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Bilinear and Bilateral Generating Functions for the Gauss' Hypergeometric Polynomials

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Abstract—The object of the present paper is to investigate several general families of bilinear and bilateral generating functions with different argument for the Gauss' hypergeometric polynomials.

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I. Introduction

IN 1994, S.D. Singh and M.S. Arora [9], gave the semi orthogonal property of the Gauss' hypergeometric polynomials with its application as follows:

$$\int_0^\infty x^{-1-b-m}(1+x)^{b-c-m}A_m^{(b,c)}(x)A_n^{(b,c)}(x)dx$$

$$= 0,\ if\ m < n$$

$$= \frac{(b)_n\ n!\ \Gamma(c)\ \Gamma(-b)\ \Gamma(1+b)}{(c)_n\ \Gamma(1+b+n)\ \Gamma(c-b)},\ if\ m = n \quad (1)$$

where $Re(c) > 0$, $Re(b) < -m$, $Re(b) > -n \implies m = n,\ b \neq -n$.

Later, in 2001, I.K. Khanna and V. Srinivasa Bhagavan [5] derive the generating functions by using the representations of the Lie group SL(2,C)(the complex special linear group).

The present paper is the extension of our earlier paper [6] in which Gauss' hypergeometric polynomials is defined by the relation

$$A_n^{(b,c)}(x) = x^n{}_2F_1\left[\begin{matrix} -n, b\ ; \\ c\ \ ; \end{matrix}\ -\frac{1}{x}\right]$$

$$= \sum_{r=0}^{\infty}(-1)^r\frac{(-n)_r(b)_r}{(c)_r\ r!}x^{n-r},\ n = 0, 1, 2, ... \quad (2)$$

provided that c is not zero nor a negative integer.

In view of the relation [see, E.D. Rainville [3], Th. 20, pp. 60],

$${}_2F_1[a,\ b;\ c;\ z] = (1-z)^{-a}{}_2F_1\left[a,\ c-b;\ c;\frac{z}{z-1}\right] \quad (3)$$

the relation (2) can be written in an elegant form as

$$A_n^{(b,c)}(x) = (1+x)^n{}_2F_1\left[\begin{matrix} -n, c-b\ ; \\ c\ ; \end{matrix}\ \frac{1}{1+x}\right] \quad (4)$$

Also, by reversing the order of summation, (2) and (4) can be written as

$$A_n^{(b,c)}(x) = \frac{(b)_n}{(c)_n}\ {}_2F_1\left[\begin{matrix} -n, 1-c-n\ ; \\ 1-b-n\ ; \end{matrix}\ -x\right] \quad (5)$$

and

$$A_n^{(b,c)}(x) = (-1)^n\frac{(c-b)_n}{(c)_n}$$

$$\times{}_2F_1\left[\begin{matrix} -n, 1-c-n\ ; \\ 1+b-c-n\ ; \end{matrix}\ 1+x\right] \quad (6)$$

Some of the definitions and notations used in the present paper are as follows:

Appell's functions of two variables are given by (see [7]).

$$F_1[a, b, b'; c; x, y] = \sum_{n,k=0}^{\infty}\frac{(a)_{n+k}(b)_n(b')_k}{n!\ k!\ (c)_{n+k}}x^n y^k \quad (7)$$

$$F_2[a, b, b'; c, c'; x, y] = \sum_{n,k=0}^{\infty}\frac{(a)_{n+k}(b)_n(b')_k}{n!\ k!\ (c)_n(c')_k}x^n y^k \quad (8)$$

$$F_3[a, a', b, b'; c; x, y] = \sum_{n,k=0}^{\infty}\frac{(a)_n(a')_k(b)_n(b')_k}{n!\ k!\ (c)_{n+k}}x^n y^k \quad (9)$$

$$F_4[a, b; c, c'; x, y] = \sum_{n,k=0}^{\infty}\frac{(a)_{n+k}(b)_{n+k}}{n!\ k!\ (c)_n(c')_k}x^n y^k \quad (10)$$

Saran's functions for three variables are given by (see [8]).

