

# Arbitrary Amplitude Ion-Acoustic Solitary Waves in Electron-Ion-Positron Plasma with Nonthermal Electrons

Basudev Ghosh, Sreyasi Banerjee

**Abstract**—Using pseudo potential method arbitrary amplitude ion-acoustic solitary waves have been theoretically studied in a collisionless plasma consisting of warm drifting positive ions, Boltzmann positrons and nonthermal electrons. Ion-acoustic solitary wave solutions have been obtained and the dependence of the solitary wave profile on different plasma parameters has been studied numerically. Lower and higher order compressive and rarefactive solitary waves are observed in presence of positrons, nonthermal electrons, ion drift velocity and finite ion temperature. Inclusion of higher order nonlinearity is shown to have significant correction to the solitary wave profile for the same values of plasma parameters.

**Keywords**—Ion-acoustic waves, Nonthermal electrons, Sagdeev potential, Solitary waves.

## I. INTRODUCTION

IN the last two decades there has been a great deal of interest in the study of nonlinear wave phenomena in electron-positron-ion (e-p-i) plasmas [1]-[6]. This is due to the fact that e-p-i plasmas occur in many astro physical environments such as active galactic nuclei [7], pulsar magnetosphere [8], polar regions of neutron stars [9], centre of our galaxy [10], the early universe [11], [12], and solar atmosphere [13]. The e-p-i plasmas have also been produced in some laboratory environments [14]-[16]. Inclusion of positrons is found to drastically modify the ion-acoustic wave supported by simple electron-ion (e-i) plasma. Popel et al [2] have reported decrease in ion-acoustic soliton amplitude in presence of positrons. The presence of non-Maxwellian electrons is common in space and astrophysical plasmas [17]. The solitary structures with density depression in the magnetosphere observed by Freja [18] and Viking [19] satellites have been explained by Cairns et al [20] by assuming a plasma with nonthermal electrons and cold ions. In fact the presence of the presence of nonthermal electrons in plasma gives rise to many interesting characteristics in nonlinear propagation of waves including the excitation of ion-acoustic solitons [10], [21]-[24] and double layers in plasma. Nonlinear ion-acoustic solitary waves in e-p-i plasma with nonthermal electrons have been considered by some authors [2], [21], [23] assuming ions to be cold. In practice ions have finite temperature and it can significantly modify the characteristics of nonlinear ion-acoustic structures [24]-[26]. The effect of

ion temperature on ion-acoustic solitary waves in e-p-i plasma with nonthermal electrons has been studied only by a few authors [27], [28]. Pakzad [27] has shown that in warm plasma solitons speed is more but solitons amplitude is less than that in cold plasma. Mamun [28] has shown that the effects of ion temperature change the minimum value of the nonthermal parameter as well as the Mach number for which compressive and rarefactive solitons co-exist and also change the width and amplitude of the solitary waves. The purpose of the present paper is to make a detailed analysis of the nonlinear propagation of small as well as large amplitude ion-acoustic solitary structures in e-p-i plasma with nonthermal electrons, warm ions and Boltzmann positrons.

## II. BASIC EQUATION

We consider an unmagnetized collisionless plasma consisting of warm positive ions, Boltzmann positrons and nonthermal electrons. The normalized basic equations governing ion dynamics for unidirectional propagation in such plasma in dimensionless form are the following [29]

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0 \quad (1)$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} + \frac{3\sigma_i}{(1-\chi)^2} n_i \frac{\partial n_i}{\partial x} = -\frac{\partial \phi}{\partial x} \quad (2)$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_p - n_i \quad (3)$$

In the above equations, the parameters  $n_i$  and  $v_i$  are respectively the concentration and velocity of the positive ions;  $n_e$  and  $n_p$  are respectively the concentration of electrons and positrons;  $\phi$  denotes the electrostatic potential, other parameters have their usual meaning; the velocities are normalized by  $\sqrt{\frac{k_B T_e}{m_i}}$ ; the densities are normalized by

equilibrium electron density  $n_{e0}$ ; all the length  $x$  by the electron Debye length  $\lambda_{De} = \sqrt{\frac{k_B T_e}{4e^2 n_{e0}}}$ ; time by  $\frac{\lambda_{De}}{C_s}$ ; ion

temperature by  $T_i$  by  $T_e$   $\left(\sigma_i = \frac{T_i}{T_e}\right)$  and the potential  $\phi$  by

