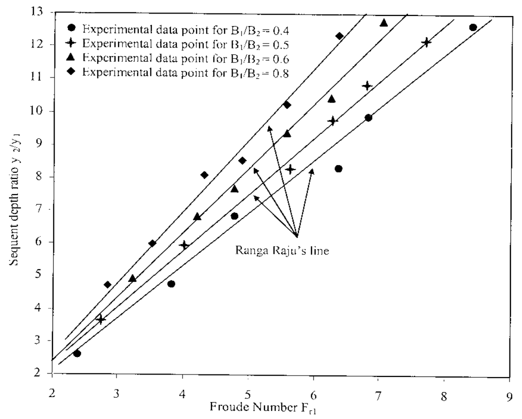


Fig. 11 Variation of Sequent depth ratio vs F_{r1} for Ranga Raju's analytical relation (8)

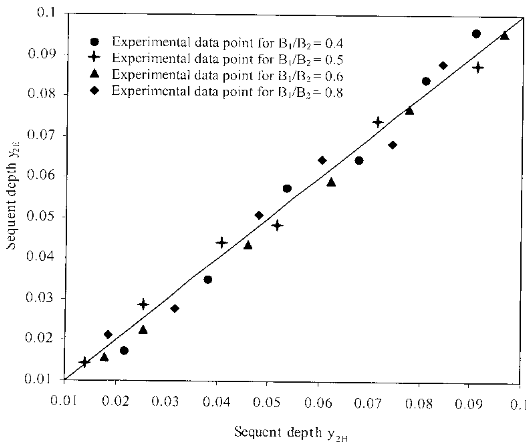


Fig. 12 Comparison of Sequent depth between Herbrand (3) and Experimental data

Following are the main inferences drawn from the present study;

1. Fig. 2 shows that the larger the B_1/B_2 ratio, the larger is the sequent depth ratio for a given F_{r1}. As the value of expansion ratio B_1/B_2 increases there is linear increment in the sequent depth ratio y_2/y_1 with F_{r1}.
2. Figs. 3, 4, and 6 predict that as smaller is the B_1/B_2 ratio, the smaller is the Efficiency E_2/E_1, Relative height h_j/E_1 and Relative post jump depth y_2/E_1 for a given F_{r1}. Efficiency E_2/E_1, Relative height h_j/E_1 and Relative post jump depth y_2/E_1 decreases non linearly with increase in F_{r1} for a given B_1/B_2 expansion ratio.
3. It is clear from Fig. 5 that smaller is the B_1/B_2 ratio larger is the Relative energy loss E_L/E_1 for a given F_{r1}. There is nonlinear increment in Relative energy loss E_L/E_1 with the increase in value of F_{r1} for a given B_1/B_2 expansion ratio.
4. It is concluded from the Fig. 7 that the value of Relative energy E_1/y_1 increases nonlinearly with the increases in expansion ratio and F_{r1}, and it also increases with the increase in expansion ratio at a given value of F_{r1}.
5. Fig. 8 shows the plot of Hagers relation for the relative length of jump (valid for $y_1B_1/B_2 = 0.1$). Experimental data shown are scattered about the Hagers line for all the expansion ratios, this because of its validity for a particular range. Relative length of jump increases non-linearly with the increase in Froude number.
6. Plot 9 shows the variation of relative length of roller L_r/y_1 for Hagers relation, which is also non-linear and increases with the Froude number. Most of the experimental data are scattered around the Hagers line, this because of the limit of Hagers relation i.e., $y_1B_1/B_2 < 0.1$.
7. Value of relative energy loss E_L/y_1 decreases with decrease of expansion ratio at a given F_{r1} and it increases non-linearly with the Froude number as shown in Fig. 10.
8. Analytical equation (8) given by Ranga Raju is also verified in Fig. 11 for B_1/B_2 = 0.4, 0.5, 0.6 & 0.8 using author's data and it is found to be highly satisfactory. The figure shows that most of the experimental data points lie within $\pm$10% of the Ranga Raju's line corresponding to respective expansion ratio.
9. Equation (3) given by Herbrand which holds good for $3.1 < F_{r1} < 9$ and gives satisfactory estimation of y_2 is also verified with experimental data of the present study for expanding channel and it is found to be in good agreement between the two. Fig. 12 shows the comparison of sequent depths between experimental data & Herbrand's empirical relation for all expansion ratios. It is predicted from the figure that most of experimental data point lies with $\pm$ 15% of the good agreement line between the two.

V. Conclusions

Larger is the B_1/B_2 ratio larger is the Sequent depth ratio y_2/y_1, Relative energy E_1/y_1 and Relative energy loss E_L/y_1 whereas, smaller is the B_1/B_2 ratio smaller is the Efficiency E_2/E_1, Relative height h_j/E_1 and Relative post jump depth y_2/E_1 and larger is the Relative energy loss E_L/E_1 for a given F_{r1}. Sequent depth ratio y_2/y_1 varies linearly with the Froude number F_{r1} where as other hydraulic jump characteristics are not.

Jump characteristics calculated from Matin's formula is varying from Hagers line for measuring relative length of jump and relative length of roller. Also, there is slight variation is seen in the experimental data from the lines drawn for Ranga Raju's analytical equation for measuring the sequent depth, but it lies within the consideration limits; it may attributed to the modified Froude number but found suitable for field use. Empirical equation proposed by Herbrand is satisfactory for predicting the sequent depth y_2. It is suggested that it can be used in the field with confidence.

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NOTATIONS

B_1 = approaching channel width
B_2 = expanded channel width
E_1 = energy before the jump
E_2 = energy after the jump
E_L = energy loss = $E_1 - E_2$
F_{r1} = incoming Froude number
F_{r1*} = modified incoming Froude number
h_j = height of jump = $y_2 - y_1$
L_r = length of roller
L_j = length of jump
r_1 =radial distance from the center of equivalent circular jump to the toe of the jump
r_2 =radial distance from the center of equivalent circular jump to the section where the hydraulic jump is complete
x =distance from the expansion section to the toe of jump
y_1 = prejump depth
y_2 = postjump depth
y_{2E} = experimental sequent depth
y_{2H} = Herbrand's sequent depth
y_{2R} = sequent depth of rectangular channel
η = r_1/r_2

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