International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:3, No:10, 2009

Approximate Method of Calculation of Inviscid Hypersonic Flow

F. Sokhanvar, A.B. Khoshnevis

***Abstract*—** In the present work steady inviscid hypersonic flows are calculated by approximate Method. Maslens' inverse method is the chosen approximate method. For the inverse problem, parabolic shock shape is chosen for the two-dimensional flow, and the body shape and flow field are calculated using Maslen's method. For the axisymmetric inverse problem paraboloidal shock is chosen and the surface distribution of pressure is obtained.

***Keywords*—** Hypersonic flow, Inverse problem method

I. INTRODUCTION

The flow field between the shock wave and the body shape is defined as the shock layer, and for hypersonic speeds this shock layer is quite thin. Analysis of this shock layer is called *"thin shock layer theory".* This can create some physical complication, such as combining of the shock wave itself with a thick, viscous boundary layer growing from body surface, a problem that can become important at low Reynolds number. At high Reynolds number where the shock layer becomes essentially inviscid, thin shock layer approximation can be used advantageously. By thin shock layer theory, what is meant is, determination of the shock shape, for the given body shape, and the flow field between them. Maslen's method is chosen here because of its simplicity, and also because of its frequent application - even today for approximate analysis of the hypersonic inviscid shock layer. This method is used for the determination of the flow field over blunt body as well as on bodies having pointed nose - and is a bridge between classical methods like Newtonian, Modified Newtonian, Shock expansion and the more modem computational methods like Time marching method or methods of Characteristics.

Hypersonic flow is a topic of current interest. Many countries are involved in the design of hypersonic vehicles. Hence understanding of hypersonic flow is important. It is interesting that in the hypersonic limit ($M \longrightarrow \infty$) certain simplifications of the flow are possible resulting in simple techniques for the calculation of the load, consisting of lift and drag. Coefficients of pressure lift and drag become independent in the above mentioned limit and as seen in the Fig. 1 are accurately predicted by the well known Newtonian law. Newtonian and modified Newtonian methods are important for the estimation of load in the preliminary stages of the design of the hypersonic vehicle. The other techniques are the Tangent wedge, Tangent Cone and Shock expansion methods. It's to be pointed out that the Shock expansion method predicts surface value of coefficient of pressure more accurately in the hypersonic regime than in supersonic regime.

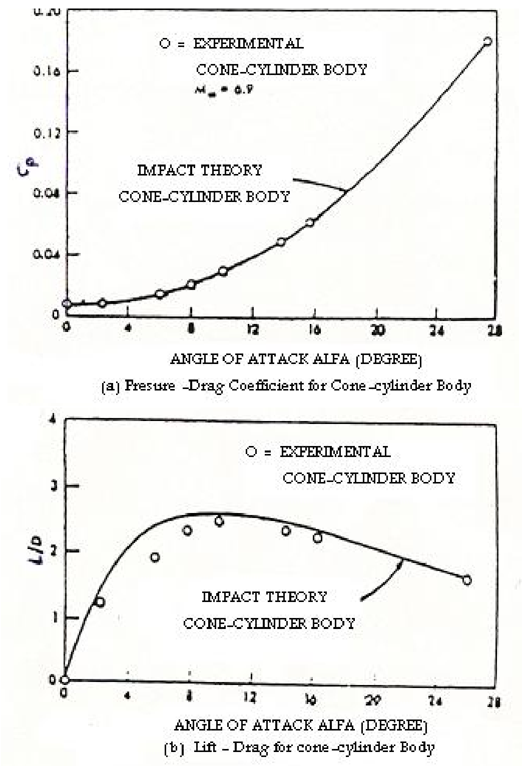

Fig. 1 CD and L/D for cone-cylinder Based on Newtonian Theory, Mach No. 6.9

Unfortunately none of the above methods can predict the shock shape, which the body generates in the hypersonic flow. Hypersonic shocks are strong even for a slender body, and are in general highly curved making the flow rotational behind the shock. Also it is important that the governing equations are nonlinear even for a slender body or thin airfoil in the hypersonic flow. This is entirely unlike where for a thin airfoil or slender body, shock waves could be replaced by Mach lines, and the flow is governed by the linear small perturbation equation.

