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Calculating Strain Energy in Multi-Surface Models of Cyclic Plasticity

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Abstract**—**When considering the development of constitutive equations describing the behavior of materials under cyclic plastic strains, different kinds of formulations can be adopted. The primary intention of this study is to develop computer programming of plasticity models to accurately predict the life of engineering components. For this purpose, the energy or cyclic strain is computed in multi-surface plasticity models in non-proportional loading and to present their procedures and codes results.

Keywords**—**Strain energy, cyclic plasticity model, multi-surface model, codes result.

I. INTRODUCTION

FATIGUE of metals plays a crucial role in many technologies, including automotive, mechatronic, power and other applications[1]-[2] .

During cyclic plasticity a certain amount of energy is stored in the material as hysteresis energy. Since this energy is irretrievable, it accumulates within the material. Whenever, the accumulated amount is enough for failure, the material fails by fatigue. Fatigue failure of mechanical components is a process of cyclic stress/strain evolutions and redistributions in the critical stressed volume. It may be imagined that due to stress concentration (notches, material defects or surface roughness) the local material yields firstly to redistribute the loading to the surrounding material, then follows with cyclic plastic deformation and finally crack initiates and the resistance is lost. Therefore, the simulations for cyclic stress/strain evolutions and redistributions are critical for predicting fatigue failure of mechanical components. As, the failure will occur when the total amount of the accumulated energy is equal to the energy required for fracture as determined by monotonic tests. Moreover, using the data obtained from that tests help to calculate the fatigue life prediction. Therefore, for the Energy-based fatigue life prediction model [3] especially under non-proportional loading, energy calculation is inevitable. It is necessary to utilize the appropriate constitutive relations for the material. These relations must be applicable to the general multi-axial loading conditions.

Moreover, the incremental plasticity approach is chosen. Among all cyclic plasticity models, multi- surface models such as Mroz [4] and Garud [5] models are considered. Also, their models briefly are explained and general feature of their computational procedure are presented. Finally, the results of the codes are compared by experimental results.

II. CYCLIC PLASTICITY MODELS

According to considerable interactive of cyclic plasticity into fatigue, fracture and deformation cases of industrial pieces of metal, it has attended by many researchers [6].

The four basic assumptions of an incremental plasticity model are as follows:

1. Yield condition
2. Flow rule
3. Consistency condition
4. Hardening rule

The yield surface for boundary recognition between elastic and elasto-plastic regions in the multi-axial loadings is utilized, and the stress point is always on the yield surface boundary. Another name of the flow rule is normality rule. Based on this important subject, the plastic increment is always normal to the yield surface. The consistency condition shows that during any plastic loading, by reversing the load direction, the behavior of the metal is always elastic. Based on this observation, it is concluded that during plastic loading, the yield surface boundary follows the stress, in stress space. In fact, the consistency condition is the root of the hardening rules. There are three common viewpoints for yield surface from the hardening rules that are, Isotropic hardening, Kinematic hardening, and combined hardening.

The most important advantage of kinematic hardening determines is when it presents the load removing or inverse loading subjects. The laboratorial results show that in inverse loading, substance enters in plastic zone sooner than forecast by isotropic hardening. Isotropic hardening is able to handle any proportional and non-proportional loading. But this hardening rule is not able to simulate the stress-strain hysteresis loop in cyclic loadings. In fact this model fails in cyclic plasticity.

A. Mroz Model [4]

Mroz for better approximation of the stress-strain curve and generalization of the plastic modulus in multiaxial case, Defined a field of different plastic modulus in stress space.

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The boundary of this field defines by some stress surfaces as is shown in "Fig. 1".

During plastic loading these stress surfaces will be active subsequently and move until the stress point meet the next stress inactive stress surface. When the stress point meets a stress surface, this surface will be active. By increasing the load, active surface and the entire previously activated surface (inner surfaces) move together, until unloading occurs.

During non proportional loading, to find the direction of the movement for active stress surfaces, these steps are performed:

1- Finding a similar point on the next surface, that has the same normal vector as the current normal vector

$$S_{ij}^{*} = \frac{R_{k+1}}{R_k}(S_{ij} - a_{ij}) + a_{ij}^{k+1}$$

2- Direction of the movement is parallel to the vector $(S_{ij}^{*} - S_{ij})$

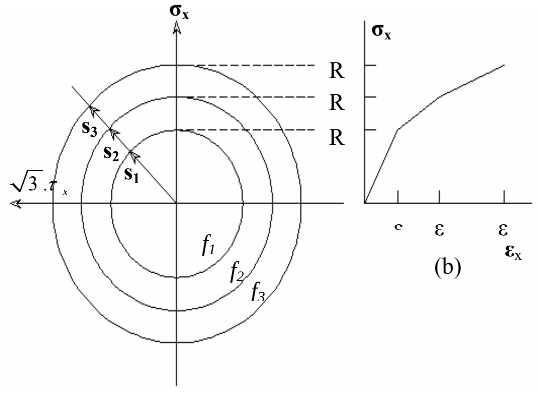
$$da_{ij}^{k} = d\eta(S_{ij}^{*} - S_{ij})$$

$d\eta$ Can be defined using consistency condition

3- Other inner surfaces $1 < k < k-1$ will be in touch with active surface during plastic loading. Then back stress of the other internal surfaces, will be

$$a_{ij}^{r} = S_{ij} - \sqrt{\frac{2}{3}}R_r n_{ij} \qquad 1 < r < k-1$$

Where R_r =radius of r^{th} stress surface, and a_{ij}^{r} =back stress of r^{th} stress surface.



