

# Bubble Growth in a Two Phase Upward Flow in a Miniature Tube

R. S. Hassani, S. Chikh, L. Tadrist, S. Radev

**Abstract**—A bubbly flow in a vertical miniature tube is analyzed theoretically. The liquid and gas phase are co-current flowing upward. The gas phase is injected via a nozzle whose inner diameter is 0.11mm and it is placed on the axis of the tube. A force balance is applied on the bubble at its detachment. The set of governing equations are solved by use of Mathematica software. The bubble diameter and the bubble generation frequency are determined for various inlet phase velocities represented by the inlet mass quality. The results show different behavior of bubble growth and detachment depending on the tube size.

**Keywords**—Two phase flow, bubble growth, minichannel, generation frequency.

## I. INTRODUCTION

**M**ICROFLUIDICS has become an emerging research field in recent decades because it is encountered in many applications of micro/nano technology and biotechnology. Two phase flow at small scale particularly bubbly flow occurs in several industrial applications such as in chemical reactors in process engineering, steam generators for energy production, photobioreactors and many others. Predicting the behavior of two phase flow in such microsystems and controlling the key parameters for a purpose of obtaining a certain flow pattern requires comprehension of bubble formation and growth in ducts of small size as well as good prediction of bubble size, generation frequency, pressure drop and flow regime transitions.

In recent decades numerous investigations on bubble formation and growth were carried out. Ramakrishnan et al. [1] proposed a theoretical model to predict the bubble size in stagnant liquid in tubes of different size. They considered two steps during the bubble formation, namely expansion and detachment. They concluded that their model was more appropriate for tubes of 3mm in diameter than for lower diameter tubes. Later, Chuang and Goldschmidt [2] developed one stage model based on the force balance for different orifice diameters under constant flow conditions in co-current upward flow. Their experimental observations showed that the bubble volume didn't depend on the gas flow rate for liquid flow velocities higher than 0.303 m/s. Predictions from this model deviate from the experimental observations. Kumar [3]

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studied the mechanism of bubble formation for different conditions and explained the various methods for measuring bubble size experimentally. They considered a two stage model. Results obtained were found to match well with experimental bubble size. Gaddis and Vogelpohl [4] posed a simple model for bubble formation in a liquid at rest in a vertical pipe. The comparison between the analytical and experimental investigations demonstrated that their theoretical model was valid in a wide range of velocities and flows. They accounted for a neck development before bubble detachment when they applied the force balance. The effects of several fluid properties namely viscosity, surface tension and density on bubble formation were investigated theoretically and experimentally by Martin et al. [5]. Their theoretical model showed good agreement with experiments with regard to bubble shape and generation time. In a more recent study, Liu et al. [6] presented a theoretical model to analyze bubble size prediction and distribution. A one stage model was developed to describe the bubble formation from an orifice exposed to liquid-cross flow. They showed that the orifice size strongly affects the bubble size and the bubble diameter could be predicted with accuracy less than 21%.

The objective of the present study is to analyze theoretically the effect of inlet conditions on bubble formation and detachment in a co-current upward flow for a vertical configuration by using a force balance for different tube diameters. The forces involved in the present model are surface tension, drag, buoyancy and inertia. Predictions of bubble diameter and bubble generation frequency are made for various inlet liquid and gas phase velocity. Results are presented in terms of a single inlet parameter represented by the inlet mass quality which depends on the flow rate of both phases.

## II. MATHEMATICAL MODEL

The process of bubble formation is governed by several parameters: flow rate of injected gas and liquid, inlet design, fluids properties, tube dimensions, etc.... A theoretical approach to describe the exact behavior of a bubble before its detachment is very complicated. In the present work, an analytical model similar to the one of Chuang and Goldschmidt [2] for the process of bubble formation in a liquid flowing upward is developed using force balance to predict the size of the bubble at detachment and its generation frequency. The bubble is assumed to keep a spherical shape during its evolution as long as its size is smaller than the tube diameter. Air is carried through the injection nozzle with an inner diameter  $d_i$  which is located along the axis of the

circular tube whose inner diameter is  $d_t$ . The liquid phase with density  $\rho_l$ , dynamic viscosity  $\mu_l$  and surface tension  $\sigma$  flows through the tube at constant flow rate  $Q_l$ . In the same direction, the gas phase with density  $\rho_g$  and dynamic viscosity  $\mu_g$  is injected at constant flow rate  $Q_g$  through the nozzle. At detachment the force balance reads as:

$$F_B + F_D = F_S + F_I \quad (1)$$

where the different forces are:

$$\text{Buoyancy force: } F_B = V_b g (\rho_l - \rho_g)$$

$$\text{Surface tension force: } F_S = \pi d_l \sigma \cos \theta$$

$$\text{Inertial force: } F_I = \frac{1}{2} \frac{d}{dt} \left( M \frac{dS}{dt} \right)$$

$$\text{Drag force: } F_D = \frac{\rho_l}{2} C_D \frac{\pi d_b^4}{4} (v_l - v_g)^2$$

$S$ ,  $(dS/dt)$  and  $M$  are the distance from the center of the bubble to the injection orifice, the equivalent velocity of the bubble during its formation and the added mass (equal to  $\rho_l V_b$ ) respectively.