F. Sokhnvar. Graduated from Sahand University of Technology, M.Sc. Student, (E-mail: fa_sokhnvar@yahoo.com).
A.B. Khoshnevis, Assistant Prof. of Mechanical Engineering Dept., Sabzevar Tarbiyat Moallem University, Sabzevar, Iran (off. Tel: +98 571 4003577; TelFax: +98 571 4411160; E-mail: khoshnevis@sttu.ac.ir)

The question may arise is finding the shock shape? To answer this question the following important fact about the hypersonic flight is to be mentioned. In low speed or hypersonic speed the aerospace vehicle or the aircraft can be conveniently divided as consisting of the wing, the lift producing unit, the fuselage for the providing the space or volume for payload, and the power plant for thrust. The performance of the above components can be evaluated independently and then combined. The total force produced by the combination will not be sum of the above individual units, since there are the forces of interaction. There are generally small, and can be taken into account. The above procedure cannot be applied for the hypersonic aircraft, since hypersonic flow is governed by nonlinear equation.

It is clear that the information regarding shock shape is necessary to locate the remaining part of engine appropriately. Also wing geometry and its location depend on the shock shape produced by the fuselage. So the determination of the shock shape becomes important.

II. Literature Survey

In the present work the method chosen for the determination of the shock is the Maslen's method [1] and Anderson [2]. This method is approximate but could be extended to the general case of body producing asymmetric shock [3]-[4]. Chemical reaction effects, which are important in the hypersonic flow, could also be accommodated in this method. Maslen's method is basically an inverse method i.e. calculate the body shape for the assumed shock critically depends on the fact that hypersonic shock layers are thin. In the present work for the case of parabolic and paraboloidal shocks, the body shapes and the flow fields are determined. As will be seen, contrary to what is found in the literature, Maslen [1] and Anderson [2] it appears that Maslen's methods is not suitable for the study of hypersonic flow past blunt bodies. As Milton Van Dyke points out in [5], hypersonic blunt body problem proved to be notoriously difficult, and the several attempts made to solve it prior to 1960 are given in [6]. However as Anderson points out, Moretti and Abbett's paper completely changed the state of art [7].

III. Description of Maslen's Method

As mentioned earlier Maslen's method is an inverse method, that is, calculation starts with assumed shock shape, and the flow field and the corresponding body shape are determined. In the following, the method is explained for the case of two-dimensional flow.

The essential fact on which Maslen's method is based is that the pressure behind the shock is more than the pressure on the surface of the body due to the curvature of the streamlines. Curvature of the streamlines generates centrifugal force field and in the Fig. 2, the pressure behind shock at the point 1 should be more than pressure on the surface of the body at the point 3. However this argument fails on the stagnation streamline oo' and hence must fail in the neighborhood of stagnation streamline, due to the continuity of the flow. As can be seen, on the stagnation streamline the pressure behind the shock P_o is less than $P_{o'}$, the pressure on the body.

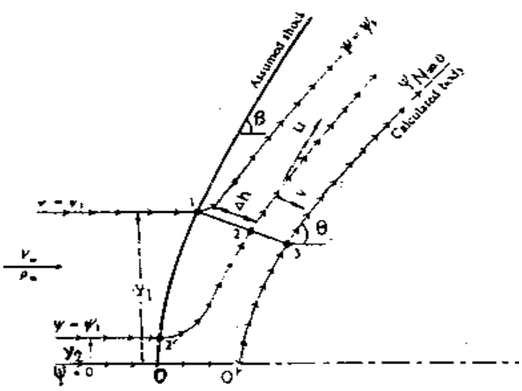

Fig. 2 2-Dimensional Geometry of Flow

Now the assumptions made by Maslen will be examined. In the shock layer, the velocity component normal to the shock is assumed to be very small compared to the tangential velocity component and is neglected. That, this is the case in the hypersonic limit, can be seen follows. The order of magnitude of the tangential and normal velocity components in the shock layer are given by.

$$u = 0(V_\infty Cos\beta) \quad (1)$$

$$v = \left(V_\infty Sin\left(\frac{\gamma - 1}{\gamma + 1} \right) \right) \quad (2)$$

In the hypersonic regime shock lies close to the body, and hence, as seen Fig. 2 Shock angle is nearly same as the angle made by the tangent to the body with the x-axis, at the corresponding point. For points away from the stagnation point is less, and hence the shock angle at the corresponding point on the shock is also less. Hence sin β becomes small. Then v << u for points far away from stagnation point, is justified. It is interesting that when high temperature effects like vibrational relaxation and chemical reaction, are taken into account $\gamma \rightarrow 1$, the assumption that v << u is even better. But this assumption that v << u breaks down on the stagnation streamline, and is not correct in the neighborhood of the stagnation streamline. In other words, the assumption that v << u is not uniformly valid in the entire flow field, however high M_∞ is.

