Burstiness Reduction of a Doubly Stochastic AR-Modeled Uniform Activity VBR Video

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Abstract—Stochastic modeling of network traffic is an area of significant research activity for current and future broadband communication networks. Multimedia traffic is statistically characterized by a bursty variable bit rate (VBR) profile. In this paper, we develop an improved model for uniform activity level video sources in ATM using a doubly stochastic autoregressive model driven by an underlying spatial point process. We then examine a number of burstiness metrics such as the peak-to-average ratio (PAR), the temporal autocovariance function (ACF) and the traffic measurements histogram. We found that the former measure is most suitable for capturing the burstiness of single scene video traffic. In the last phase of this work, we analyse statistical multiplexing of several constant scene video sources. This proved, expectedly, to be advantageous with respect to reducing the burstiness of the traffic, as long as the sources are statistically independent. We observed that the burstiness was rapidly diminishing, with the largest gain occuring when only around 5 sources are multiplexed. The novel model used in this paper for characterizing uniform activity video was thus found to be an accurate model.

Keywords—AR, ATM, burstiness, doubly stochastic, statistical multiplexing.

I. INTRODUCTION

SOURCE modeling and traffic characterization is rapidly gaining importance in broadband network analysis. Such networks are designed to carry traffic that are heterogeneous, statistically bursty, and with very large bit rates [1]. While large bit rates can be compressed using modern source coding techniques, it is the burstiness of the traffic that poses a serious challenge to the efficient operation of broadband networks. An accurate source model is thus indispensable for network optimization, design, and resource allocation, for performance evaluation and quality of service (QoS) provisioning, and for congestion avoidance and control.

In this work, we study a class of traffic known as video streaming. We chose this particular type of traffic because it is known to pose a serious challenge to the optimal performance of broadband multimedia networks because of its burstiness and variable bit rate statistical characteristics. To better support video services on high speed and integrated networks,

an understanding of the characteristics of VBR video traffic is thus required.

Video traces with low levels of scene activity have exponentially decaying temporal correlations with respect to time-lag, whereas video traces with non-uniform scene activity have frame sizes that change slowly over long time intervals [2]. The autocorrelation function for these types of video traces decays slowly or does not reach zero even for long lag intervals. VBR video traffic with uniform scene activity can also exhibit long-term correlation. To model video traffic, methods that efficiently capture the switching between these levels of activity are required [3].

II. A STOCHASTIC MODEL FOR UNIFORM ACTIVITY VIDEO

In this section, we first survey stochastic models for uniform activity level video sources in ATM networks, then we propose a new model and stochastically analyze it by deriving its probability density function (pdf) and ACF.

A. A Literature Survey of Video Models

The autoregressive (AR) model has been widely used to model broadcast-video traces generated by a DPCM-based coding algorithm without motion compensation. In the AR(1) model (of order 1), a finite-state Markov chain is used to generate a sequence of states. These states are used to determine the frame sizes [4]. This model requires only that the mean, variance, and autocorrelation coefficient of intra-scene frames be determined. However, the AR(1) model was not found to be accurate for all video traces tested, however. For the video conference traces, the AR(1) proved to be a good source model, mainly because video conferencing data belongs to the same scene. This is referred to as *uniform activity level* video [5].

A time-varying AR process was applied to model DPCM/DCT-coded full motion video. The number of frames used to generate the model was 500 frames and 2 arbitrary thresholds were selected from bit-rate histogram of the video trace. An enhanced Markov chain (MC) based approach has also been successfully used to analyze traffic from single and 2 layer MPEG-2 coders.

A scenic model based on the AR(1) model has also been used in the literature to model VBR traffic. Scene changes were estimated using differences in the number of bits between consecutive frames rather than by using MC. To discern the scene changes, the VBR video trace was first passed through a median filter having a length of 0.5 secs. Using the output, the short-time mean was calculated using

the 5 frame average filter. The short-time average value exhibited a significant change in the value at a scene boundary. Tests of the model using full-motion video showed that for large buffers, the scenic model estimated cell-loss rate more accurately than AR(1) [6].

