

# Backstepping Sliding Mode Controller Coupled to Adaptive Sliding Mode Observer for Interconnected Fractional Nonlinear System

D. Elleuch and T. Damak

**Abstract**—Performance control law is studied for an interconnected fractional nonlinear system. Applying a backstepping algorithm, a backstepping sliding mode controller (BSMC) is developed for fractional nonlinear system. To improve control law performance, BSMC is coupled to an adaptive sliding mode observer have a filtered error as a sliding surface. The both architecture performance is studied throughout the inverted pendulum mounted on a cart. Simulation result show that the BSMC coupled to an adaptive sliding mode observer have stable control law and eligible control amplitude than the BSMC.

**Keywords**—Backstepping sliding mode controller, interconnected fractional nonlinear system, adaptive sliding mode observer.

## I. INTRODUCTION

NONLINEAR systems are more complex. To modeling new system a complicate nonlinear term appears. A fractional nonlinear system is one of complicate nonlinear forms which both state and parameter are nonlinear modeling like the robot, the pendulum [1],[2]. However, to solve tracking trajectory and stabilization problem, many controller are developed to diver's nonlinear form. The classical nonlinear controllers are not usually performance as sliding mode controller and backstepping controller.

A sliding mode controller is mostly studied in literature [3],[4],[5]. It is known as one as robust nonlinear controller to reject uncertainty and perturbation. But the major sliding mode problem is the chattering phenomena. To eliminate this problem many solutions are presented in literature like high order sliding mode [6] and the combination between sliding mode and backstepping controller, the new controller is named backstepping sliding mode controller (BSMC) [7],[8]. The BSMC controller is extended to an adaptive form which is developed for an uncertain nonlinear system transformed into parametric semi strict feedback system [9] and electrical machine [10].

The backstepping control solves both the problem of stabilization and tracking trajectory of nonlinear systems [9].

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In [11], the author proves that the backstepping control does not guarantee the tracking trajectory asymptotic convergence in the presence of measurement noise on the state and the output and input constraints.

The adaptive backstepping sliding mode controller (ABSMC) ensures robustness to tracking trajectory and feasibility to construct a Lyapunov function.

The ABSMC is compared to sliding mode controller and to adaptive backstepping controller. The authors shown that the ABSMC is more robust to tracking trajectory and the chattering phenomena are rejected [12],[13],[14],[17]. The ABSMC study is limited to an uncertain nonlinear system transformed into a parametric semi strict feedback

In [15], the authors developed a direct adaptive control for a triangular nonlinear system. The studied controller showed robustness to tracking trajectory. Using the same assumption as [15], the BSMC developed to an interconnected fractional nonlinear system is studied.

In [16], the authors present an adaptive sliding mode observer having a filtered error designed to an interconnected fractional nonlinear system which shows a good performance to estimate both unmeasured state and unknown parameter.

To improve BSMC robustness, it is coupled to an adaptive sliding mode observer having a filtered error as a sliding surface. The combination stability is studied throughout the Lyapunov function. Both BSMC and the BSMC coupled to adaptive sliding mode observer having a filtered error as a sliding surface performance are tested throughout the inverted pendulum mounted on a cart.

## II. SYSTEM CLASS

Let nonlinear system

$$\ddot{x} = f(x, \dot{x}) + g(x, u) \quad (1)$$

The system (1) is transformed into  $m$  interconnected fractional nonlinear system having the following form:

$$S_j = \begin{cases} \dot{x}_{1j} = x_{2j} \\ \dot{x}_{2j} = \frac{f_j(x) + g_j(x)u}{\beta(x)} \\ y_j = x_{1j} \end{cases} \quad (2)$$

where:  $j = 1 \dots m$

For each subsystem:  $y_j = x_{1j}$  : subsystem output ,

$y = [y_1, y_2, \dots, y_m] = [x_{11}, x_{12}, \dots, x_{1m}]$  : system output,  $x = [x_{11}, x_{21}, x_{12}, x_{22}, \dots, x_{2m}] \in R^n$  : state vector and  $\theta \in R^p$  : unknown parameter vector