The other major fact to be noted is that, in Maslen's method the centrifugal equation $\frac{dp}{dn} = \frac{\rho V^2}{R}$ is integrated along the normal to the shock and not along normal to the streamline. This is correct in the hypersonic limit, but again not uniformly. This assumption breakdowns in the stagnation streamline and its neighborhood. New Maslen's method will be explained in detail. The other assumption made in the method will be examined. In Fig. 2 O1 is the assumed shock shape and O' 3 is the body shape to be found , 1-3 is the normal to the shock and 1 is just behind the shock and 3 is on the body surface. 1-N (in the Fig. N=2) is divided into number of intervals N .1-2, 2-3 aren't equispace intervals but are such that ψ_2 -ψ_1 = ψ_3-ψ_2 = $\Delta\psi$ where ψ is the stream function. OO'3 is stagnation streamline on the body surface. It may be noted that:

$$\psi_1 = \rho_\infty V_\infty y_1 \tag{3}$$

And ψ_n = the stream function on the body, is zero.

STEP 1:
$\Delta\psi$ is calculated: as,

$$\Delta\psi = \frac{\psi_1}{N} = \frac{\rho_\infty V_\infty y_1}{N} \tag{4}$$

At point 1 which is behind the shock flow quantities are known from the following oblique shock relations.

$$\frac{P_1}{P_\infty} = 1 + \frac{2\gamma}{\gamma+1}(M_\infty^2 Sin^2\beta - 1) \tag{5}$$

$$\frac{\rho_1}{\rho_\infty} = 1 + \frac{(\gamma+1)M_\infty^2 Sin^2\beta}{(\gamma-1)M_\infty^2 Sin^2\beta + 2} \tag{6}$$

$$\frac{T_1}{T_\infty} = \frac{(P_1/P_\infty)}{(\rho_1/\rho_\infty)} \tag{7}$$

$$u_1 = V_\infty . Cos\beta \tag{8}$$

STEP 2:
Pressure at point 2 is calculated as follows. The momentum equation normal to a streamline is,

$$\frac{dp}{dn} = \frac{\rho V^2}{R} \tag{9}$$

And

$$\frac{d\psi}{N} = \rho V \tag{10}$$

From (9) & (10) we find that

$$dp = \frac{V d\psi}{R} \tag{11}$$

As mentioned earlier V is approximated by u.

Maslen further assumes that R the radius of curvature of any streamlines lying between I and 3 to be constant and equal to the radius of curvature of the shock at 1, which is intuitively obvious in the hypersonic limit. Hence equation (11) reduces to,

$$dp = \frac{u d\psi}{R_s} \tag{12}$$

A further assumption made by Maslen's for integration of equation along the normal 1-3 is that u=u_s =u_1, where u_s is tangential velocity behind the shock. This is difficult to justify, but it simplifies the calculation. Then (12) can be integrated as:

$$p - p_1 = \frac{u_1}{R_s}(\psi - \psi_1) \tag{13}$$

It is to be mentioned that only for getting pressure in (13) u is assumed to be constant equal u_s. Subsequently u is treated as a variable and is calculated. In (13) making ψ =ψ_2 and p=p_2, results in:

$$p_2 = p_1 + \frac{u_1}{R_s}(\psi_2 - \psi_1) \tag{14}$$

From the assumed equation of the shock, curvature of the shock at the point 1 is found. Hence p_2 is known.

STEP 3:
As is clear from Fig. 2, 2-2' being streamline $\psi_2' = \psi_2$, ψ_2 is known. From Fig. 2 we find that

$$y_2 = \frac{\psi_2}{\rho_\infty V_\infty} \tag{15}$$

Hence the location of the intersection point of streamline 2-2' and shock is known. Using oblique shock theory, the entropy $S_{2'}$ is found. 2-2' being streamline S_2=S_2'.

$$\Delta s = s_2' - s_\infty = s_2 - s_\infty \tag{16}$$

From the first and second laws of thermodynamics we find that [8],

$$\Delta s = C_p \ln\left(\frac{T_2}{T_\infty}\right) - R\left(\frac{p_2}{p_\infty}\right) \tag{17}$$

From the known values of Δs from (17), and p_2 from (14), T_2 is found from the following thermodynamic relation. That is,

$$\frac{T_2}{T_\infty} = \exp\left\{\frac{\Delta s R \ln(P_2/p_\infty)}{C_p}\right\} \tag{18}$$

Then the density ρ_2 is found from the equation of state as,

$$\rho_2 = \frac{p_2}{RT_2} \tag{19}$$

Since the total enthalpy is the same for the entire flow field, finally u_2 is determined from

$$C_p T_2 + \frac{u_2^2}{2} = C_p T_\infty + \frac{V_2^2}{2} \tag{20}$$

At this stage all the flow variables are known at 2, except its coordinates, which will be found in the next step.