Research was also conducted on self-similar (SS) models, which were developed to estimate the long-range dependence (LRD) and were used in conjunction with short range dependence (SRD) for VBR full-motion video traces. In such work, SS traffic models were used to match the LRD, SRD, and PDF. In queueing simulations, the model underestimated ATM cell-loss rates when compared to the actual trace. The presence of a scene change in an MPEG video was determined using the difference in the frame bit count between 2 consecutive I-frames. Two thresholds were used as a measure of the scene activity. Intra-scene fluctuations for I-frames were estimated using an AR(2) process. Each frame type was fit to a lognormal distribution using the histogram. Composition of each frame type according to group of picture (GOP) format generated a video trace possessing the characteristics of VBR video traffic [7 - 9].

B. A Doubly Stochastic Autoregressive Model

The autoregressive model describes the average cell as a function of the frame number. This function is a linear combination that consists of an addition between the delayed average cell rate and a white Gaussian noise. This leads to a model which is jointly Gaussian distributed.

The classical 1st order AR model describing the average cell rate as a function of the frame number is

$$\lambda(n) = a\lambda(n-1) + bw(n) . \qquad (1)$$

Higher order AR models have been successfully tested in the literature for different video traces with different types of activities. Since the chosen order number depends on the statistical nature of the video source, we propose a *doubly stochastic* AR model where the order is allowed to be random. This model is described by

$$\lambda(n)\big|(M=m) = \sum_{k=1}^{m} a_k\lambda(n-k) + bw(n) , \qquad (2)$$

$$\begin{aligned}\lambda(n) &= E_M\left(\lambda(n)\big|M\right) \\ &= \sum_{m=1}^{\infty}\left(\sum_{k=1}^{m} a_k\lambda(n-k) + bw(n)\right) p_M(m),\end{aligned} \qquad (3)$$

where the order M is randomized and drawn from the negative binomial distribution with parameters $(1, \rho)$

$$p_M(m) = \rho(1-\rho)^{m-1}, 0 \le \rho \le 1 . \qquad (4)$$

Such a probability mass function (pmf) for the AR model M is consistent with stochastic modeling of random point processes [10, 11] in time and is reasonable if video frames are conceived as "points" randomly occurring in time (known as time epochs or occurance times).

Under this doubly stochastic model, the conditional distribution is Gaussian since M is fixed (yielding a simple AR(M) model) and the joint frame rate distribution is Gaussian averaged over a Pascal distribution.

III. SIMULATION AND RESULTS

A. Single Video Source

Simulations have been conducted using Matlab. When using simulations, it is important to determine the sample size and number of repetitions over which to average estimates. In our simulation, the sample size was determined to be 4096 by starting out with a fairly small number, calculating the mean, doubling the number, recalculating the mean, and repeating the process until the change in the mean was less than about 1%. The number of repetitions or realizations used to average estimates was calculated to be about 1000 by taking the cumulative average of the mean and peak as a function of the number of realizations. The results are plotted in Figs. 1 and 2.
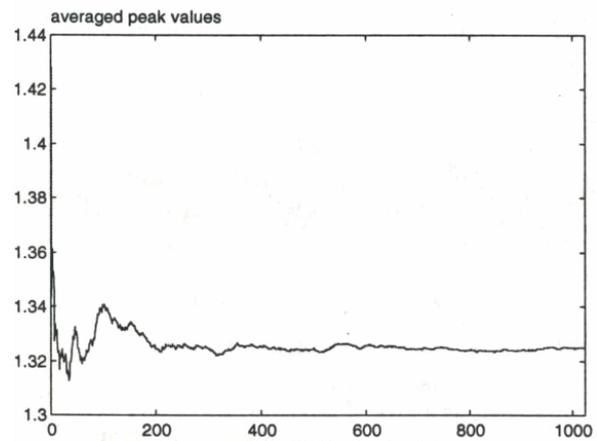

Fig. 1 Averaged peak values of frame rates as a function of the number of simulated realizations

We observe from the graphs of Figs. 1 and 2 that about 1000 realisations, with the number of points in each realization equal to 4096, will produce a fairly accurate result. We also note that both the mean and peak converge as the number of realizations becomes large. For the case of a single video source, the plot of $\lambda(n)$ as a function of the number of frames appear to follow a profile that is peaky. Fig. 3 highlights the different peaks. Theoretically, the nature of uniform activity video sources is bursty, which jusitifes the domination of the bursty peaks of the video data. $\lambda(n)$ is in a linear relation with the white Gaussian noise, so it has the same statistical properties (conditionally).