The nonlinearity functions  $f_j(x)$ ,  $g_j(x)$  and  $\beta$  are expressed such as:

$$f_j(x) = f_{0j}(x) + \theta^T W_{fj}(x)$$

$$g_j(x) = g_{0j}(x) + \theta^T W_{gj}(x)$$

$$\beta(x) = \beta_0(x) + \theta^T W_\beta(x)$$

With  $x \in R^n, \theta \in R^p, f_j(x), g_j(x)$  and  $\beta(x) \in C^1,$

$W_{fj}(x), W_{gj}(x)$  and  $W_\beta(x) \in R^p$

The function  $W_{fj}(x, u)$  and  $W_\beta(x, u) \in R^p$  must satisfy the following assumption

*Assumption:*

1.  $W_\beta(x) < W_{fj}(x)$
2.  $W_{fj}(x) < \varpi$ ; with  $\varpi \in R^n$
3.  $W_{fj}(x)$  and  $W_\beta(x)$  have the same sign
4.  $W_{gj}(x)$  is invertible

### III. BACKSTEPPING SLIDING MODE CONTROLLER (BSMC)

If The BSMC has the same form as a sliding mode control law as follow:

$$u = l + u_{disc}$$

$l$ : nominal control

$u_{disc}$  : discontinues control

Due simplicity to construct a lyapunov function, the backstepping algorithm is applied to construct a nominal control  $l$ . Using the interconnected fractional nonlinear system (2), a BSMC is developed to nonlinear system (1).

The BSMC robustness is validated throughout an inverted pendulum mounted on car system. The simulation result is presented in next section.

*Theorem:*

The backstepping sliding mode controller is robust to tracking trajectory and to estimate unknown parameter if the control law satisfies they condition:

$$u_n = \left( g_{0j}(x) + \hat{\theta}^T W_{gj}(x) \right)^{-1} \left[ \left( \beta_0(x) + \hat{\theta}^T W_\beta(x) \right) (-\ddot{y}_r + c_1(-c_1 z_1 + z_2) + z_1 + c_2 z_2) - \left( f_{0j}(x) + \hat{\theta}^T W_{fj}(x) \right) \right]$$

$$\mu = -kz_1$$

With

$$c_i > 0, i = 1, 2$$

$$z_1 = x_{1j} - y_r$$

$$z_2 = x_{2j} - \dot{y}_r + c_1 z_1$$

*Proof: annex 1*

To improve the BSMC performance, it is coupled to an adaptive sliding mode observer. Next, stability BSMC coupled to adaptive sliding mode observer is analyzed; the combination is developed to interconnected fractional

nonlinear system.

### IV. ADAPTIVE SLIDING MODE OBSERVER HAVING A FILTERED ERROR AS SLIDING SURFACE

Use A sliding mode observer is widely studied. Many research have compare it to an others adaptive observer which proved that the sliding mode observer is one as robust observer. In [16], the authors present an adaptive sliding mode observer having a filtered error as sliding surface designed to interconnected fractional nonlinear systems and a theorem is announced.

**Theorem:** Adaptive sliding mode observer having a filtered error [16]

Let the nonlinear system (2), the assumption (3) is satisfied and the WCLF function [15] is verified, the adaptive sliding mode observer having a filtered error as a sliding surface have a good performance if:

Adaptive observer has the following form

$$S_j = \begin{cases} \hat{x}_{1j} = \hat{x}_{2j} - s_{1j} \\ \hat{x}_{2j} = \frac{f_j(\hat{x}) + g_j(\hat{x})u}{\beta(\hat{x})} - s_{2j} \\ y_j = x_{1j} \end{cases} \quad (4)$$

;  $j = 1 \dots m$

The observer is stable and converges if

$$\sum_{i=1}^{i=2} L_{ij} \geq \sum_{i=1}^{i=2} \gamma_{ij} \quad (5)$$

$$|\tau| > 1 \quad (6)$$

For each subsystem the adaptation law has the following form

$$\hat{\theta} = \Gamma^{-1} \left[ \text{sup} \left( W_{fj}(x_{1j}, e_{sj} + l_j) \right) W_\beta^T(\hat{x}) - W_\beta(x_{1j}, l_j) \text{sup} \left( W_{fj}^T(\hat{x}) \right) + \varpi \right] \hat{\theta}$$

with:

$\Gamma^{-1} \in R$  : Diagonal matrix

$\alpha$  : Constant vector

Let state variable error:

$$e_j = [x_{1j} - \hat{x}_{1j} \quad x_{2j} - \hat{x}_{2j}] \quad \text{where } j = 1 \dots m$$

Filtered error:  $e_{sj} = \dot{e}_{1j} + \tau e_{1j}$

Sliding surface:  $s_{ij} = -L_{ij} e_{sj} + \gamma_{ij} \text{sat}(e_{sj}), i = 1, 2; j = 1 \dots m$