International Journal of Mechanical, Industrial and Aerospace Sciences
ISSN: 2517-9950
Vol:3, No:10, 2009

STEP 4:
From the continuity equation it follows that:

$$\Delta\psi = \frac{\rho_1 u_1 + \rho_2 u_2}{2}\Delta n \qquad \text{(Fig. 2)}$$

Hence,

$$\Delta n = \frac{2\times(\psi_1 - \psi_2)}{(\rho_1 u_1 + \rho_2 u_2)} \tag{21}$$

The only unknown Δn is found at this stage. Δn Being normal to the shock shape, and since the shock shape is known (assumed). x_2 and y_2 can be found from following equation:

$$mp_1 = \frac{-1}{y_1} \tag{22}$$

$$x_2 = x_1 - \frac{\Delta n}{\sqrt{1+mp'^2_1}} \tag{23}$$

$$y_2 = y_1 - \frac{mp'_1\,\Delta n}{\sqrt{1+mp'_1}} \tag{24}$$

Having found the flow quantities at 2, the method described above is applied to calculate flow variables at the next grid point on the normal. The procedure is repeated till the body surface is reached (for which ψ=0).
In the same fashion, calculations are carried out along various normal. Hence the entire flow field and body shape are determined.

IV. Axisymmetric Flow

(paraboloidal shock)

In this section the flow field and the body shape are determined for the case of paraboloidal shock. In this case M_∞ is assumed to be ∞. This case has been solved by Maslen and the result of present calculations can be compared with the ones available in [1]. Equation of the shock is,

$$r_s' = (2x')^{1/2} \tag{25}$$

Noting that $M_\infty = \infty$, the limiting non dimensional flow variables behind the shock are given by,

$$p_s' = \frac{2}{\gamma+1}\left(\frac{1}{2x'+1}\right) \tag{26}$$

$$\rho_s' = \frac{\gamma+1}{\gamma-1} \tag{27}$$

$$u_s' = \left(\frac{2x'}{1+2x'}\right)^{1/2} \tag{28}$$

$$\psi_s' = x' \tag{29}$$

Starting from the known values of flow variables behind the shock, the values at any point are determined, by marching from the shock to the body, along the normal to the shock. As the details are same as two- dimensional flow, are not given for this case. The equations for the various flow quantities are

$$p' = p_s'\left[1+k\left(\frac{\psi'}{\psi_s'}-1\right)\right] \tag{30}$$

Where;

$$k = \frac{\gamma+1}{4}\left(\frac{2x'}{1+2x'}\right) \tag{31}$$

$$\rho = \frac{\gamma+1}{\gamma-1}\left[\frac{\gamma+1}{2}\rho_s'(1+2\psi')\right]^{1/\gamma} \tag{32}$$

$$u = \left[\frac{4\gamma}{(\gamma+1)^2} - \frac{2\gamma}{\gamma-1}\frac{p}{\rho}\right]^{1/2} \tag{33}$$

The radial coordinate (Fig. 3) of the point to which the above flow quantities belong, is determined by integration of the continuity equation, which is given by,

$$r' = \left[2x' - 2u_s\int_{\psi'}^{\psi_s'}\frac{d\psi'}{\rho u}\right]^{1/2} \tag{34}$$

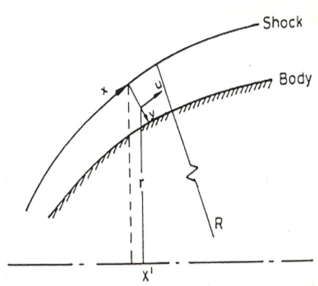

Fig. 3 Axisymmetric Geometry of Flow

All the above equations are given in [1]. The equations were re-derived, and found to be free of mistakes.