$$F_E\left[\alpha_1,\ \alpha_1,\ \alpha_1,\ \beta_1,\ \beta_2,\ \beta_2;\ \gamma_1,\ \gamma_2,\ \gamma_3;\ x, y, z\right]$$

$$= \sum_{m,n,p=0}^{\infty}\frac{(\alpha_1)_{m+n+p}\ (\beta_1)_m\ (\beta_2)_{n+p}}{(\gamma_1)_m\ (\gamma_2)_n\ (\gamma_3)_p\ m!\ n!\ p!}x^m\ y^n\ z^p \quad (11)$$

$$F_G\left[\alpha_1,\ \alpha_1,\ \alpha_1,\ \beta_1,\ \beta_2,\ \beta_3;\ \gamma_1,\ \gamma_2,\ \gamma_2;\ x, y, z\right]$$

$$= \sum_{m,n,p=0}^{\infty}\frac{(\alpha_1)_{m+n+p}\ (\beta_1)_m\ (\beta_2)_n\ (\beta_3)_p}{(\gamma_1)_m\ (\gamma_2)_{n+p}\ m!\ n!\ p!}x^m\ y^n\ z^p \quad (12)$$

$$F_S\left[\alpha_1,\ \alpha_2,\ \alpha_2,\ \beta_1,\ \beta_2,\ \beta_3;\ \gamma_1,\ \gamma_1,\ \gamma_1;\ x, y, z\right]$$

$$= \sum_{m,n,p=0}^{\infty}\frac{(\alpha_1)_m(\alpha_2)_{n+p}(\beta_1)_m(\beta_2)_n(\beta_3)_p}{(\gamma_1)_{m+n+p}\ m!\ n!\ p!}x^m y^n z^p \quad (13)$$

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Lauricella's hypergeometric functions for n variables is defined by (see [4]).

$$F_C^{(n)}\left[a, b;\ c_1, ..., c_n;\ x_1, ..., x_n\right] = \sum_{m_1,...,m_n=0}^{\infty} \frac{(a)_{m_1+...+m_n}\ (b)_{m_1+...+m_n}}{(c_1)_{m_1}...(c_n)_{m_n}} \times \frac{x_1^{m_1}}{m_1!}...\frac{x_n^{m_n}}{m_n!} \tag{14}$$

$$F_D^{(n)}\left[a, b_1, ..., b_n;\ c\ ;\ x_1, ..., x_n\right] = \sum_{m_1,...,m_n=0}^{\infty} \frac{(a)_{m_1+...+m_n}\ (b_1)_{m_1}...(b_n)_{m_n}}{(c)_{m_1+...+m_n}} \times \frac{x_1^{m_1}}{m_1!}...\frac{x_n^{m_n}}{m_n!} \tag{15}$$

Confluent form of Lauricella's functions for n variables is defined by (see [4]).

$$\psi_2^{(n)}\left[a, c_1, ..., c_n;\ x_1, ..., x_n\right] = \sum_{m_1,...,m_n=0}^{\infty} \frac{(a)_{m_1+...+m_n}}{(c_1)_{m_1}...(c_n)_{m_n}} \frac{x_1^{m_1}}{m_1!}...\frac{x_n^{m_n}}{m_n!} \tag{16}$$

Similarly, a general triple hypergeometric series $F^{(3)}[x,y,z]$ (see [4], pp. 69) is defined as

$$F^{(3)}[x,y,z] = F^{(3)}\left[\begin{array}{l} (a) :: (b); (b'); (b'')\ : (c); (c'); (c'')\ ; \\ (e) :: (g); (g'); (g'') : (h); (h'); (h''); \end{array} x, y, z\right] = \sum_{m,n,p=0}^{\infty} \Lambda(m,n,p) \frac{x^m}{m!} \frac{y^n}{n!} \frac{z^p}{p!} \tag{17}$$

where for convenience

$$\Lambda(m,n,p) = \frac{\prod_{j=1}^{A}(a_j)_{m+n+p} \prod_{j=1}^{B}(b_j)_{m+n} \prod_{j=1}^{B'}(b'_j)_{n+p}}{\prod_{j=1}^{E}(e_j)_{m+n+p} \prod_{j=1}^{G}(g_j)_{m+n} \prod_{j=1}^{G'}(g'_j)_{n+p}} \times \frac{\prod_{j=1}^{B''}(b''_j)_{p+m} \prod_{j=1}^{C}(c_j)_m \prod_{j=1}^{C'}(c'_j)_n \prod_{j=1}^{C''}(c''_j)_p}{\prod_{j=1}^{H}(h_j)_m \prod_{j=1}^{H'}(h'_j)_n \prod_{j=1}^{H''}(h''_j)_p \prod_{j=1}^{G''}(g''_j)_{p+m}}$$