Deriving the filtered error:

$$\dot{e}_{sj} = \frac{1}{\beta_\alpha(x, \hat{x})} [\beta(\hat{x}) f_j(x_{2j}) - \beta(x_{2j}) f_j(\hat{x})] + v_j$$

where  $\beta_\alpha(x_{1j}, \sigma + l_j, \hat{x}_{2j}) = \beta(x_{1j}, \sigma + l_j) \beta(\hat{x}_{2j})$  with  $\alpha(x)$  is  $\beta(\hat{x}_{2j})$

The derivative appears new variable defined by:  $v_j = \tau(s_{1j} + e_j) + s_{2j}$

Unmeasured state can be expressed as follow

$$x_{2j} = e_{sj} + l_j$$

Where  $l_j = \hat{x}_{2j} - [\tau \ 0]e_j; j = 1 \dots m$

For each subsystem, the state vector is:  $x_j = (x_{1j}, e_{sj} + l_j)$

V. BSMC COUPLED TO ADAPTIVE SLIDING MODE OBSERVER HAVING A FILTERED ERROR AS A SLIDING SURFACE

The BSMC depend to unmeasured state and unknown parameter, for that to improve BSMC performance, the BSMC is coupled to an adaptive sliding mode observer having a filtered error as a sliding surface. In literature, the authors shown that the separation principal isn't usually robust. In this part, a stability combination between BSMC and adaptive sliding mode observer having a filtered error as a sliding surface designed to interconnected fractional nonlinear system.

Theorem

The Combination between a BSMC and an adaptive sliding mode observer having a filtered error as a sliding surface is stable and robust if the adaptive sliding mode observer have the form (4) and satisfy the condition (5) and (6) besides:

BSMC law has the following expression:

$$u = (g_{0j}(x) + \hat{\theta}^T W_{gj}(x))^{-1} \left[ (\beta_0(x) + \hat{\theta}^T W_\beta(x)) (\ddot{y}_r - z_2(c_1 + c_2) - z_1(1 - c_1^2)) \right] - (f_{0j}(x) + \hat{\theta}^T W_{fj}(x)) + (\mu + kz_1) \text{sat}(z_1)$$

$$\mu = -k z_1$$

which:

$$z_1 = \hat{x}_{11} - y_r$$

$$z_2 = \hat{x}_{21} - \dot{y}_r + c_1 z_1$$

Let an adaptive sliding mode observer having a filtered error as a sliding surface (4) and satisfy the conditions (5) and (6)

The variable error  $z_1 = \hat{x}_{11} - y_r$  with  $y_r$  : reference

$$\dot{z}_1 = \dot{\hat{x}}_{11} - \dot{y}_r = \hat{x}_{21} + L_1 e_{s1} - \gamma_1 \text{sat}(e_{s1}) - \dot{y}_r$$

However, according to the observer stability

$$\text{If } t \rightarrow \infty \text{ then } L_1 e_{s1} - \gamma_1 \text{sat}(e_{s1}) \rightarrow 0$$

$$\dot{z}_1 = \hat{x}_{21} - \dot{y}_r + c_1 z_1 - c_1 z_1$$

$$\dot{z}_1 = -c_1 z_1 + z_2$$

$$\text{with } z_2 = \hat{x}_{21} - \dot{y}_r + c_1 z_1$$

Let  $V_{z_1}$  lyapunov function depending to  $z_1$

$$V_{z_1} = \frac{1}{2} z_1^2$$

The derivative lyapunov function is:

$$\dot{V}_{z_1} = z_1 \dot{z}_1 = z_1(-c_1 z_1 + z_2) = -c_1 z_1^2 + z_1 z_2$$

The  $z_2$  derivative is:

$$\dot{z}_2 = \dot{\hat{x}}_{2j} - \ddot{y}_r + c_1 \dot{z}_1$$

$$\dot{z}_2 = \frac{f_j(\hat{x}) + g_j(\hat{x})u}{\beta(\hat{x})} + L_2 e_{s2} - \gamma_2 \text{sat}(e_{s2}) - \ddot{y}_r + c_1(-c_1 z_1 + z_2)$$

