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Calibration of the Radical Installation Limit Error of the Accelerometer in the Gravity Gradient Instrument

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***Abstract*—**Gravity gradient instrument (GGI) is the core of the gravity gradiometer, so the structural error of the sensor has a great impact on the measurement results. In order not to affect the aimed measurement accuracy, limit error is required in the installation of the accelerometer. In this paper, based on the established measuring principle model, the radial installation limit error is calibrated, which is taken as an example to provide a method to calculate the other limit error of the installation under the premise of ensuring the accuracy of the measurement result. This method provides the idea for deriving the limit error of the geometry structure of the sensor, laying the foundation for the mechanical precision design and physical design.

***Keywords*—**Gravity gradient sensor, radial installation limit error, accelerometer, uniaxial rotational modulation.

I. Preface

ANYOBJECT on the surface of the earth is subjected to the action of gravity, which is the resultant force of the gravitation of the earth and centrifugal inertia force caused by the rotation of the earth. Gravity gradient is widely researched not only in geoscience, but also in improving the accuracy of autonomous navigation, the precision of guidance, sensing and breaking the threats from aircrafts or unmanned underwater vehicles (UUVs) [1]-[4]. The performance of traditional gradient measurement method is poor, meanwhile superconductivity and atomic technology have not been practically applied [5]-[7]. The United States Bell Aerospace company (now incorporated into Lockheed Martin) developed rotary accelerometer full-scale gravity gradient meter Air-FTGTM, which have made a certain application with high efficiency both in the military and commercial. Rotary accelerometer gravimetric analyzer is composed of a variety of mechanical devices, among which error in the installation of the accelerometer for gravitational gradient sensor have a great impact on the accuracy of the gravity gradient signal measurement [8]. Therefore, it is very important to calibrate the gravity gradient sensor installation limit error to guarantee the accuracy of the measurement results. Taking the radial installation in the accelerometer for instance, on the premise of ensuring the accuracy, this paper provides a method to calculate the installation limit error within the tolerance bounds.

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II. Concept of Gravity Gradient and the Uniaxial Rotational Modulation Gravity Gradiometer

A. Conception of Gravitational Gradient

The gravitational gradient is the second derivative of the gravitational force W, used to describe the change in the gravity component due to the position.

$$g_x(r+\delta r) = g_x(r) + \frac{\partial^2 W}{\partial x^2}\delta x + \frac{\partial^2 W}{\partial y \partial x}\delta y + \frac{\partial^2 W}{\partial z \partial x}\delta z = g_x(r) + \Gamma_{xx}\delta x + \Gamma_{yx}\delta y + \Gamma_{zx}\delta z \tag{1}$$

Gravity gradient tensor Γ is described as following:

$$\Gamma = \begin{vmatrix} \Gamma_{xx} & \Gamma_{yx} & \Gamma_{zx} \\ \Gamma_{xy} & \Gamma_{yy} & \Gamma_{zy} \\ \Gamma_{xz} & \Gamma_{yz} & \Gamma_{zz} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 W}{\partial x^2} & \frac{\partial^2 W}{\partial y \partial x} & \frac{\partial^2 W}{\partial z \partial x} \\ \frac{\partial^2 W}{\partial x \partial y} & \frac{\partial^2 W}{\partial y^2} & \frac{\partial^2 W}{\partial z \partial y} \\ \frac{\partial^2 W}{\partial x \partial z} & \frac{\partial^2 W}{\partial y \partial z} & \frac{\partial^2 W}{\partial z^2} \end{vmatrix} \tag{2}$$

There are nine components of the secondary derivatives of gravity:

$$\begin{vmatrix} \Gamma_{xx} & \Gamma_{yx} & \Gamma_{zx} \\ \Gamma_{xy} & \Gamma_{yy} & \Gamma_{zy} \\ \Gamma_{xz} & \Gamma_{yz} & \Gamma_{zz} \end{vmatrix},$$

where Γ_{xx} is the spatial rate of change of g_x in the X direction, Γ_{xy} is the spatial change rate of g_x in the Y direction, Γ_{yy} is the spatial change rate of g_y in the Y direction, and so on. Continuously derived by partial derivative order,

$$\Gamma_{xy} = \Gamma_{yx}, \Gamma_{yz} = \Gamma_{zy}, \Gamma_{zx} = \Gamma_{xz} \tag{3}$$