V. Numerical Results

(For two dimensional flow)

The flow field and body shape are calculated for parabolic shock shape, given by the equation: y^2 =x. The free stream conditions correspond to ones at the altitude 20 km. they are given by:ρ_∞ = 0.0886 Kg /m^3, P_∞= 5480 N/m^2, T_∞ = 216.65 K. The free stream Mach no is assumed to be 5.8. For this free stream condition and assumed parabolic shock shape, the flow field and the corresponding body shape are calculated.

A program computer is used for the calculation. The results presented in Fig. 4 shows the assumed parabolic shock shape and the calculated body shape. The shock shape is divided into 30 points; the corresponding number of points on the body also is 30. It is important to note that the body shape obtained is not blunt as it should be. In a way this is expected because as seen earlier the important assumptions made in the Maslen's method break down on the stagnation streamline and its neighborhood.

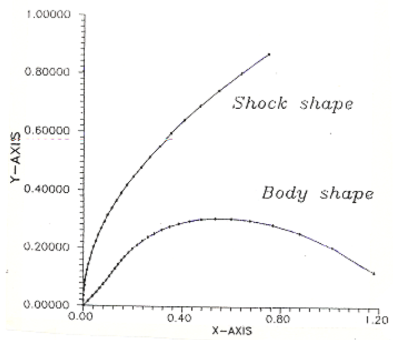

Fig. 4 Assumed shock shape and corresponding body shape, Mach No. 5.8

As mentioned for the assumed shock shape, the flow quantities are obtained by numerical integration marching from the shock shape to the body. Integration stops where ψ = 0, which is on the body. In the Fig. 5 the grid points are shown on the various normal to the shock. In this figure each normal is divided into 20 intervals. All flow variables are calculated and the hence are known at the 20 grid points. In Fig. 6 the shock shape, calculated body shape, and the sonic line are shown. Above the sonic line flow is supersonic, and is subsonic below the sonic line. In Fig. 7 convergence of x and y coordinates of a point on the body is examined.

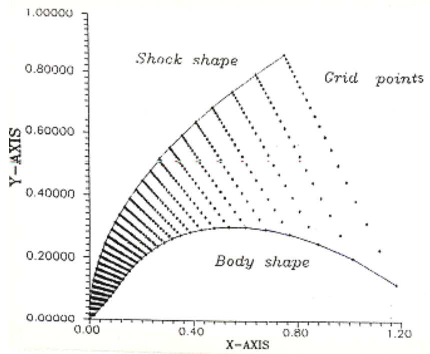

Fig. 5 Grid Point Distribution, Mach No. 5.8

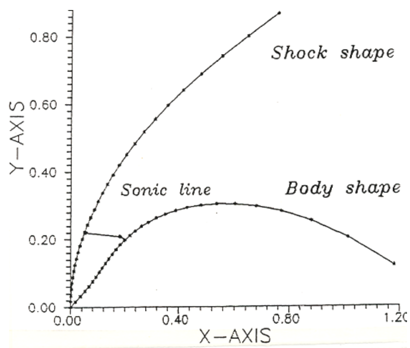

Fig. 6 Shock shape, body shape and the sonic line, Mach No. 5.8

The corresponding point on the shock is such that the shock angle is 40. It is seen that for the division of normal into intervals, both x, y coordinates have converged. The same thing is shown in Fig. 8 where the only charge- is that the location of the point on the body is such that, for the corresponding point on the shock, shock angel is 80. Unlike for, the β=80, and it is seen that division of normal into 20 intervals is required for the convergence. In Figs. 9 & 10 Mach number independence of C_p is verified, for two points on the converged body shape. These two are the same as the ones chosen earlier, in Fig. 7 and Fig. 8, for the convergent study. As can be seen Cp becomes independent of the free stream Mach number, when it is about 10. Since the point on the body in Fig. 10 is closer to the nose, than the point in the Fig. 9., the asymptotic value of Cp in Fig. 10 is 1.55. In Fig. 11 surface value of P/P_o is plotted as a function of x.

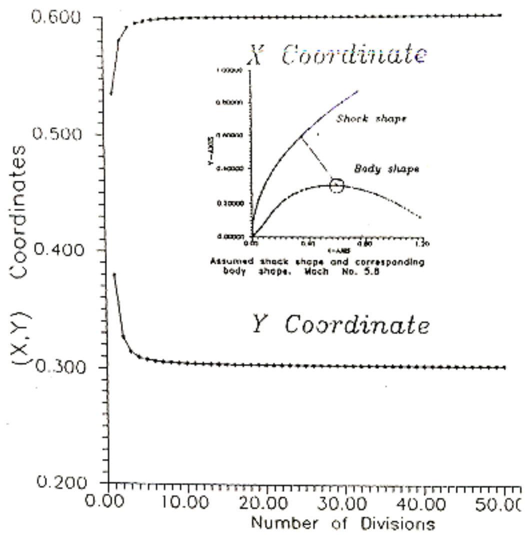

Fig. 7 Convergence of x and y coordinate of the Body Shape for the Shock Angle Beta, Mach No. 5.8