$z_2)$

$$\text{then } t \rightarrow \infty \text{ } L_2 e_{s2} - \gamma_2 \text{sat}(e_{s2}) \rightarrow 0$$

$$\dot{z}_2 = \frac{f_j(\hat{x}) + g_j(\hat{x})u}{\beta(\hat{x})} - \ddot{y}_r + c_1(-c_1 z_1 + z_2) + z_1 - z_1 + c_2 z_2 - c_2 z_2$$

$$c_2 z_2 - c_2 z_2$$

$$\dot{z}_2 = -c_2 z_2 - z_1 + \left( \frac{f_j(\hat{x}) + g_j(\hat{x})u}{\beta(\hat{x})} - \ddot{y}_r + c_1(-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

Let  $V_{z_2}$  is lyapunov function for error variable depending to

$z_2$

$$V_{z_2} = \frac{1}{2} z_2^2$$

The derivative lyapunov function is

$$\dot{V}_{z_2} = \dot{z}_2 z_2$$

$$\dot{V}_{z_2} = -c_2 z_2^2 - z_2 z_1 + z_2 \left( \frac{f_j(\hat{x}) + g_j(\hat{x})u}{\beta(\hat{x})} - \ddot{y}_r + c_1(-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

System lyapunov function is

$$V = V_{z_1} + V_{z_2}$$

then

$$\dot{V} = \dot{V}_{z_1} + \dot{V}_{z_2}$$

$$\dot{V}_z = -c_1 z_1^2 - c_2 z_2^2 + z_2 \left( \frac{f_j(\hat{x}) + g_j(\hat{x})u}{\beta(\hat{x})} - \ddot{y}_r + c_1(-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

thus

$$\dot{V}_z =$$

$$-c_1 z_1^2 - c_2 z_2^2 + z_2 \left( \frac{f_{0j}(\hat{x}) + \hat{\theta}^T W_{fj}(\hat{x}) + (g_{0j}(\hat{x}) + \hat{\theta}^T W_{gj}(\hat{x}))u}{\beta_0(\hat{x}) + \hat{\theta}^T W_\beta(\hat{x})} - \ddot{y}_r + c_1(-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

$u$  has the following form

$$u = u_n + (\mu + kz_1) \text{sat}(z_1)$$

which

$u_n$ : Nominal control

$(\mu + kz_1) \text{sat}(z_1)$  : discontinues control

$$\dot{V}_z =$$

$$-c_1 z_1^2 - c_2 z_2^2 + z_2 \left( \frac{f_{0j}(\hat{x}) + \hat{\theta}^T W_{fj}(\hat{x}) + (g_{0j}(\hat{x}) + \hat{\theta}^T W_{gj}(\hat{x}))u_n}{\beta_0(\hat{x}) + \hat{\theta}^T W_\beta(\hat{x})} - \ddot{y}_r + c_1(-c_1 z_1 + z_2) + z_1 + c_2 z_2 + \frac{(g_{0j}(\hat{x}) + \hat{\theta}^T W_{gj}(\hat{x}))(\mu + kz_1) \text{sat}(z_1)}{\beta_0(\hat{x}) + \hat{\theta}^T W_\beta(\hat{x})} \right)$$

$$\dot{V}_z = -c_1 z_1^2 - c_2 z_2^2 \leq 0$$

with

$$c_1 > 0 \text{ and } c_2 > 0$$

$$u_n =$$

$$\left( g_{0j}(x) + \hat{\theta}^T W_{gj}(x) \right)^{-1} \left[ (\beta_0(x) + \hat{\theta}^T W_\beta(x)) (\ddot{y}_r - z_2(c_1 + c_2) - z_1(1 - c_1^2)) \right] - (f_{0j}(x) + \hat{\theta}^T W_{fj}(x))$$

$$\text{and } \mu = -k z_1$$

To test the BSMC and the BSMC coupled to an adaptive sliding mode observer having a filtered error as a sliding surface performance, these architecture are applied to an inverted pendulum mounted on a cart.

