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Bit Error Rate Analysis of Mobile Communication Network in Nakagami Fading Channel: Interference Considerations

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Abstract**—**Co-channel interference is one of the major problems in wireless systems. The effects of co-channel interference in a Nakagami fading channel on the ABER (Average Bit Error Rate) with static nodes are well analyzed. In this paper, we derive the ABER with the presence of mobile nodes. ABER is also derived for mobile systems in the presence of co-channel interference.

Keywords**—**ABER, co-channel interference, Nakagami fading.

I. Introduction

THERE is a steady rise in the number of mobile system users in the recent times. In order to accommodate more users in a given bandwidth, the spectral efficiency needs to be improved. So, frequency reuse technique is being adopted to increase spectral efficiency [1]. However, frequency reuse causes inter channel and co-channel interference which is more than additive white Gaussian noise of many system [2]. Due to the presence of ICI (inter channel interference) and co-channel interference, the capacity of the system is reduced.

Outage probability and BER are the two performance measures which are usually adopted to evaluate the performance of mobile communication systems [3]. The outage probability and BER of a wireless system in a Rayleigh fading channel have been studied in [4], [5]. The performance measures have also been studied for more generalized Nakagami fading channel in [6], [7]. However, these performances are studied for wireless static networks. So, there is a need to study the variation of performance measures for a mobile wireless network where the nodes are mobile.

Usually nodes in a static wireless network are stationary and the distance between nodes is constant. So, the received signal in a static wireless network is characterized by the short term fading model such as Rayleigh, Rician, Nakagami. It is well known that the received signal follows exponential distribution and Gamma distribution for Rayleigh fading channel and Nakagami fading channel respectively [1], [3]. However, in mobile wireless networks, the nodes are mobile and the distance between nodes is random in nature [8]. So, in these networks, the received signal from a Nakagami fading channel no more follows Gamma distribution. Mobility of nodes, multipath fading model, and distance dependent path loss results in the time varying nature of the received signal power. The time varying nature of the received power affects the performance analysis of the mobile wireless network to a great extent [9]. It is appropriate to study the statistics of the received power in order to understand the effect of mobility on the mobile wireless networks.

Distribution of node in a static environment usually modeled as uniform distribution to study connectivity properties. However, in the mobile networks like MANET (Mobile Ad Hoc Network), the nodes are mobile and it requires a more realistic model to depict the position of nodes. In order to study the distribution of received power, several mobility models are adopted to depict actual mobile wireless network environment. A few of the mobility models which are used to depict mobile environment are random walk (RW), random way point (RWP) and random direction (RD) mobility model [10]. One of the mobility model, which is usually used for the analysis on of mobile networks is Random way point [11].

In a recent work [12], authors have studied the effect of co-channel interference on performance measures in a Nakagami fading channel for mobile environment. However, the study is limited to the computation of outage probability only. Thus, it is important to analyze the effect of co channel interference on the ABER at the receiver of a mobile link in a Nakagami environment. The main contribution of the paper is.

a. The ABER expression is derived for a mobile system in a Nakagami fading channel.
b. The effect of fading parameters on ABER is studied for a mobile system.
c. The ABER expression is derived for a mobile system in a Nakagami fading channel in the presence of co-channel interference.

Rest of this paper is organized as follows. Section II describes the effect of mobility on ABER. The effect of co-channel interference is derived in Section III. Conclusion of the work is presented in Section IV.

II. Effect of Mobility on ABER

It is known that in a static wireless system the received signal power (r) for Nakagami fading environment follows a

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gamma distribution. The probability density function (PDF) of the received power is presented as

$$p(r) = \left(\frac{m_q}{P_A}\right)^{m_q} \frac{r^{m_q-1}}{\Gamma(m_q)} exp\left(-\frac{m_q}{P_A}\right) \quad (1)$$

where 'P_A' is the average signal power and 'm_q' is Nakagami fading parameter.