II. Bilinear Generating Functions

By using the definition (2) and the Gaussian hypergeometric transformation (see, Rainville [3], Th. 21, pp. 60]

$${}_2F_1[a,\ b;\ c;\ z] = (1-z)^{c-a-b}{}_2F_1\left[c-a,\ c-b;\ c; z\right] \tag{18}$$

We thus obtain the bilinear generating function

$$\sum_{n=0}^{\infty} \frac{(c+b)_n (c+m)_n}{(1+d)_n\ n!} A_{m+n}^{(-b-n,c)}(x) A_n^{(-d-n,e)}(y) t^n = (1+x)^m \left(\frac{x}{1+x}\right)^{-b-c} F_c^{(3)}\left[c+m\ , c+b\ ; c\ , e\ , 1+d\ ; -\frac{1}{x}, -\frac{(1+x)^2 t}{x}, \frac{(1+x)^2 yt}{x}\right] \tag{19}$$

where $F_c^{(3)}$ denote the Lauricella's function defined by (14, with $n = 3$). An interesting special case of the generating function (19) would occurs when we set, $m = 0$, $d = b$, $e = c$, and appealing the hypergeometric reduction formula (see, B.L. Sharma [1], pp. 716, (2.4)).

$$F_c^{(3)}[\alpha+\beta+1, \beta+1\ ;\ \alpha+1, \beta+1, \beta+1\ ;\ x, y, z] = (1+x-y-z)^{-\alpha-\beta-1} \times F_4\left[\frac{\alpha+\beta+1}{2}, \frac{\alpha+\beta+2}{2}; \alpha+1, \beta+1; X, Y\right] \tag{20}$$

where, $X = \frac{4x}{(1+x-y-z)^2}$, $Y = \frac{4yz}{(1+x-y-z)^2}$ yields the generating relation

$$\sum_{n=0}^{\infty} \frac{(c+b)_n (c)_n}{(1+b)_n\ n!} A_n^{(-b-n,c)}(x) A_n^{(-b-n,c)}(y) t^n = \{1+(1+x)(1+y)t\}^{-b-c} \times F_4\left[\frac{1}{2}(c+b), \frac{1}{2}(c+b+1); 1+b, c; \xi, \zeta\right] \tag{21}$$

where, $\xi = \frac{4xyt}{(1+(1+x)(1+y)t)^2}$, $\zeta = \frac{4t}{(1+(1+x)(1+y)t)^2}$ and F_4 is the Appell's function defined by (10).

Another bilinear generating function are obtained by using (2), which in conjunction with ([6], (2.25)),

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(1+b+c)_n\ n!} A_{m+n}^{(b,-c-n)}(x) t^n = \frac{(1+b+c-m)_m}{(1+c-m)_m} x^m (1-xt)^{-\lambda} \times F_1\left[b, -m,\ \lambda;\ 1+b+c-m; \frac{1+x}{x}, -\frac{(1+x)t}{1-xt}\right] \tag{22}$$

readily gives the relation

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(1+b+c)_n\ n!} A_n^{(b,-c-n)}(x) A_n^{(d,e)}(y) t^n = (1-xyt)^{-\lambda} \sum_{n=0}^{\infty} \frac{(\lambda)_n (d)_n (b)_n}{(e)_n (1+b+c)_n\ n!} (\chi)^n \times F_2\left[\lambda+n, d+n, b+n; e+n, 1+b+c+n; \psi, \omega\right] \tag{23}$$

where, $\frac{\chi}{-(1+x)} = \frac{\psi}{x} = \frac{\omega}{-(1+x)y} = \frac{t}{1-xyt}$ and F_2 is the Appell's function defined by (8)

The second member of (23) can indeed be written in terms of Srivastava triple hypergeometric series $F^{(3)}[x,y,z]$ defined by (17), and we thus obtain the alternative form of the bilinear generating function (23) as,

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n (1+c)_n}{(1+b+c)_n\ n!} A_n^{(b,-c-n)}(x) A_n^{(d,e)}(y) t^n = (1-xyt)^{-\lambda} \times F^{(3)}\left[\begin{array}{l} \lambda\ :: d\ ; ...;\qquad b\qquad : ...; ...; ...; \\ ... :: e\ ; ...; 1+b+c : ...; ...; ...; \end{array} \chi, \psi, \omega\right] \tag{24}$$