$\frac{k_B T_e}{e}$ ; where  $k_B$  is the Boltzmann's constant. The nonthermal electron density is given by;

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$$n_e = (1 - \beta\phi + \beta\phi^2) \exp(\phi) \quad (4)$$

where  $\beta = \frac{4\delta}{1+3\delta}$  measures the deviation from the thermalized state and  $\delta$  determines the presence of nonthermal electrons inside the plasma. The density of Boltzmann positrons is given by,

$$n_p = \chi \exp(-\sigma_p \phi) \quad (5)$$

where  $\chi = \frac{n_{p0}}{n_{e0}}$  is the ratio between the unperturbed positron and electron number densities and  $\sigma_p = \frac{T_e}{T_p}$  is the ratio

between electron and positron temperatures. The equilibrium charge neutrality condition in normalized form is given by

$$\chi + n_{i0} = 1 \quad (6)$$

Using (4) and (5), (3) is re-written as

$$\frac{\partial^2 \phi}{\partial x^2} = (1 - \beta + \beta\phi^2) \exp(\phi) - \chi \exp(-\sigma_p \phi) - n_i \quad (7)$$

### III. SAGDEEV POTENTIAL

In order to derive the Sagdeev potential we use the following Galilean transformation

$$\eta = x - Vt \quad (8)$$

where  $V$  is the phase velocity of the solitary waves.

We also use the following boundary conditions:

As

$$x \rightarrow \infty, \quad v \rightarrow v_{i0}, \quad n \rightarrow n_{i0} = (1 - \chi), \quad \phi \rightarrow 0 \quad (9)$$

Now using (8) in (1) and (2) we get,

$$n_i = \frac{(1 - \chi)}{2\sqrt{3}\sigma_i} \left[ \sqrt{(A - B - 2\phi)} - \sqrt{(A + B - 2\phi)} \right] \quad (10)$$

where,

$$A = 3\sigma_i + (V - v_{i0})^2 \quad (11)$$

and

$$B = \sqrt{12\sigma_i} (V - v_{i0})^2 \quad (12)$$

Now from (7) we get,

$$-\frac{d^2 \phi}{d\eta^2} = \left( 1 + \phi + \frac{\phi^2}{2} + \frac{\phi^3}{6} - \beta\phi - \beta\frac{\phi^3}{2} - \beta\frac{\phi^5}{24} + \beta\phi^3 \right) - \left( \chi + \chi\sigma_p\phi - \chi\frac{\sigma_p^2\phi^2}{2} + \chi\frac{\sigma_p^3\phi^3}{2} \right) - \frac{(1 - \chi)}{2\sqrt{3}\sigma_i} \left[ \sqrt{(A - B - 2\phi)} - \sqrt{(A + B - 2\phi)} \right] \quad (13)$$

Equation (13) can be rewritten as:

$$\frac{d^2 \phi}{d\eta^2} = -\frac{\partial \psi_s}{\partial \phi} = -\psi_s'(\phi) \quad (14)$$

where,  $\psi_s$  is known as Sagdeev Potential.

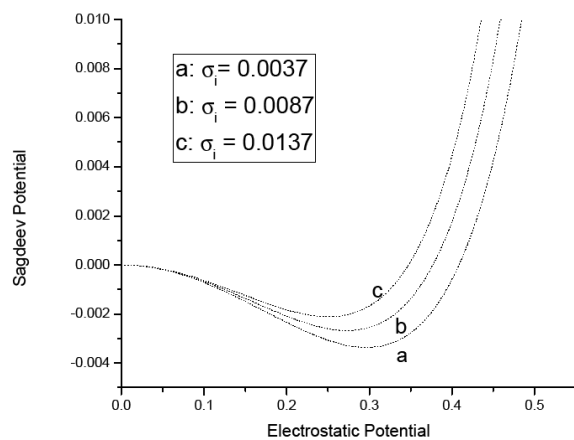


Fig. 1 Profile of Sagdeev potential by varying ion temperature ( $\sigma_i = 0.0037, 0.0087$  and  $0.0137$ ) where, nonthermal parameter  $\beta=0.15$ , positron temperature  $\sigma_p = 0.01$ , positron concentration  $\chi=0.01$ , phase velocity  $V=1.705$ , initial velocity  $v_{i0}=0.5$ .