In a mobile network, since the transmitter-receiver distance is a random variable, the PDF of the received power does not follow gamma distribution. We assume a mobile network where the mobility of the nodes is presented by random waypoint mobility model. PDF of the received signal power at the receiving node for a one dimensional wireless network is expressed as [12]

$$p(r) = \frac{6}{\beta\Gamma(m_q)}\left(\frac{m_q}{P_A}\right)^{-\frac{2}{\beta}} r^{-\left(\frac{2}{\beta}+1\right)} \times \left[\gamma\left(m_q + \frac{2}{\beta}, \frac{m_q}{P_A} r\right) - \left(\frac{m_q}{P_A}\right)^{-\frac{1}{\beta}} r^{-\frac{1}{\beta}} \gamma\left(m_q + \frac{3}{\beta}, \frac{m_q}{P_A} r\right)\right] \quad (2)$$

where, β is the path loss component and $\gamma(\ , \ )$ is the lower incomplete gamma function as defined in [13, eq. 8.350.1].

Next, we derive the expression of performance metric ABER for the analysis of wireless mobile networks.

A. ABER

The conditional BER corresponding to digital modulation scheme such as non-coherent frequency shift keying (NCFSK) or differentially coherent phase shift keying (DPSK) for a given SNR (y) is given by [7]

$$P_B(y) = \frac{1}{2} exp(-ky) \quad (3)$$

where $k = 1$, for BPSK (Binary phase shift keying).

Considering the noise power to be unity, the conditional BER is given as

$$P_B(y) = P_B(r) = \frac{1}{2} exp(-kr) \quad (4)$$

Now, the ABER is evaluated using the expression as [3].

$$\overline{P_B} = \int_0^\infty P_B(r) p(r) dr \quad (5)$$

Using (2) and (4) in (5), the expression for ABER is rewritten as:

$$\overline{P_B} = \frac{3}{\beta\Gamma(m_q)}\left(\frac{m_q}{P_A}\right)^{-\frac{2}{\beta}} \times \left[\int_0^\infty exp(-kr) r^{-\frac{2}{\beta}+1} \gamma\left(m_q + \frac{2}{\beta}, \frac{m_q}{P_A} r\right) dr - \int_0^\infty exp(-kr)\left(\frac{m_q}{P_A}\right)^{-\frac{1}{\beta}} r^{-\frac{3}{\beta}+1} \gamma\left(m_q + \frac{3}{\beta}, \frac{m_q}{P_A} r\right) dr\right] \quad (6)$$

The integral terms in (5) are further simplified using results in [13, eq. 6.455.2] and represented in the form of gauss hyper geometric function ${}_2F_1$(-;-;-) [14] as:

$$\overline{P_B} = 3 \times \left(\frac{m_q}{m_q+P_A}\right)^{m_q} \times \left[\frac{1}{2+m_q\beta} \times {}_2F_1\left(1, m_q; m_q + \frac{2}{\beta} + 1; \frac{m_q}{m_q+kP_A}\right) - \frac{1}{3+m_q\beta} \times {}_2F_1\left(1, m_q; m_q + \frac{3}{\beta} + 1; \frac{m_q}{m_q+kP_A}\right)\right] \quad (7)$$

The ABER performance of the mobile wireless network vs transmitted power for differentially coherent BPSK modulation in Nakagami fading channel is presented in Fig. 1.