Again, when we set $\lambda = 1 + b + c$ in (24), along with ([7], pp. 35, (10))

$$F_2[a, b, b'; a, c'; x, y] = (1-x)^{-b} F_1\left[b', b, a-b; c'; \frac{y}{1-x}, y\right] \tag{25}$$

Moreover, the power series identity ([4], 1.6(2)).

$$\sum_{m,n=0}^{\infty} f(m+n)\frac{x^m}{m!}\frac{y^n}{n!} = \sum_{N=0}^{\infty} f(N)\frac{(x+y)^N}{n!} \tag{26}$$

We obtain generating function in the form

$$\sum_{n=0}^{\infty} \frac{(1+c)_n}{n!} A_n^{(b,-c-n)}(x) A_n^{(d,e)}(y) t^n = (1+yt)^{-b}(1-xyt)^{-c-1} \times F_1\left[d, b, 1+c; e; -\frac{1}{1+yt}, \frac{xt}{1-xyt}\right] \tag{27}$$

where F_1 is the Appell's function defined by (7).

In view of the definition (2) and (4), which in conjunction with (18), we obtain some more bilinear generating function for $A_n^{(b,c)}(x)$ as given below:

$$\sum_{n=0}^{\infty} \frac{(c+m)_n(1+e)_n}{(\lambda)_n\ n!} A_{m+n}^{(b,c)}(x) A_n^{(-d-n,-e-n)}(y) t^n = (1+x)^m \left(\frac{x}{1+x}\right)^{b-c} \times F_G\left[c+m,\ c+m,\ c+m,\ c-b,\ 1+e,\ d-e; c,\ \lambda,\ \lambda; -\frac{1}{x},\ (1+x)(1+y)t, (1+x)t\right] \tag{28}$$

Alternatively, equivalently using (2) along with (5), we obtain

$$\sum_{n=0}^{\infty} \frac{(1+e)_n}{n!} A_{m+n}^{(b-n,c-n)}(x) A_n^{(d,-e-n)}(y) t^n = \frac{(b)_m}{(c)_m}(1+x)^{c+m-1} \times F_G\left[1-b,\ 1-b,\ 1-b,\ 1-c-m,\ 1+e,\ d; 1-b-m,\ 1-c,\ 1-c;\ \frac{x}{1+x},\ yt, -t\right] \tag{29}$$

where in (28) and (29) F_G are the Saran's function defined by (12).

Further, we obtain some more bilinear generating function by using the relation (2) along with (3) in an elegant form as

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(c+m)_n}{(1+d)_n\ n!} A_{m+n}^{(b,c)}(x) A_n^{(-d-n,e)}(y) t^n = (1+x)^m \left(\frac{x}{1+x}\right)^{b-c} \times F_E\left[c+m,\ c+m,\ c+m,\ c-b,\ \lambda,\ \lambda; c,\ e,\ 1+d; -\frac{1}{x}, -(1+x)t,\ (1+x)yt\right] \tag{30}$$

or, equivalently

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(1-c)_n}{(1+d)_n\ n!} A_{m+n}^{(b-n,c-n)}(x) A_n^{(-d-n,e)}(y) t^n = \frac{(b)_m}{(c)_m}(1+x)^{c+m-1} \times F_E\left[1-b,\ 1-b,\ 1-b,\ 1-c-m,\ \lambda,\ \lambda; 1-b-m,\ e,\ 1+d;\ \frac{x}{1+x}, -t,\ yt\right] \tag{31}$$

where in (30) and (31) F_E is the Saran's function defined by (11).

III. Bilateral Generating Functions

The polynomials $A_n^{(b,c)}(x)$ admits several bilateral generating functions. Firstly, we introduce three bilateral generating function by using the relation (2), each of which involved the Gaussian hypergeometric ${}_2F_1$ function in terms of the Lauricella's triple hypergeometric series F_4, F_8 and F_7 (which, in the notation used by Saran's [8], are F_E, F_G, F_S respectively) are as follows:

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(\mu)_n}{(1+b)_n\ n!} A_n^{(-b-n,c)}(x)\ {}_2F_1\left[\lambda+n,\ \beta;\ \gamma;\ y\right] t^n = F_E\left[\lambda,\ \lambda,\ \lambda,\ \beta,\ \mu,\ \mu;\ \gamma,\ 1+b,\ c;\ y,\ xt, -t\right] \tag{32}$$