From the graph in Fig. 3, we can calculate the burstiness coefficient, characterized by the PAR (peak-to-average ratio) metric, as PAR = 1.1588/0.4537 = 2.554. The fact that the PAR is greater than 1 verifies that the uniform activity video source produces bursty data.
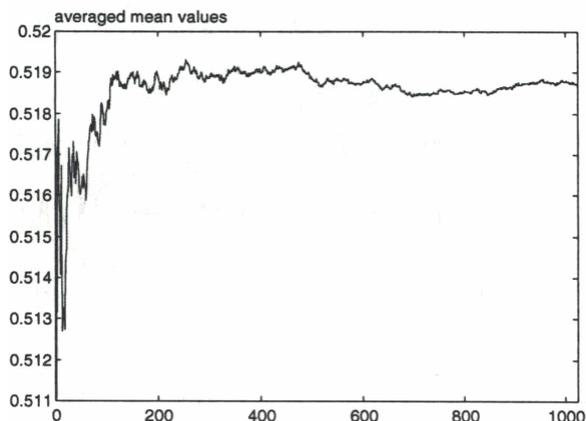
Fig. 2 Averaged mean values of frame rates as a function of the number of simulated realizations
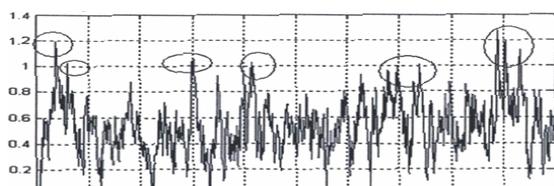

Fig. 3 Cell rate in terms of the frame number for a single source based on a doubly stochastic AR model
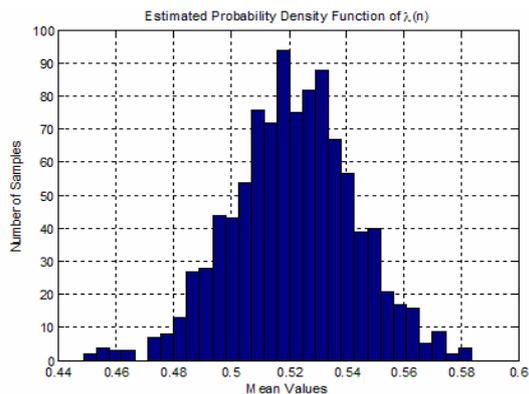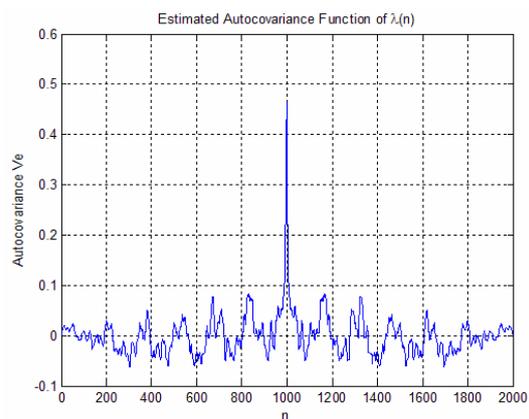

Fig. 4 Estimated pdf of $\lambda(n)$ for a single video source
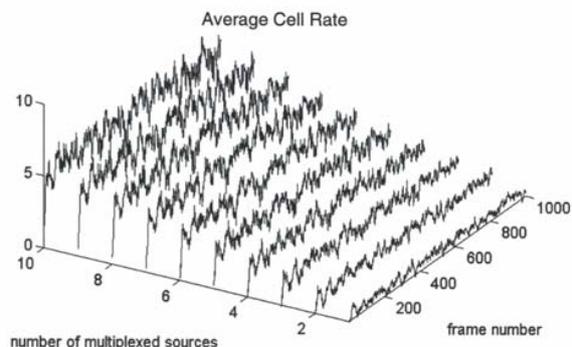

Fig. 5 Simulated ACF of a single video source

Fig. 6 Multiplexed cell rate for a different no. of sources

The histogram (representative of the pdf) of $\lambda(n)$ was calculated using the histogram function built-in Matlab from 1025 representative points of a single realization. The results are shown in Fig. 4. We note that the "envelope" of the pdf appears to be Gaussian. The actual pdf, however, is not exactly Gaussian since it is not perfectly symmetric about the mean. This is caused by the averaging of a number of Gaussian curves over a Pascal-negative binomial distribution as dictated by the doubly stochastic model of $\lambda(n)$.