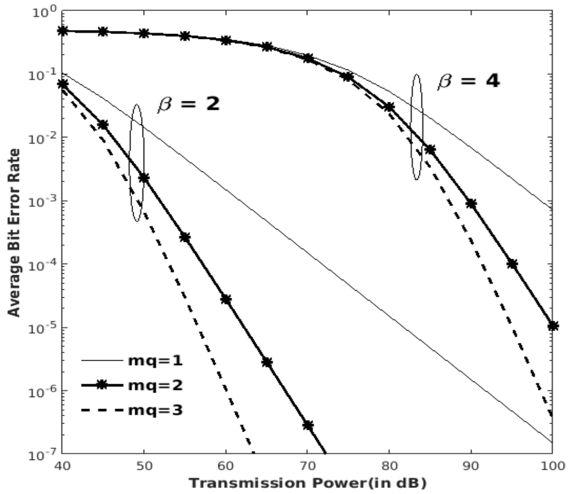

Fig. 1 Transmission power Vs ABER for 1D network with RWP mobility model

It is observed for β =2 that the ABER gradually decreases with transmission power for m_q =1. However, with increase in fading parameter from m_q =1 to m_q=3, the ABER seems to decrease sharply with increase in transmission power. It is also noticed that the ABER for higher fading parameter is better than lower fading parameter. In case of higher path loss exponent (β=4) there is no significant change in ABER for lower transmission power up to 70 dB even though the fading parameter changes. Further increase in transmission power results in a rapid decrease in ABER for different fading parameter. In Fig. 1 it is also seen that the ABER for β=4 is higher than β =2 for all type of fading environment.

III. Effect of Interference

In a wireless network where more frequencies are reused to increase capacity, interferences (co-channel) are more significant. The effect of interference is more severe than AWGN (Additive white Gaussian noise) in some systems. Now, ignoring the effects of AWGN, signal to interference plus noise ratio (SINR) is approximated as signal to interference ratio (SIR) only. Thus, the ABER at a receiving node in such a scenario depends on SIR. In this paper, we consider the effect of 'N' i.e. number of mutually independent and identically distributed Nakagami interferers at the receiving node. The interferers are considered to be of equal distance from the receiving node and stationary in nature for the analysis of interference on ABER. The average received powers from all the interferers are same as they are at equal distance from the receiving node. Thus, the PDF of the sum of all the interference power (s) at the receiving node due to the Nakagami interferers is given by [6]

$$p(s) = \left(\frac{m_i}{P_I}\right)^{m_i N} \frac{s^{m_i N-1}}{\Gamma(m_i N)} exp\left(-\frac{m_i}{P_I}s\right) \tag{8}$$

where 'P_I' is the average interference power and 'm_i' is the fading parameter related to interfering signals.

Let the variable 'z' defines the SIR (i.e. $z = r/s$) . The PDF of z is given as:

$$p(z) = \int_0^\infty s\, p(zs)p(s)ds = \frac{6}{\beta\Gamma(m_q)\Gamma(m_i N)}\left(\frac{m_q}{P_A}\right)^{-\frac{2}{\beta}}\left(\frac{m_i}{P_I}\right)^{m_i N} z^{-\left(\frac{2}{\beta}+1\right)}\times \left[\int_0^\infty s^{m_i N-\frac{2}{\beta}-1}\gamma\left(m_q+\frac{2}{\beta},\frac{m_q z s}{P_A}\right)exp\left(\frac{m_i}{P_I}s\right)ds - \left(\frac{m_q}{P_A}\right)^{-\frac{1}{\beta}} z^{-\frac{1}{\beta}}\times \int_0^\infty s^{m_i N-\frac{3}{\beta}-1}\gamma\left(m_q+\frac{3}{\beta},\frac{m_q z s}{P_A}\right)exp\left(\frac{m_i}{P_I}s\right)ds\right] \tag{9}$$

Evaluating the integral terms in (9) using [13, eq. 6.455.2], the PDF of SIR (z) is further simplified as

$$p(z) = \frac{6\Gamma(m_q+m_i N)}{\Gamma(m_q)\Gamma(m_i N)}\left(\frac{m_i}{P_I}\right)^{m_i N}\left(\frac{m_q}{P_A}\right)^{m_q}\times\frac{z^{m_q-1}}{\left(\frac{m_q z}{P_A}+\frac{m_i}{P_I}\right)^{m_q+m_i N}}\times \left[\frac{1}{2+m_q\beta}\,{}_2F_1\left(1,m_q+m_i N;m_q+\frac{2}{\beta}+1;\frac{m_q z/P_A}{\frac{m_q z}{P_A}+\frac{m_i}{P_I}}\right) - \frac{1}{3+m_q\beta}\,{}_2F_1\left(1,m_q+m_i N;m_q+\frac{3}{\beta}+1;\frac{m_q z/P_A}{\frac{m_q z}{P_A}+\frac{m_i}{P_I}}\right)\right] \tag{10}$$