$(v_l - v_g)$  is the relative velocity of the bubble and  $C_D$  is the drag coefficient obtained from the following correlation taken from Bird et al. [7]:

$$C_D = 18.5 \text{Re}^{-3/5} \quad (2)$$

Substituting all these forces in (1), it becomes:

$$\frac{d_b^3}{6} g \rho_l + \frac{\rho_l d_b^2}{2 \cdot 4} \frac{18.5 v^{3/5}}{\left( U_l - \frac{d_b f}{6} \right)^{3/5}} \left( U_l - \frac{d_b f}{6} \right)^2 = d_l \sigma + \frac{\rho_l f^2 d_b^4}{54} \quad (3)$$

where  $U_l$  is the relative velocity of the bubble and  $f$  is the bubble generation frequency which is related to the bubble diameter by the following equation:

$$\frac{d d_b}{dt} = \frac{d_b f}{3} \quad (4)$$

The set of (3) and (4) are to be solved for the bubble diameter  $d_b$  and the bubble generation frequency.

The Mathematica software is utilized for this purpose and the obtained results are presented in the next section.

### III. RESULTS AND DISCUSSION

Results are presented in terms of dimensionless bubble diameter  $d_b^*$  and bubble generation frequency versus inlet mass quality. The dimensionless bubble diameter plotted in Figs. 1, 2 is defined as  $d_b^* = d_b / d_i$  where  $d_i$  in the present study is taken equal to 0.11 mm.

Figs. 1 and 2 display the equivalent dimensionless bubble diameter versus inlet mass quality in a microtube of 0.5mm in diameter and a minitube of 3mm in diameter for several values

of liquid flow rate. It is depicted that bubble diameter increases with increasing inlet mass quality for the different values of liquid velocity used for both tubes. The horizontal line plotted in Fig. 1 represents the limiting value (4.54 corresponding to the end of a bubbly flow). Above this value, the equivalent bubble diameter becomes higher than the tube diameter and hence a slug flow is obtained. In the minitube of larger diameter (3mm, Fig. 2), for the smaller liquid velocities but the same range of inlet mass quality the slug pattern has not been reached.

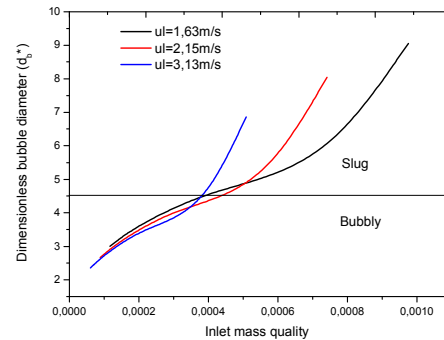


Fig. 1 Dimensionless bubble diameter versus inlet mass quality for tube diameter  $d_t = 0.5\text{mm}$

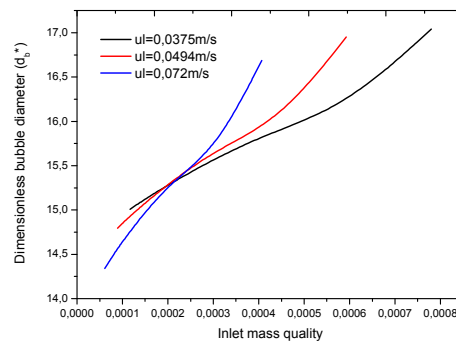


Fig. 2 Dimensional bubble diameter versus inlet mass quality for tube diameter  $d_t = 3\text{mm}$

From Fig. 1 it is clear that the limit of the bubbly flow occurs at an inlet mass quality that depends on the liquid velocity.

Bubble generation frequency is plotted versus inlet mass quality in Fig. 3. It illustrates the case of a minitube of 3mm in diameter. It shows the frequency of generating bubbles that increases with the increasing inlet mass quality. It is exhibited that a higher liquid velocity yields a higher generation frequency. In the range of explored flow rates (i.e. inlet mass quality) the bubble diameter remains much lower than the minitube diameter ( $d_t = 3\text{mm}$ ) and thus the effect of tube wall on bubble size is not very significant. Consequently the detachment frequency keeps increasing. It is also evidenced that the threshold of slug flow depends directly on tube size and mass quality for a given liquid flow rate.

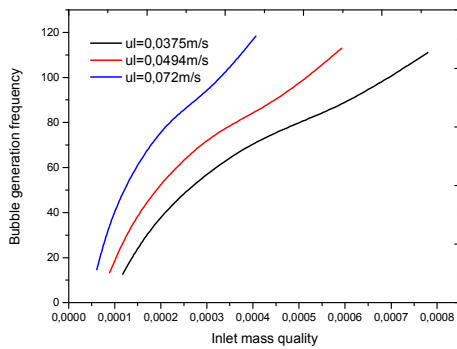


Fig. 3 Bubble generation frequency versus inlet mass quality for tube diameter  $d_t = 3\text{mm}$

#### IV. CONCLUSION

A bubbly flow in a vertical miniature tube is analyzed theoretically. The bubble size and the detachment frequency are determined by an analytical approach by solving a force balance applied on a bubble in a small size tube. Results show different behavior in a microtube whose diameter is 0.5mm or in a minitube whose diameter is 3mm.

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