In Fig. 1, we plot Sagdeev potential for different values of ion temperatures. It shows that the depth of Sagdeev potential decreases as ion temperature increases.

### IV. SOLITARY WAVES

For small amplitude case ( $|\phi| < 1$ ) Sagdeev potential may be expanded as follows [30]:

$$\frac{d^2 \phi}{d\eta^2} = C^{(1)}\phi + C^{(2)}\phi^2 + C^{(3)}\phi^3 \dots \quad (15)$$

where,

$$C^{(1)} = 1 - \beta + \chi\sigma_p + \frac{(1 - \chi)}{2\sqrt{3}\sigma_i} \left( \frac{1}{\sqrt{A+B}} - \frac{1}{\sqrt{A-B}} \right) \quad (16)$$

$$C^{(2)} = -\frac{1}{2} + \frac{\chi\sigma_p^2}{2} - \frac{(1 - \chi)}{4\sqrt{3}\sigma_i} \left\{ \frac{1}{(A+B)^{3/2}} + \frac{1}{(A-B)^{3/2}} \right\} \quad (17)$$

$$C^{(3)} = \frac{1}{6} - \frac{\beta}{2} + \frac{\chi\sigma_p^3}{6} + \frac{(1-\chi)}{4\sqrt{3}\sigma_i} \left\{ \frac{1}{(A+B)^{5/2}} - \frac{1}{(A-B)^{5/2}} \right\} \quad (18)$$

Lowest order solitary wave solution is

$$\phi^{(1)} = \frac{3C^{(1)}}{2C^{(2)}} \operatorname{sech}^2 \left( \frac{\eta}{\Delta} \right) \quad (19)$$

And higher order solitary wave solution is [30],

$$\phi^{(2)} = \frac{2\gamma^{(1)}}{\left\{ (\gamma^{(2)})^2 - 4\gamma^{(1)}\gamma^{(3)} \right\}^{1/2} (2 \cosh^2 \theta - 1) + \gamma^{(2)}} \quad (20)$$

where,

$$\Delta^{(1)} = \frac{2}{\sqrt{C^{(1)}}}, \quad \gamma^{(1)} = C^{(1)}, \quad \gamma^{(2)} = \frac{2C^{(2)}}{3}, \quad \gamma^{(3)} = -\frac{C^{(3)}}{2} \quad (21)$$

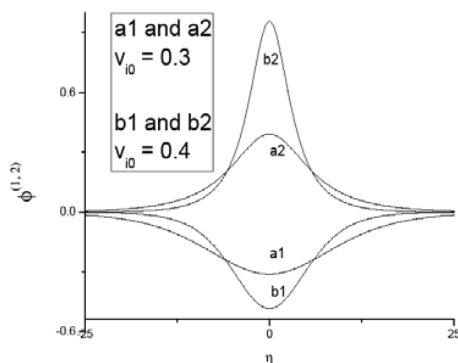


Fig. 2 Lower and higher order solitary structures by varying ion drift velocity ( $v_{i0}=0.3$  and  $0.4$ ) where nonthermal parameter  $\beta=0.025$ , ion temperature  $\sigma_i = 0.015$ , positron temperature  $\sigma_p=0.015$ , positron concentration  $\chi=0.028$ , phase velocity  $V=1.65$ .

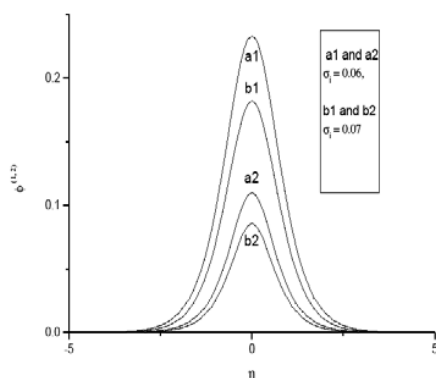


Fig. 3 Lower and higher order solitary structures by varying ion temperature ( $\sigma_i=0.06$  and  $0.07$ ), where nonthermal parameter  $\beta=0.055$ , positron temperature  $\sigma_p=0.015$ , positron concentration  $\chi=0.028$ , phase velocity  $V=1.45$ , initial velocity  $v_{i0}=0.5$ .