Using the concept defined in (4) and (5), the ABER is expressed as

$$\overline{P_B} = \frac{1}{2}\int_0^\infty p(z)\,exp(-kz)\,dz \tag{11}$$

Incorporating (10) in (11), and substituting $\frac{m_q}{P_A}/\frac{m_i}{P_I} = t$ the expression for ABER is written as

$$\overline{P_B} = \frac{3\Gamma(m_q+m_i N)}{\Gamma(m_q)\Gamma(m_i N)}\times t^{m_q}\times\left[\int_0^\infty \frac{z^{m_q-1}exp(-kz)}{(2+m_q\beta)(1+tz)^{m_q+m_i N}}\times {}_2F_1\left(1,m_q+m_i N;m_q+\frac{2}{\beta}+1;\frac{tz}{1+tz}\right)dz - \int_0^\infty \frac{z^{m_q-1}exp(-kz)}{(3+m_q\beta)(1+tz)^{m_q+m_i N}}\times {}_2F_1\left(1,m_q+m_i N;m_q+\frac{3}{\beta}+1;\frac{tz}{1+tz}\right)dz\right] \tag{12}$$

Implementing the equivalent relation between the gauss hyper geometric function defined in [13, eq. 9.131.1, pg. 1008], (12) is further expressed as given below.

$$\overline{P_B} = \frac{3\Gamma(m_q+m_i N)}{\Gamma(m_q)\Gamma(m_i N)}\times t^{m_q}\times\left[\frac{1}{2+m_q\beta}\int_0^\infty z^{m_q-1}exp(-kz)\times {}_2F_1\left(1,m_q+m_i N;m_q+\frac{2}{\beta}+1;-tz\right)dz - \frac{1}{3+m_q\beta}\int_0^\infty z^{m_q-1}exp(-kz)\times {}_2F_1\left(1,m_q+m_i N;m_q+\frac{3}{\beta}+1;\frac{tz}{1+tz}\right)dz\right] \tag{13}$$

Using [13], the 1st integral term in (13) is further simplified as

$$\int_0^\infty z^{m_q-1}exp(-kz)\times {}_2F_1\left(1,m_q+m_i N;m_q+\frac{2}{\beta}+1;-tz\right)dz = \frac{\Gamma\left(m_q+\frac{2}{\beta}+1\right)k^{-m_q}}{\Gamma(m_q+m_i N)\Gamma\left(m_q+\frac{2}{\beta}\right)}\times E\left(m_q+m_i N,m_q+\frac{2}{\beta},m_q;m_q+\frac{2}{\beta}+1;\frac{k}{t}\right) \tag{14}$$

where, E(-;-;-) is defined as Mac Robert's E-function [15, p. 425]. Expressing the Mac Robert's E-function in terms of equivalent Meijer's G-function [15] (for computing in MATLAB), (14) is rewritten as

$$\int_0^\infty z^{m_q-1}exp(-kz)\times {}_2F_1\left(1,m_q+m_i N;m_q+\frac{2}{\beta}+1;-tz\right)dz = \frac{\Gamma\left(m_q+\frac{2}{\beta}+1\right)k^{-m_q}}{\Gamma(m_q+m_i N)\Gamma\left(m_q+\frac{2}{\beta}\right)} G_{2\,3}^{3\,1}\left(\left.\begin{matrix}1,m_q+\frac{2}{\beta}+1\\ m_q+m_i N,m_q+\frac{2}{\beta},m_q\end{matrix}\right|\frac{k}{t}\right) \tag{15}$$