## VI. SIMULATION

$$S_1 \begin{cases} \dot{x}_{11} = x_{21} \\ \dot{x}_{21} = \frac{u - F_x x_{21} + j^{-1}(ml)^2 g \sin(x_{12}) \cos(x_{12}) - (ml)(j^{-1}F_\theta x_{22} \cos(x_{12}) - x_{22}^2 \sin(x_{12}))}{(m+M) - j^{-1}(ml)^2 \cos^2(x_{12})} \end{cases}$$

$$S_2 \begin{cases} \dot{x}_{12} = x_{22} \\ \dot{x}_{22} = \frac{j^{-1} \left[ \begin{array}{l} (m+M)F_\theta x_{22} - (ml)^2 x_{22}^2 \sin(x_{12}) \cos(x_{12}) + \\ (ml)F_x x_{21} \cos(x_{12}) + \\ (m+M)(ml)g \sin(x_{12}) - (ml) \cos(x_{12})u \end{array} \right]}{(m+M) - j^{-1}(ml)^2 \cos^2(x_{12})}$$

Adaptive Sliding Mode Observer Having a Filtered Error as a Sliding Surface:

$$S_1 \begin{cases} \dot{x}_{11} = x_{21} - s_{11} \\ \dot{x}_{21} = \frac{\begin{bmatrix} u - F_x x_{21} + \\ j^{-1}(ml)^2 g \sin(x_{12}) \cos(x_{12}) \\ - (ml) \left( j^{-1} F_\theta x_{22} \cos(x_{12}) - x_{22}^2 \sin(x_{12}) \right) \end{bmatrix}}{(m+M) - j^{-1}(ml)^2 \cos^2(x_{12})} - s_{21} \end{cases}$$

$$S_2 \begin{cases} \dot{x}_{12} = x_{22} - s_{12} \\ \dot{x}_{22} = \frac{\begin{bmatrix} (m+M)F_\theta x_{22} - \\ (ml)^2 x_{22}^2 \sin(x_{12}) \cos(x_{12}) + \\ (ml)F_x x_{21} \cos(x_{12}) + \\ (m+M)(ml)g \sin(x_{12}) \\ - (ml) \cos(x_{12})u \end{bmatrix}}{(m+M) - j^{-1}(ml)^2 \cos^2(x_{12})} - s_{22} \end{cases}$$

with

$$s_{11} = -L_{11} e_{s1} + \gamma_{11} \text{sat}(e_{s1}); s_{21} = -L_{21} e_{s1} + \gamma_{21} \text{sat}(e_{s1});$$

$$s_{12} = -L_{12} e_{s2} + \gamma_{12} \text{sat}(e_{s2}); s_{22} = -L_{22} e_{s2} + \gamma_{22} \text{sat}(e_{s2});$$

$$\alpha = [21000 \quad -80 \quad 1000 \quad 1600]^T$$

$$e_{11} = x_{11} - \hat{x}_{11}; e_{12} = x_{12} - \hat{x}_{12}; x_{22} = e_{s2} + l_2$$

$$\theta = \begin{bmatrix} \theta_1 = -(ml)^2 \\ \theta_2 = -ml \\ \theta_3 = (m+M) \\ \theta_4 = (ml)(m+M) \end{bmatrix}$$

The Adaptation Law

$$\begin{aligned} \dot{\hat{\theta}}_1 &= \Gamma_1^{-1} e_{s2} [j^{-2} \cos^2(\hat{x}_{12}) ((e_{s2} + l_2)^2 - \hat{x}_{22}^2) \hat{\theta}_1 - \\ & j^{-2} F_x \hat{x}_{21} \hat{\theta}_2 + (-j^{-2} F_\theta \cos^2(\hat{x}_{12}) + j^{-1} (e_{s2} + l_2)^2) \hat{\theta}_3 - \\ & j^{-2} g \cos^2(\hat{x}_{12}) \hat{\theta}_4 + 21000] \\ \dot{\hat{\theta}}_2 &= \Gamma_2^{-1} e_{s2} [-j^{-2} F_x \hat{x}_{21} \cos^2(\hat{x}_{12}) \hat{\theta}_1 + j^{-1} F_x \hat{x}_{21} \hat{\theta}_3 - 80] \\ \dot{\hat{\theta}}_3 &= \Gamma_3^{-1} e_{s2} [(j^{-2} F_\theta \cos^2(\hat{x}_{12}) (e_{s2} + l_2) - j^{-1} \hat{x}_{22}^2) \hat{\theta}_1 + \\ & j^{-1} F_x \hat{x}_{21} \hat{\theta}_2 + (-j^{-2} F_\theta \hat{x}_{22} \cos^2(\hat{x}_{12}) + j^{-1} F_\theta ((e_{s2} + l_2) - \\ & \hat{x}_{22})) \hat{\theta}_3 - j^{-1} g \hat{\theta}_4 + 1000] \\ \dot{\hat{\theta}}_4 &= \Gamma_4^{-1} e_{s2} [j^{-2} g \cos^2(\hat{x}_{12}) \hat{\theta}_1 + j^{-1} g \hat{\theta}_4 + 1600] \end{aligned}$$