Figs. 2 and 3 show the soliton profiles for lower and higher order solitary wave solutions. a1 and b1 represents solitary structures for lowest order and a2 and b2 represents higher order solitary structures. From the profiles we find that the nature of solitary structures depend significantly on various plasma parameters like nonthermal parameter, ion temperature, positron concentration and ion drift velocity.

## V. RESULTS

1. The depth of Sagdeev potential decreases as ion temperature increases.
2. The amplitude of compressive solitons decreases significantly as we include higher order nonlinearity.
3. In case of rarefactive solitons inclusion of higher order nonlinearity may change the nature of the soliton.

## REFERENCES

- [1] P K Shukla, A A Mamun *J.Phys.* IV France 5 C6-43(1995).
- [2] S I Popel, S V Valmdimirov, P K Shukla *Phys. Plasma* 2 716 (1995).
- [3] Y N Nejoh. *Phys. Plasma* 3 1447 (1996).
- [4] M K Mishra, A. K. Arora and R.S Chhabra, *Phys Rev E*, 66, 46402 (2002).
- [5] S Ghosh, R Bharuthram *Astrophys Space Sci* 314 ,121 (2008).
- [6] H R Pakzad *Astrophys Space Sci* 323 345 (2009).
- [7] H R Miller, P J Witta *Active Galactic Nuclei*, P. 202 *Springer Berlin* (1978).
- [8] F C Michel *Rev Mod Phys.* 54 1 (1982).
- [9] F C Michel *Theory of Neutron star Magnetosphere*, Chicago University Press Chicago (1991).
- [10] M I Barns *Positron electron pairs in Astrophysics*, New York: American Institute of Physics (1983).
- [11] W K Misner, S Thorne, J A Wheeler *Gravitation*, P. 763 Freeman San Francisco (1973).
- [12] M J Rees, G W Gibbons, S W Hawking, S Siklaseds *The Early Universe*, Cambridge University Press, Cambridge (1983).
- [13] Tandberg, E Hansen, A G Emslie *The physics of solar Flares*, Cambridge University Press Cambridge 1988 P. 124.
- [14] C M Surko, M Leventhal, A Passner *Phys. Rev. Lett* 62 601 (1989).
- [15] H Bochmer, M Adams, N Rynn *Phys. of Plasmas* 2 4369 (1995).
- [16] E P Liang, S C Wilks, M Tabak *Phys. Rev Lett* 81 4887 (1998).
- [17] R A Cairns, A A Mamun, R Bingham, P K Shukla *Phys.Scripta* 763 80 (1996).
- [18] P O Dovner, A I Erikson, R Bostorm, B Holdack *Geophys. Res. Lett* 21 1827 (1994).
- [19] R Bostrom, G Gustafsson, B Holback, G Holmgren, H Koskinen, P Kintner *Phys. Rev Lett.* 61 82 (1988)
- [20] R A Cairns, R Bingham, R O Dendy, C M C Nairn, R Bostrom *Geophys. Res. Lett.* 22 2709 (1995)
- [21] M Salahuddin, H Saleem, M Saddiq *Phys. Rev E* 66 036407 (2002)
- [22] T S Gill, C Bedi, A S Bains *Phys. Scr* 81 055503 (2010).
- [23] T K Balaku, M A Helberg *Plasma Phys. and Control Fusion* 53 095007 (2011).
- [24] G Murtaza and M Salahuddin, *Plasma phys.* 24 451 (1981).
- [25] Yashvir, T N Bhatnagar and S R Sharma, *Plasma Phys. and Controlled fusion* 26 1303 (1984).
- [26] Y N Nejoh. *Phys. Plasma* 3 1447 (1996).
- [27] A A Mamun *Phys Rev E* 55 1852 (1997).
- [28] H R Pakzad *Astrophysics Space Sci* 323 345 (2009).
- [29] A. S Bains, N S Saini, T S Gill *Astro Phys. Sci* 343, 293 (2013).
- [30] B Ghosh, S Banerjee and S N Paul *Indian J. Pure and Appl. Phys.* 51 488 (2013).



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