According to the knowledge of the gravity field, the gravity is continuous and limited everywhere outside the earth; thus, gravity potential meets the Laplace equation below:

$$W_{xx} + W_{yy} + W_{zz} = 0 \tag{4}$$

In conclusion, any five independent values in the second derivative of the gravitational components leads to the full gradient information.

B. Conception of Uniaxial Rotation Modulation

Earth's gravity gradient signal is extraordinarily weak. In case of the measurement accuracy of 1E, the equivalent gravity difference between two points 10cm away from each other in the space reaches $10^{-11}g$ [9]. However, for the gravity gradient meter based on the concept, the accuracy of the existing accelerometer cannot meet the requirements. Therefore, Bell Aerospace Textron Company of the United States uses the concept of uniaxial rotation modulation, breaking through the performance limits of the accuracy of the accelerometer, so as to achieve the goal of measuring the weak gravitational gradient signal. The company and Australia jointly developed the product has been successfully put into use [3], and rotary accelerometer gravimeter is currently the only successful practical application, suitable for airborne, ship borne gradient measuring instruments [10].

GGI, shown in Fig. 1, is a sensitive gravity gradient sensor. GGI is composed of two pairs of rotating symmetrical and orthogonal high-precision accelerometer, installed on the turntable. Based on the rotation of the disc, the gravity gradient signal is modulated at twice the speed frequency. In conclusion, this method makes GGI eliminate the noise caused by the carrier motion and so on.

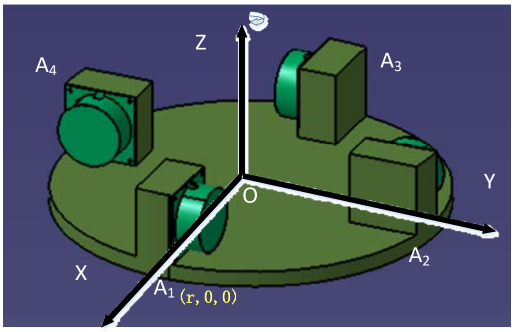

Fig. 1 Basic structure of the GGI

C. Concept of Uniaxial Rotation Modulation

Derived from the same working principle, literature [8], [11]-[14] comes out with different working model. In this paper, we focus on a relatively perfect model which is based on the principle of equal distribution of errors to establish the limit error of the radial installation, laying a good foundation for the analysis of structural error, and mechanical precision design.

According to the force analysis of the accelerometer in GGI, and it can be deduced by the principle of Taylor unfolds,

$$\begin{aligned} a_{1x} &= a_{0x} + \Gamma_{xx}\Delta x + \Gamma_{xy}\Delta y = a_{0x} + \Gamma_{xx} r\cos\omega t - \Gamma_{xy} r\sin\omega t \\ a_{1y} &= a_{0y} - \Gamma_{yy}\Delta y + \Gamma_{yx}\Delta x = a_{0x} - \Gamma_{yy} r\sin\omega t + \Gamma_{yx} r\cos\omega t \\ \Gamma_{xy} &= \Gamma_{yx} \end{aligned} \quad (5)$$

where r is the theoretical distance from the accelerometer to the center of the disc, a_{1x} is the acceleration of the accelerometer in the direction x marked as 1, a_{0x} is the acceleration of the center of the disc in the x direction. The angular speed of the turntable is ω, then ωt is the angle at which the accelerometer 1 is turned in the gravitational gradient.