Backstepping Sliding Mode Control

$$u_{BSMC} = \beta(\hat{x}) [\ddot{y}_r - z_1(1 - c_1^2) - z_2(c_1 + c_2)] - f_1(\hat{x}) + (\mu + kz_1) \text{sat}(z_1)$$

Parameters Simulation

$$m = 0.535 \text{ Kg}; M = 3.2 \text{ Kg}; l = 0.365 \text{ m}; g = 9.8 \text{ m/s}^2$$

$$j = 0.062 \text{ Kg m}^2; F_x = 6.2 \text{ Kg/s}; F_\theta = 0.009 \text{ Kg/m}^2;$$

$$\theta = \begin{bmatrix} \theta_1 = -(ml)^2 \\ \theta_2 = -ml \\ \theta_3 = (m+M) \\ \theta_4 = (ml)(m+M) \end{bmatrix}$$

$$\hat{\theta}_1(0) = -10^{-4}; \hat{\theta}_2(0) = 0; \hat{\theta}_3(0) = 2; \hat{\theta}_4(0) = 0.5;$$

$$L_{11} = 0.1; L_{21} = 1; L_{12} = 0.01; L_{22} = 1;$$

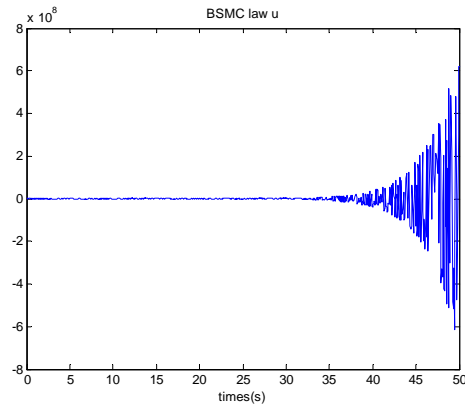
$$\gamma_{11} = \gamma_{12} = \gamma_{21} = 0.001; \gamma_{22} = -8; \Gamma_1^{-1} = 10^{-9};$$

$$\Gamma_2^{-1} = 8 * 10^{-4}; \Gamma_4^{-1} = 10^{-4}; \Gamma_2^{-1} = 2 * 10^{-5};$$

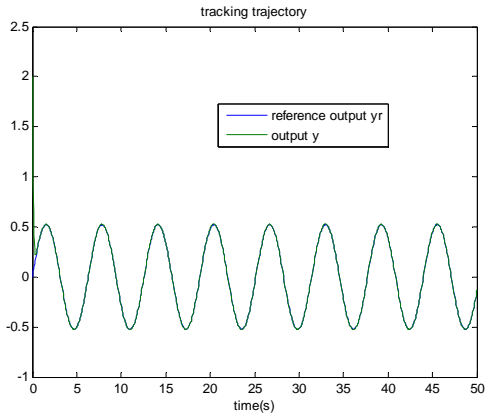
$$\tau = 150; y_r = \frac{\pi}{6} \sin(t)$$

$$c_1 = 6; c_2 = 50; \mu = 2; k = 4$$

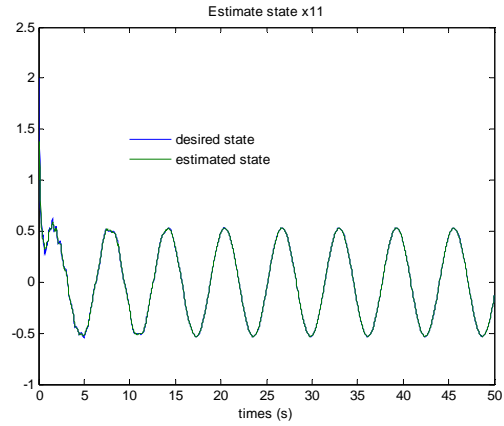
The simulation results show that the BSMC have a good performance to tracking trajectory but the control law amplitude increase and it is instable (Fig. 1). This inconvenient control law can be damaged the system.