(a)

Fig. 1 Plastic modulus field in the Multi surface model

B. Garud Model [5]

Movement of the surface in Garud model is dependent on the stress direction. The following steps are needed to determine the movement direction of the yield surface based on the Garud model.

1- Finding the stress point on the next inactive surface (S_{ij}^{B}), by extending the current stress increment.

$$S_{ij}^{B} = S_{ij} + kdS_{ij}$$

2- Finding normal vector on the next surface at (S_{ij}^{B}).

$$n_{ij}^{B} = \sqrt{\frac{3}{2}}\frac{S_{ij}^{B} - a_{ij}^{k+1}}{R_{k+1}}$$

3- Dtermining a similar point on the active surface, which has the same normal vector $n_{ij}^{B} = n_{ij}^{*}$

$$S_{ij}^{*} = \sqrt{\frac{2}{3}}R_k n_{ij}^{*} + a_{ij}^{k}$$

4- Movement direction is as follow:

$$da_{ij}^{k} = d\eta(S_{ij}^{B} - S_{ij}^{*})$$

5- Other inner surfaces $1 < k < k-1$ will be in touch with active surface during plastic loading. Then back stress of the other internal surfaces, will be

$$a_{ij}^{r} = S_{ij} - \sqrt{\frac{2}{3}}R_r n_{ij} \qquad 1 < r < k-1$$

III. METHODOLOGY

A computer program is developed to compute the model numerically in order to calculate strain energy based on the back stress values in the plasticity model. The elasto-plastic stiffness tensor is used in the computer program for incremental loading. By using increments of load, the total and plastic strains are calculated for different values of stress. After these results are obtained, the cyclic strain energy will be calculated by integrating both elastic and plastic components of the energy from cyclic hysteresis loops. The stress-strain curve or hysteresis loop for different types of loading is plotted. Then, values of fatigue life in different loading condition are calculated by the energy method and they are listed in the table. The plots and tables obtained from the mentioned models are analyzed and compared with the experimental results and another existing multiaxial model.

IV. RESULTS

To evaluate the strain energy from developed codes in plasticity models, several sets of fatigue data on 1% Cr-Mo-V steel material reported in the literature [7] have been studied. The data include proportional and non-proportional tension–torsion tests with several of paths loading (φ) and strain ratios (λ). "Figs. 2,3" illustrate some typical hysteresis loops together with plots of both axial and torsional strain- stress under loading path, φ=90, λ=4, $\varepsilon_a = 0.51$.

The strain energy or plastic work per cycle under non-proportional loading has been calculated with integrating both loadings hysteresis loops, and the values are written in Table I.

TABLE I
COMPARISON OF PREDICTED AND EXPERIMENTAL OF STRAIN ENERGY
(MJ/m^3)

Experimental	Mroz code	Garud code
30.87	28.93	29.13

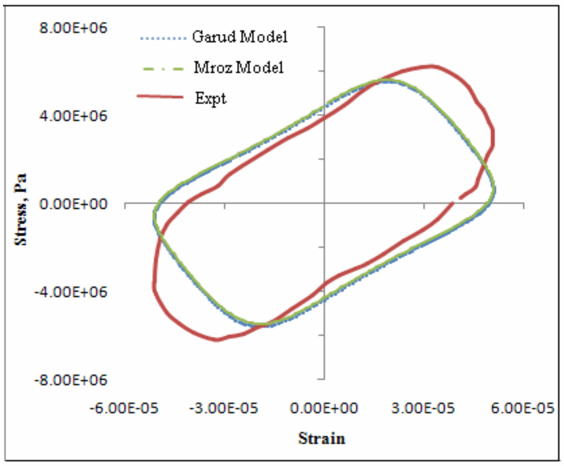


Fig. 2 Axial hysteresis loop

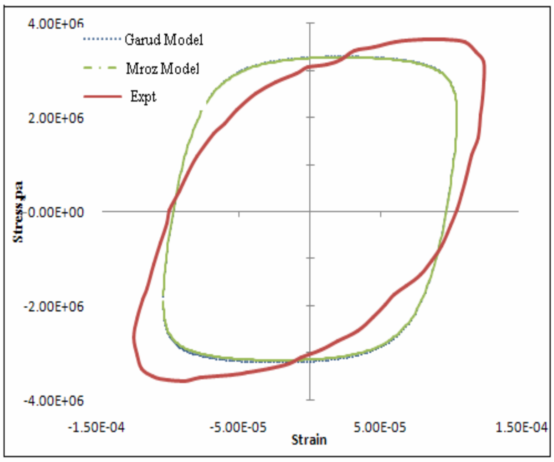


Fig. 3 Torsional hysteresis loop

Comparison stress-strain responses shows value of plastic strain energy more depends on torsional loading for this loading path. Figures obtained of codes results of two models with experimental results aren't similar to each other perfectly, but they are equivalent each other numerically, as is depicted in Table I. The table is shown that values of the plasticity models are close by together, as this matter on the curves of figures is observable distinctly. In this case, the result of the Garud model is closer to experimental results than another model.

Therefore, fatigue life of material comfortably is predicted by obtained values of plastic work per cycle.

Table II predicted fatigue lives is shown using results of plasticity models and experimental data.

TABLE II
COMPARISON OF PREDICTED AND EXPERIMENTAL FATIGUE LIVES (CYCLE)

Experimental	Mroz code	Garud code
366	343	345

V. CONCLUSION

At expressed, calculating the strain energy is needed to satisfy four assumptions which are yield and consistency conditions and flow and hardening rules. For this purpose, utilizing programming codes to calculate fast and accurately it is inevitable. Furthermore, the numerical values of these programming codes are close to the result of experimental data. Consequently, the fatigue life of material will be predicted with approximation of logical and desired.

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