Accelerometer can only output sensitive shaft ratio, so the No. 1 accelerometer output ratio f_1 is:

$$\begin{aligned} f_1 &= -a_{1x}\sin\omega t - a_{1y}\cos\omega t = -a_{0x}\sin\omega t - a_{0y}\cos\omega t \\ &+ r\left(\Gamma_{yy} - \Gamma_{xx}\right)\sin\omega t\cos\omega t + \Gamma_{xy} r(\sin\omega t)^2 - \Gamma_{yx} r(\cos\omega t)^2 \end{aligned} \quad (6)$$

The signal output equation measured by four accelerometers:

$$\begin{aligned} \overline{a_1}\bullet\overline{v_1} &= -a_{0x}\sin\omega t - a_{0y}\cos\omega t - \frac{1}{2}r\left(\Gamma_{xx} - \Gamma_{yy}\right)\sin 2\omega t - \Gamma_{xy} r\cos 2\omega t \\ \overline{a_3}\bullet\overline{v_3} &= a_{0x}\sin\omega t + a_{0y}\cos\omega t - \frac{1}{2}r\left(\Gamma_{xx} - \Gamma_{yy}\right)\sin 2\omega t - \Gamma_{xy} r\cos 2\omega t \\ \overline{a_2}\bullet\overline{v_2} &= a_{0x}\cos\omega t + a_{0y}\sin\omega t + \frac{1}{2}r\left(\Gamma_{xx} - \Gamma_{yy}\right)\sin 2\omega t + \Gamma_{xy} r\cos 2\omega t \\ \overline{a_4}\bullet\overline{v_4} &= -a_{0x}\cos\omega t - a_{0y}\sin\omega t + \frac{1}{2}r\left(\Gamma_{xx} - \Gamma_{yy}\right)\sin 2\omega t + \Gamma_{xy} r\cos 2\omega t \end{aligned} \quad (7)$$

Among them, the angle rotated by accelerometer 1 is ωt, while those of number 2, 3, 4 are $\omega t + \frac{\pi}{2}$, $\omega t + \pi$, $\omega t + \frac{3\pi}{2}$, respectively.

The relationship between $\overline{v_1}, \overline{v_2}, \overline{v_3}, \overline{v_4}$, being the directional vector of the four accelerometers, is as following:

$$\begin{aligned} \overline{v_1} &= \left[-\sin\omega t \quad -\cos\omega t \quad 0\right] \\ \overline{v_3} &= \left[\sin\omega t \quad \cos\omega t \quad 0\right] = -\overline{v_1} \\ \overline{v_2} &= \left[-\cos\omega t \quad \sin\omega t \quad 0\right] \\ \overline{v_4} &= \left[\cos\omega t \quad -\sin\omega t \quad 0\right] = -\overline{v_2} \end{aligned} \quad (8)$$

Using the following equation, the signal $\Gamma_{xy}, \Gamma_{xx} - \Gamma_{yy}$ comes out.

$$\begin{aligned} &\left((\overline{a_1}\bullet\overline{v_1} + \overline{a_3}\bullet\overline{v_3}) - (\overline{a_2}\bullet\overline{v_2} + \overline{a_4}\bullet\overline{v_4})\right) \\ &= -2r\left(\Gamma_{xx} - \Gamma_{yy}\right)\sin 2\omega t - 4r\Gamma_{xy}\cos 2\omega t \end{aligned} \quad (9)$$

Equation (9) is a basic mathematical model of the working principle of GGI. We can get a relatively perfect model, if the acceleration among the systems, the centripetal acceleration, and the Coriolis force are considered during the establishment of the model.