(a)



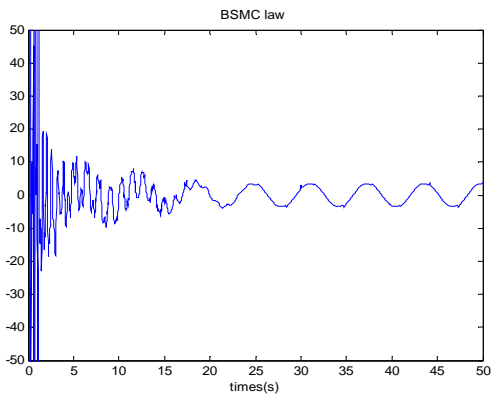
(b)



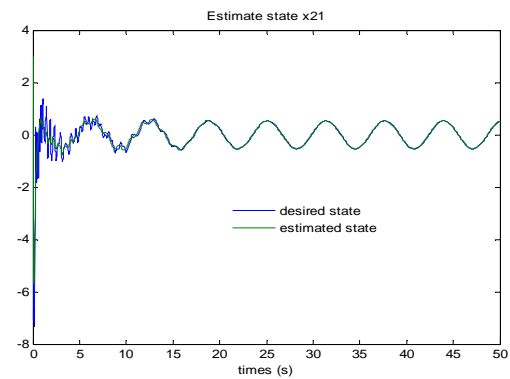
(c)

Fig. 1 Backstepping sliding mode control (BSMC) (a) Control law  
(b) Tracking trajectory

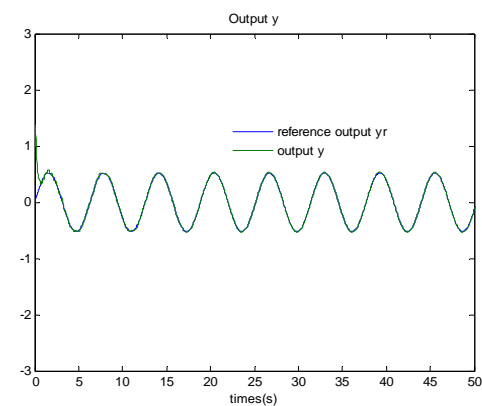
To improve the control law performance, the BSMC is coupled to an adaptive sliding mode observer having a filtered error as a sliding surface. In Fig. 2, the control law is stable and has eligible amplitude. Also the adaptive conserve his performance to estimate the state as in [16].



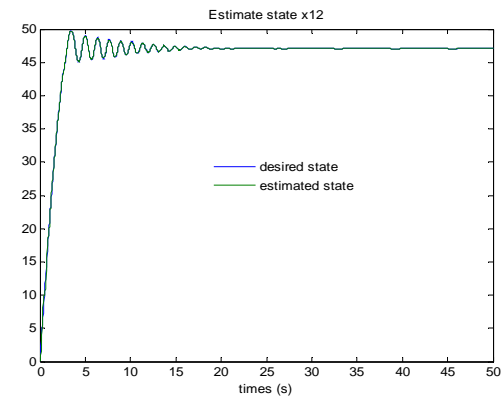
(a)



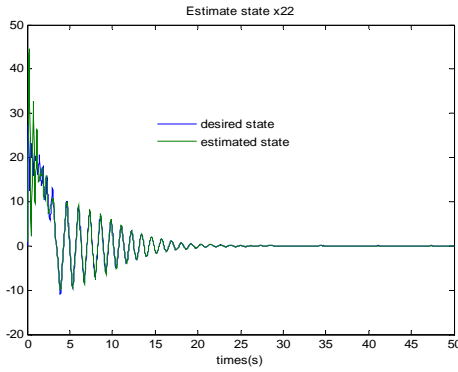
(d)



(b)



(e)



(f)

Fig. 2 BSMC coupled to an adaptive sliding mode observer having a filtered error as a sliding surface (a) Control law (b) Tracking trajectory (c) Estimate  $x_{11}$  (d) Estimate  $x_{21}$  (e) Estimate  $x_{12}$  (f) Estimate  $x_{22}$

## VII. CONCLUSION

In this paper, a backstepping sliding mode controller (BSMC) is developed for interconnected fractional nonlinear system. The system has both state and parameter nonlinear. To improve controller performance, BSMC is coupled to an adaptive sliding mode observer having a filtered error as a sliding surface. Applying the BSMC algorithm to the proposed adaptive observer, the combination stability is studied and the sufficient conditions are presented. To conclude, the BSMC and the combination performance are tested throughout the inverted pendulum mounted on a cart system. The simulation results prove that the BSMC coupled to adaptive observer and the BSMC have the same performance to tracking trajectory but the combination gives a best control law characteristic than the BSMC.