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(1+c)_n}{(\mu)_n\ n!} A_n^{(b,-c-n)}(x)\ {}_2F_1\left[\lambda+n,\ \beta;\ \gamma;\ y\right] t^n = F_E\left[\lambda,\ \lambda,\ \lambda,\ \beta,\ 1+c,\ b;\ \gamma,\ \mu,\ \mu;\ y,\ xt, -t\right] \tag{33}$$

and

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n(1+c)_n}{(\mu)_n\ n!} A_n^{(b,-c-n)}(x)\ {}_2F_1\left[\beta,\ \gamma;\ \mu+n;\ y\right] t^n = F_S\left[\beta,\ \lambda,\ \lambda,\ \gamma,\ 1+c,\ b;\ \mu,\ \mu,\ \mu;\ y,\ xt, -t\right] \tag{34}$$

Now, by using the definition (2) along with Laguerre polynomials (see [3], pp. 200, (1)), yields the generating function in the form

$$\sum_{n=0}^{\infty} \binom{m+n}{n} \frac{L_{m+n}^{(\alpha)}(x) A_n^{(-b-n,c)}(y)}{(1+b)_n} t^n = \binom{\alpha+m}{m} e^x \times\ \psi_2^{(3)}\left[\alpha+m+1;\ \alpha+1,\ c,\ 1+b; -x, -t,\ yt\right] \tag{35}$$

$$\sum_{n=0}^{\infty} \binom{m+n}{n} L_{m+n}^{(\alpha)}(x)\ A_n^{(b,c)}(y) t^n = \binom{\alpha+m}{m} e^x (1-yt)^{-\alpha-m-1} \times \psi_1\left[\alpha+m+1,\ b;\ c,\ 1+\alpha; \frac{t}{1-yt}, -\frac{x}{1-yt}\right] \tag{36}$$

Alternatively, equivalently using (5), we obtain

$$\sum_{n=0}^{\infty}\binom{m+n}{n}\frac{(1-c)_n}{(1-b)_n}L_{m+n}^{(\alpha)}(x)\,A_n^{(b-n,c-n)}(y)t^n$$

$$=\binom{\alpha+m}{m}e^x(1-t)^{-\alpha-m-1}$$

$$\times\psi_1\left[\alpha+m+1,1-c;1-b,1+\alpha;\frac{yt}{1-t},\frac{-x}{1-t}\right] \quad (37)$$

where, in (35) $\psi_2^{(3)}$ is the confluent form of Lauricella's function defined by (16), with $n=3$ and in (36) and (37) ψ_1 is the confluent hypergeometric function of two variables (see [4], pp. 59, (41)).

The generalized heat polynomials $P_{n,\nu}(x,u)$ defined by (Haimo [2], p.736, (2.1)).

$$P_{n,\nu}(x,u)=\sum_{k=0}^{n}2^{2k}\binom{n}{k}\frac{\Gamma(\nu+n+\frac{1}{2})}{\Gamma(\nu+n-k+\frac{1}{2})}x^{2n-2k}u^k \quad (38)$$

By reversing the order of summation, (38) can be written as

$$P_{n,\nu}(x,u)=(4u)^n\left(\nu+\frac{1}{2}\right)_n\sum_{k=0}^{n}\frac{(-n)_k}{(\nu+\frac{1}{2})_k\;k!}\left(\frac{-x^2}{4u}\right)^k$$

$$=(4u)^n\;n!\;L_n^{(\nu-\frac{1}{2})}\left(-\frac{x^2}{4u}\right) \quad (39)$$

Further, involving the relation (39) with (2) and (5), another form of generating function equivalent to (35), (36) and (37) are obtained,

$$\sum_{n=0}^{\infty}\frac{P_{m+n,\nu}(x,u)A_n^{(-b-n,\;c)}(y)}{(1+b)_n\;n!}t^n$$

$$=(4u)^m\left(\nu+\frac{1}{2}\right)_m exp\left(\frac{-x^2}{4u}\right)$$

$$\times\psi_2^{(3)}\left[\nu+m+\frac{1}{2};\nu+\frac{1}{2},c,1+b;\frac{x^2}{4u},-4ut,4uyt\right] \quad (40)$$