In this situation, the theoretical output is S_{out}:

$$S_{out}=\left((\overline{a_1}\bullet\overline{v_1}+\overline{a_3}\bullet\overline{v_3})-(\overline{a_2}\bullet\overline{v_2}+\overline{a_4}\bullet\overline{v_4})\right)=-4r\cos 2\omega t[\Gamma_{xy}-(\omega_{Ex}+\omega_{Px}+\omega_{Tx})(\omega_{Ey}+\omega_{Py}+\omega_{Ty})]$$
$$+2r\sin 2\omega t[\Gamma_{xx}-\Gamma_{yy}-(\omega_{Ex}-\omega_{Ey}+\omega_{Px}-\omega_{Py}+\omega_{Tx}-\omega_{Ty})(\omega_{Ex}+\omega_{Ey}+\omega_{Px}+\omega_{Py}+\omega_{Tx}+\omega_{Ty})] \quad (10)$$

where the parameters are all the components of the angular velocities of the self-rotation of earth, carrier platform, and the dick of the sensor, respectively, shown as following.

$$\overline{\omega_E}=\left(\omega_{Ex},\omega_{Ey},\omega_{Ez}\right)$$
$$\overline{\omega_P}=\left(\omega_{Px},\omega_{Py},\omega_{Pz}\right)$$
$$\overline{\omega_T}=\left(\omega_{Tx},\omega_{Ty},\omega_{Tz}\right) \quad (11)$$

In the output of the two corresponding accelerometers in the gravity gradient sensor turntable, there are two corresponding difference terms which can offset each other. The signal is modulated to twice the angular velocity of the turntable, where the gravity gradient signal can be demodulated.

III. Modeling of Output of GGI with the Radial Installation Error

The error is $\Delta r_n = r_n - r(n=1,2,3,4)$, where r and r_n are theoretical and actual length of the radius. Take accelerometer number 1 as an example, assuming that the coordinate of theoretical radial vector is $(r,0,0)$, while the actual position is $(r_1,0,0)$. Replace the theoretical radius by $r_n = \Delta r_n + r$, where n equals 1, 2, 3, 4respectively:

$$\overline{a_{\Delta r_1}}\bullet\overline{v_1}=-(a_{0x}-g_{0x})\sin\omega t-(a_{0y}-g_{0y})\cos\omega t$$
$$+r_1[\alpha_{PTz}-\omega_{Py}\omega_{Tx}-\omega_{Ey}(\omega_{Px}+\omega_{Tx})+\omega_{Px}\omega_{Ty}+\omega_{Ex}(\omega_{Py}+\omega_{Ty})]$$
$$-\frac{1}{2}r_1\sin 2\omega t[\Gamma_{xx}-\Gamma_{yy}$$
$$-(\omega_{Ex}-\omega_{Ey}+\omega_{Px}-\omega_{Py}+\omega_{Tx}-\omega_{Ty})(\omega_{Ex}+\omega_{Ey}+\omega_{Px}+\omega_{Py}+\omega_{Tx}+\omega_{Ty})]$$
$$-r_1\cos 2\omega t[\Gamma_{xy}-(\omega_{Ex}+\omega_{Px}+\omega_{Tx})(\omega_{Ey}+\omega_{Py}+\omega_{Ty})] \quad (12)$$

The actual output $S_{out\Delta r}$ is:

$$S_{out\Delta r}=\left((\overline{a_{\Delta r_1}}\bullet\overline{v_1}+\overline{a_{\Delta r_3}}\bullet\overline{v_3})-(\overline{a_{\Delta r_2}}\bullet\overline{v_2}+\overline{a_{\Delta r_4}}\bullet\overline{v_4})\right)$$
$$=-(4r+\sum_{n=1}^{4}\Delta r_n)\cos 2\omega t[\Gamma_{xy}-(\omega_{Ex}+\omega_{Px}+\omega_{Tx})(\omega_{Ey}+\omega_{Py}+\omega_{Ty})]$$
$$+(2r+\frac{1}{2}\sum_{n=1}^{4}\Delta r_n)\sin 2\omega t[\Gamma_{xx}-\Gamma_{yy}$$
$$-(\omega_{Ex}-\omega_{Ey}+\omega_{Px}-\omega_{Py}+\omega_{Tx}-\omega_{Ty})(\omega_{Ex}+\omega_{Ey}+\omega_{Px}+\omega_{Py}+\omega_{Tx}+\omega_{Ty})]$$
$$(\Delta r_1+\Delta r_2-\Delta r_3-\Delta r_4)\{\alpha_{PTz}+[-\omega_{Py}\omega_{Tx}+\omega_{Px}\omega_{Ty}$$
$$+\omega_{Ex}(\omega_{Py}+\omega_{Ty})-\omega_{Ey}(\omega_{Px}+\omega_{Tx})]\} \quad (13)$$