## VIII. ANNEXES

Let  $z_1 = x_{1j} - y_r$   
with  $y_r$  : reference output

$$\dot{z}_1 = \dot{x}_{1j} - \dot{y}_r = x_{2j} - \dot{y}_r$$

$$\dot{z}_1 = x_{2j} - \dot{y}_r + c_1 z_1 - c_1 z_1 = -c_1 z_1 + z_2$$

then  $z_2 = x_{2j} - \dot{y}_r + c_1 z_1$

Let  $V_{z_1}$  lyapunov function  $z_1$

$$V_{z_1} = \frac{1}{2} z_1^2$$

The derivative lyapunov function is:

$$\dot{V}_{z_1} = z_1 \dot{z}_1 = z_1 (-c_1 z_1 + z_2) = -c_1 z_1^2 + z_1 z_2$$

The  $z_2$  derivative is:

$$\dot{z}_2 = \dot{x}_{2j} - \dot{y}_r + c_1 \dot{z}_1$$

$$\dot{z}_2 = \frac{f_j(x) + g_j(x)u}{\beta(x)} - \dot{y}_r + c_1 (-c_1 z_1 + z_2)$$

$$\dot{z}_2 = \frac{f_j(x) + g_j(x)u}{\beta(x)} - \dot{y}_r + c_1 (-c_1 z_1 + z_2) + z_1 - z_1 + c_2 z_2 - c_2 z_2$$

$$\dot{z}_2 = -c_2 z_2 - z_1 + \left( \frac{f_j(x) + g_j(x)u}{\beta(x)} - \dot{y}_r + c_1 (-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

Let  $V_{z_2}$  is lyapunov function for error variable  $z_2$

$$V_{z_2} = \frac{1}{2} z_2^2$$

The derivative lyapunov function is

$$\dot{V}_{z_2} = \dot{z}_2 z_2$$

$$\dot{V}_{z_2} = -c_2 z_2^2 - z_2 z_1 + z_2 \left( \frac{f_j(x) + g_j(x)u}{\beta(x)} - \dot{y}_r + c_1 (-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

Let the lyapunov function is

$$V = V_{z_1} + V_{z_2}$$

$$\dot{V} = \dot{V}_{z_1} + \dot{V}_{z_2}$$

$$\dot{V}_z = -c_1 z_1^2 - c_2 z_2^2 + z_2 \left( \frac{f_j(x) + g_j(x)u}{\beta(x)} - \dot{y}_r + c_1 (-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

$$\text{Or } f_j(x) = f_{0j}(x) + \theta^T W_{fj}(x)$$

$$g_j(x) = g_{0j}(x) + \theta^T W_{gj}(x)$$

$$\beta(x) = \beta_0(x) + \theta^T W_{\beta}(x)$$

$$\dot{V}_z = -c_1 z_1^2 - c_2 z_2^2$$

$$z_2 \left( \frac{f_{0j}(x) + \theta^T W_{fj}(x) + (g_{0j}(x) + \theta^T W_{gj}(x))u}{\beta_0(x) + \theta^T W_{\beta}(x)} - \dot{y}_r + c_1 (-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

$u$  has the following form

$$u = l + (\gamma + k z_1) \text{sat}(z_1)$$

which

$l$ : nominal control

$(\gamma + k z_1) \text{sat}(z_1)$  : discontinues control

$$\dot{V}_z =$$

$$-c_1 z_1^2 - c_2 z_2^2 + z_2 \left( \frac{f_{0j}(x) + \theta^T W_{fj}(x) + (g_{0j}(x) + \theta^T W_{gj}(x))(u_n + (\mu + k z_1) \text{sat}(z_1))}{\beta_0(x) + \theta^T W_{\beta}(x)} - \dot{y}_r + c_1 (-c_1 z_1 + z_2) + z_1 + c_2 z_2 \right)$$

$$\dot{V}_z = -c_1 z_1^2 - c_2 z_2^2 \leq 0$$

with:

$$c_1 > 0 \text{ and } c_2 > 0$$

$$u_n = (g_{0j}(x) + \theta^T W_{gj}(x))^{-1} \left[ (\beta_0(x) + \theta^T W_{\beta}(x)) (-\dot{y}_r + c_1 (-c_1 z_1 + z_2) + z_1 + c_2 z_2) - (f_{0j}(x) + \theta^T W_{fj}(x)) \right]$$

$$\mu = -k z_1$$

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