$$\sum_{n=0}^{\infty}P_{m+n,\nu}(x,u)A_n^{(b,c)}(y)\frac{t^n}{n!}$$

$$=(4u)^m\left(\nu+\frac{1}{2}\right)_m exp\left(\frac{-x^2}{4u}\right)(1-4uyt)^{-\nu-m-\frac{1}{2}}$$

$$\times\psi_1\left[\nu+m+\frac{1}{2},\;b;\;c,\;\nu+\frac{1}{2};\frac{4ut}{1-4uyt},\frac{x^2}{4u(1-4uyt)}\right] \quad (41)$$

$$\sum_{n=0}^{\infty}\frac{(1-c)_n}{(1-b)_n}P_{m+n,\nu}(x,u)A_n^{(b-n,c-n)}(y)\frac{t^n}{n!}$$

$$=(4u)^m\left(\nu+\frac{1}{2}\right)_m exp\left(\frac{-x^2}{4u}\right)(1-4ut)^{-\nu-m-\frac{1}{2}}$$

$$\times\psi_1\left[\nu+m+\frac{1}{2},\;1-c;1-b,\;\nu+\frac{1}{2}\right.$$

$$\left.;\frac{4uyt}{1-4ut},\frac{x^2}{4u(1-4ut)}\right] \quad (42)$$

Again using the definition (2), along with Jacobi polynomials (see [3], (1), pp. 254), which in conjunction with (3) yields the generating relations

$$\sum_{n=0}^{\infty}\frac{(1+\alpha+\beta)_n}{(1+b)_n}A_n^{(-b-n,c)}(x)P_n^{(\alpha,\beta)}(y)t^n$$

$$=\left(\frac{1+y}{2}\right)^{-\alpha-\beta-1}F_c^{(3)}\left[1+\alpha+\beta,\;1+\alpha;\right.$$

$$\left.1+b,\;c,\;1+\alpha;-\frac{2t}{1+y},\frac{y-1}{1+y},\frac{2xt}{1+y}\right] \quad (43)$$

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n}{(1+b)_n}A_n^{(-b-n,c)}(x)P_n^{(\alpha,\beta-n)}(y)t^n$$

$$=\left(\frac{1+y}{2}\right)^{-\alpha-\beta-1}F_E\left[1+\alpha,\;1+\alpha,\;1+\alpha,\right.$$

$$\left.1+\alpha+\beta,\;\lambda,\;\lambda;\;1+\alpha,\;1+b,\;c\;;\frac{y-1}{y+1},xt,-t\right] \quad (44)$$

$$\sum_{n=0}^{\infty}\frac{(1+c)_n}{(\lambda)_n}A_n^{(b,-c-n)}(x)P_n^{(\alpha,\beta-n)}(y)t^n$$

$$=\left(\frac{1+y}{2}\right)^{-\alpha-\beta-1}F_G\left[1+\alpha,\;1+\alpha,\;1+\alpha,;\right.$$

$$\left.1+\alpha+\beta,\;1+c,\;b1+\alpha,\;\lambda,\;\lambda;\frac{y-1}{y+1},xt,-t\right] \quad (45)$$

Next, some more generating functions are expressed by using (2), which in conjunction with Lauricella's triple hypergeometric function $F_C^{(S)}$ and $F_D^{(S)}$.

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n(\mu)_n}{(1+b)_n\;n!}A_n^{(-b-n,c)}(x)$$

$$\times F_C^{(S)}\left[\lambda+n,\;\mu+n\;;\rho_1,...,\rho_s\;;z_1,...,z_s\right]t^n$$

$$=F_C^{(S+2)}\left[\lambda,\mu;\rho_1,...,\rho_s,c,1+b;z_1,...,z_s,-t,xt\right] \quad (46)$$

and

$$\sum_{n=0}^{\infty}\frac{(\lambda)_n(1+c)_n}{(\mu)_n\;n!}A_n^{(b,-c-n)}(x)$$

$$\times F_D^{(S)}\left[\lambda+n\;,\nu_1,....,\nu_s\;;\mu+n\;;z_1,....,z_s\right]t^n$$

$$=F_D^{(S+2)}\left[\lambda,\nu_1,...,\nu_s,b,1+c;\mu;z_1,...,z_s,-t,xt\right] \quad (47)$$

where in (46) and (47) $F_C^{(S+2)}$ and $F_D^{(S+2)}$ denote the Lauricella's triple hypergeometric function defined by (14) and (15) with $n=s+2$.

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