where $\overline{a_{\Delta r_1}},\overline{a_{\Delta r_2}},\overline{a_{\Delta r_3}},\overline{a_{\Delta r_4}}$ are the real accelerations in this situation and $\frac{d(\overline{\omega_P}+\overline{\omega_T})}{dt}=\overline{\alpha_P}+\overline{\alpha_T}=\left(\alpha_{PTx},\alpha_{PTy},\alpha_{PTz}\right)$ is the sum of the angular acceleration of platform and the disk.

IV. Calculation of the Radial Installation Limit Error

Based on the assumption that there is no error besides the radial error of the gravity gradient sensor and the random error of the speed (i.e., ignore the signal output process error, signal measurement error, signal demodulation error, etc.), the non-determined system error of the gravity gradient sensor structure and the random error distribution are treated equally in the synthesis and distribution; therefore, the distribution method is exactly the same. Based on the principle of equally distribution of errors, there is an error function:

$$\Gamma=\Gamma_{xx}-\Gamma_{yy}=\frac{2\left(A_{\sin}-\alpha_{2\omega}\right)}{R_1+R_2+R_3+R_4}+\omega_c \quad (14)$$

where $\omega_c=(\omega_{Ex}-\omega_{Ey}+\omega_{Px}-\omega_{Py}+\omega_{Tx}-\omega_{Ty})(\omega_{Ex}+\omega_{Ey}+\omega_{Px}+\omega_{Py}+\omega_{Tx}+\omega_{Ty})$ is a constant.

According to the equal action principle:

$$\delta_\alpha=\frac{\delta_\Gamma}{\sqrt{N}}\frac{1}{\partial\Gamma/\partial\alpha_{2\omega}}=\frac{10E}{\sqrt{5}}\frac{1}{R_1+R_2+R_3+R_4}=\frac{\sqrt{5}E}{2r} \quad (15)$$

$$\delta_{R_1}=\frac{\delta_\Gamma}{\sqrt{N}}\frac{1}{\partial\Gamma/\partial R_1}=\frac{10E}{\sqrt{5}}\frac{1}{-\frac{2\left(A_{\sin}-\alpha_{2\omega}\right)}{\left(R_1+R_2+R_3+R_4\right)^2}}=\frac{16\sqrt{5}r^2E}{\left(\alpha_{2\omega}-A_{\sin}\right)} \quad (16)$$

$$\delta_{R_1}=\delta_{R_2}=\delta_{R_3}=\delta_{R_4} \quad (17)$$
$$N=5$$

δ_α, δ_{R_1}, δ_{R_2}, δ_{R_3}, δ_{R_4} are the limit errors of the coefficient of $\alpha_{2\omega}$ and $\Delta R_1,\Delta R_2,\Delta R_3,\Delta R_4$.

If the error caused by rotating is ignored, thus $N=4$, here comes out:

$$\delta_{R_1}=\frac{\delta_\Gamma}{\sqrt{N}}\frac{1}{\partial\Gamma/\partial R_1}=\frac{10E}{\sqrt{4}}\frac{1}{-\frac{2\left(A_{\sin}\right)}{\left(R_1+R_2+R_3+R_4\right)^2}}=\frac{40r^2E}{\left(-A_{\sin}\right)} \quad (18)$$

V. Conclusion

Based on the principle of measuring the gravity gradient using the rotary accelerometer, the radial installation error model of the accelerometer of the gravity gradient sensor is established. The model reflects the coupling of the error source with the signal in the output, and then the limit error of the radial installation is deduced under the premise of the defined measurement accuracy. This method provides some ideas for further deriving the limit error of other structures of the sensor, laying the foundation for the mechanical precision design and physical design.

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