The autocovariance function ACF of $\lambda(n)$ is a good measure of burstiness as it captures and follows the variation of the peakedness in the signal and can support a high volume of traffic which cannot be handled by the PAR metric. Therefore the variation between the samples is well captured. The estimated autocovariance curve of $\lambda(n)$ is depicted in Fig. 5. We observe that the ACF has a maximum at $n = 10000$ which is the maximum number of frames.

B. Statistical Multiplexing of Doubly Stochastic AR Videos

By multiplexing many sources together (superposition) and repeating the analysis, we find that the pdf becomes narrower and the burstiness coefficient decreases as the number of sources is increased. Fig. 6 shows an example simulation run where $\lambda(n)$ versus n is plotted between 1 and 10 multiplexed sources. More insight can be gained if we normalize the cell rate by dividing by the number of sources for each line in the graph. This is shown in Fig. 7.

Figs. 6 and 7 clearly indicate that the burstiness coefficient reduces dramatically as the number of sources increases. Averaged data from 1024 realisations is tabulated in Table I and graphed in Fig. 8.

It is clear from Fig. 8 that as the number of sources is increased beyond 5, a law of diminishing return takes place since the improvement of the burstiness coefficient is almost stagnant. The optimum number of sources to multiplex can thus be considered to be 5. The effect of statistical multiplexing is also illustrated in Fig. 9 by showing the means and peaks separately and as a ratio.

Fig. 10 shows the normalized histogram for 10 multiplexed sources from a representative sample of 1025 points. The pdf is consistent with a Gaussian distribution. This is a result of the central limit theorem where the individual statistics of the random variables (rvs) do not affect the final statistics of the

summed rvs, especially when the rvs are independent and their number is large. This means that the averaging effect of the binomial distribution of the AR order is transparent to the multiplexing statistics for a relatively large number of sources. Clearly, the width of the pdf has been considerably reduced as shown in Fig. 10. This is yet another way of indicating that the burstiness of the process is much lower. The width of the pdf can also serve a burstiness measure.
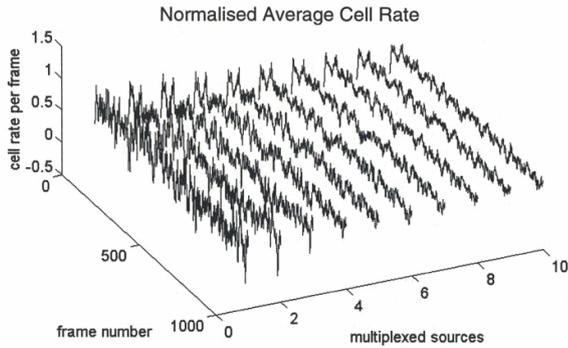

Fig. 7 normalized multiplexed cell rate as a function of the frame number for a different number of sources

TABLE I
PAR VERSUS THE NUMBER OF MULTIPLEXED SOURCES

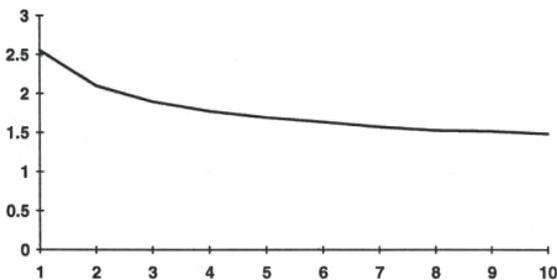| Number of multiplexed sources	B = peak/mean
1	2.553705
2	2.101162
3	1.898157
4	1.775751
5	1.695251
6	1.636071
7	1.575440
8	1.531043
9	1.518870
10	1.486915