In a similar way, the 2nd integral term in (13) is expressed as

$$\int_0^\infty z^{m_q-1}exp(-kz)\times {}_2F_1\left(1,m_q+m_i N;m_q+\frac{3}{\beta}+1;-tz\right)dz = \frac{\Gamma\left(m_q+\frac{3}{\beta}+1\right)k^{-m_q}}{\Gamma(m_q+m_i N)\Gamma\left(m_q+\frac{3}{\beta}\right)} G_{2\,3}^{3\,1}\left(\left.\begin{matrix}1,m_q+\frac{3}{\beta}+1\\ m_q+m_i N,m_q+\frac{3}{\beta},m_q\end{matrix}\right|\frac{k}{t}\right) \tag{16}$$

Substituting (15) and (16) in (13), the ABER is expressed in the form of Meijer's G-function as:

$$\overline{P_B} = \frac{3t^{m_q}}{\Gamma(m_q)\Gamma(m_i N)k^{m_q}}\times \left[\frac{\Gamma\left(m_q+\frac{2}{\beta}+1\right)k^{-m_q}}{\Gamma(m_q+m_i N)\Gamma\left(m_q+\frac{2}{\beta}\right)} G_{2\,3}^{3\,1}\left(\left.\begin{matrix}1,m_q+\frac{2}{\beta}+1\\ m_q+m_i N,m_q+\frac{2}{\beta},m_q\end{matrix}\right|\frac{k}{t}\right) - \frac{\Gamma\left(m_q+\frac{3}{\beta}+1\right)k^{-m_q}}{\Gamma(m_q+m_i N)\Gamma\left(m_q+\frac{3}{\beta}\right)} G_{2\,3}^{3\,1}\left(\left.\begin{matrix}1,m_q+\frac{3}{\beta}+1\\ m_q+m_i N,m_q+\frac{3}{\beta},m_q\end{matrix}\right|\frac{k}{t}\right)\right] \tag{17}$$

Analysis of average BER for different number of interferers is presented using (17). Fig. 2 shows the effects of transmission power on ABER in the presence of interferers in a Nakagami fading channel. In order to analyze the effect of interference at the receiving node we consider the distance ratio between interferer-receiver and transmitter-receiver to be 1.5. The interfering nodes are assumed to use a fixed transmission power equal to 80 dB with fading parameter m_i = 1. It is seen from Fig. 2 that the ABER increases with the addition of interference even though the transmission power of the transmitting node increases. It is also noticed that with increase in number of interferers the degradation in BER performance is very less for transmission power up to 65 dB. However, for higher value of transmission power the variation in ABER is of significant for different value of interferers. Similar trend of ABER with increase in transmission power is also observed for different value of fading parameter in an interference limited system. However, the ABER for fading parameter m_q=3 is better than m_q=1 in the presence of interferers.

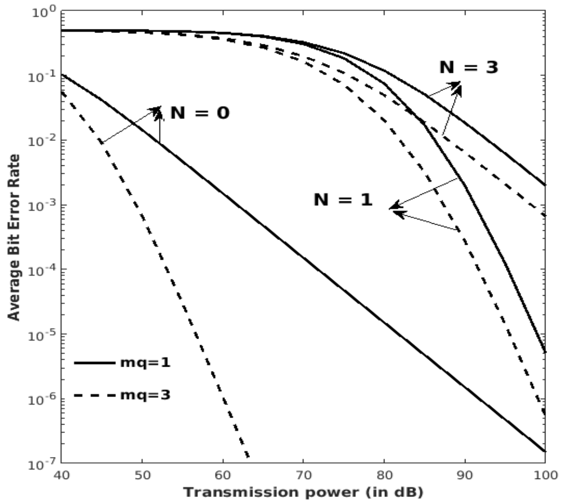

Fig. 2 Transmission power Vs ABER for interference limited system with m_i =1,β=2 and N={0,1,3}

IV. CONCLUSION

In this paper. we derived an analytical closed form expression for computation of ABER at the receiving node of a mobile network following RWP mobility in a Nakagami environment. The effect of co-channel interference on the ABER performance is also studied with the help of a derived closed form mathematical expression. Comparison of BER performance obtained from numerical results indicates that the interference from the non-desired co-channel interfering transmitters in a mobile scenario degrades the BER at the receiver. The effect is found to be severe with increase in number in interferes.

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