Fig. 8 Burstiness coefficient versus number of sources

The histograms produced from a series of 4096 long sample functions for a number of multiplexed sources between 1 and 10 are illustrated in Fig. 11. The trend is that the pdf becomes narrower as the number of mux-sources increases. The largest noticeable difference occurs between 1 and 5 sources.
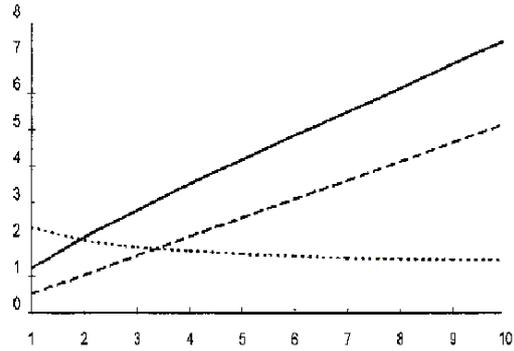

Fig. 9 Effect of statistical multiplexing as a function of the number of multiplexed sources
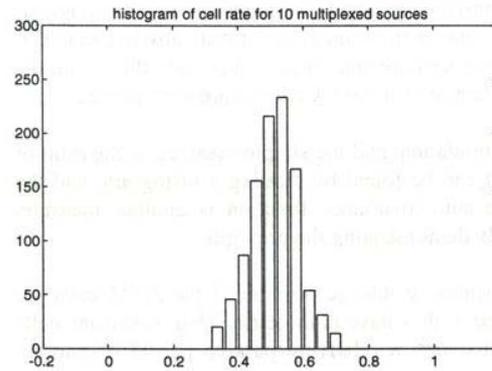

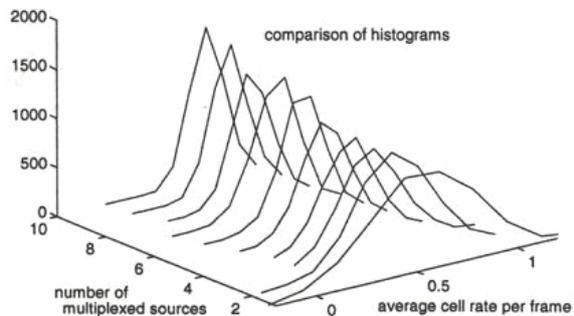Fig. 10 The histogram of $\lambda(n)$ for 10 multiplexed sources

Fig. 11 Simulated histograms for a number of multiplexed sources

The autocovariance curves are plotted in Fig. 12 for 500 sample functions and averaged for 100 realisations.
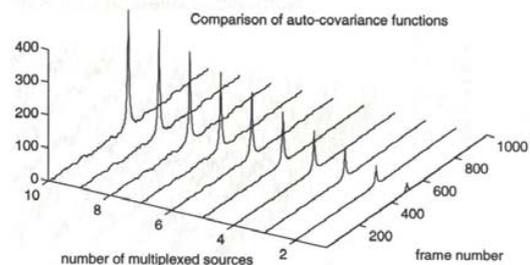

Fig. 12 Simulated ACFs for a number of multiplexed sources

Fig. 12 shows that the width of the covariance function is reduced as the number of multiplexed sources increases, as does the peak (the variance). This is because as more sources are multiplexed, the correlation between samples is reduced. However, as the variance increases, the mean increases even faster so that the ratio of standard deviation-to-average ratio (SAR) is reduced. This is expected since the SAR also serves as a measure of burstiness.

IV. CONCLUSION

In this paper, we developed a model for uniform activity level video sources in ATM networks using a doubly stochastic autoregressive model. The model was consistent with statistical multiplexing theory and proved to be accurate. A number of burstiness metrics were studies, particularly the peak-to-average ratio (PAR), the temporal autocovariance function (ACF), and the traffic measurements histogram (in terms of its width). We found that the former measure is most suitable for capturing the burstiness of single scene video traffic.

The results of multiplexing several constant scene video sources proved to be advantageous with respect to reducing the burstiness of the traffic, as long as the sources are statistically independent. Significant reduction in the burstiness rapidly diminished as the number of multiplexed sources was increased. The largest gain occured when around 5 sources were multiplexed.

One would expect that even higher reductions in burstiness would be possible if the ATM cells are multiplexed asynchronously, rather than just being added, that is, shifting cells associated with a highly active period from one source into a relatively passive period of another source. This is out of the scope of this paper and will be considered in future work.

Also as future work, we propose to investigate a new approach to statistically demultiplex video sources using a novel "spectral queueing" theory and blind decomposition techniques.

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