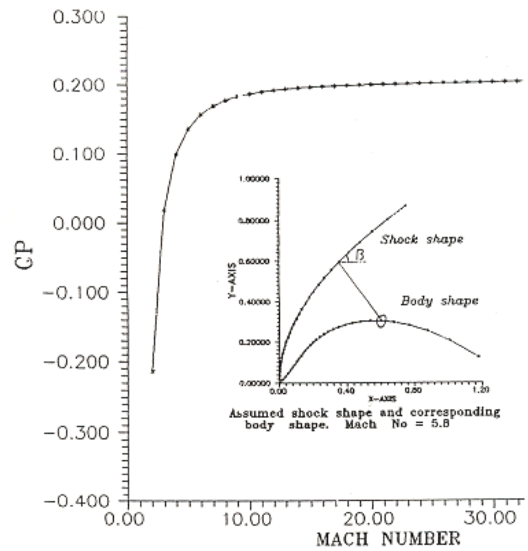

Fig. **8** Convergence of x and y coordinate of the Body Shape, for the Shock Angle Beta=80 Mach. No. 5.8

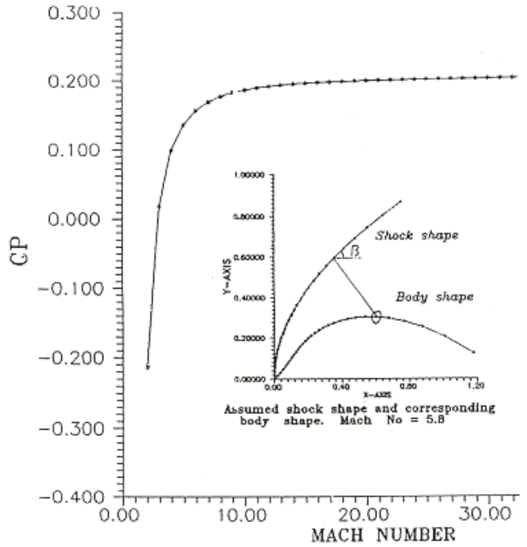

Fig. 9 Variation of C_p on the Body Surface for X=0.355 Y= 0.596 (Mach. No. Independence)

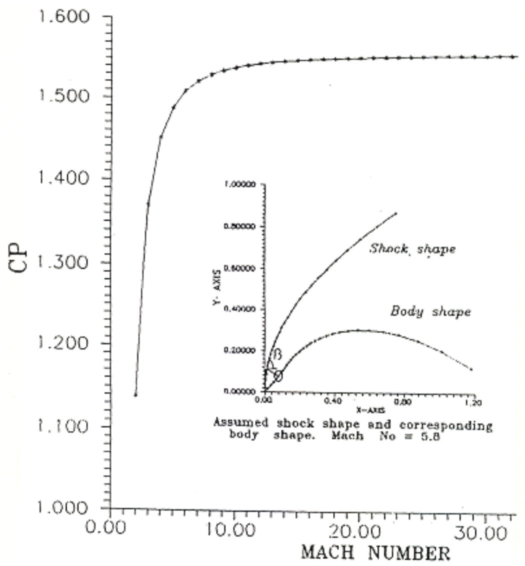

Fig. 10 Variation of C_p on the Body Surface for X=0.00778 Y= 0.088 (Mach. No. Independence)

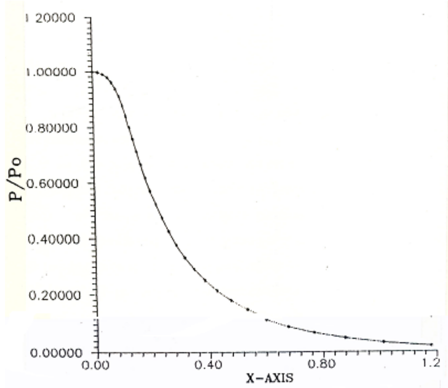

Fig. 11 Variation of Surface Pressure / Stagnation Pressure , Mach No. 5.8

VI.

NUMERICAL RESULTS FOR AXISYMMETRIC METHOD

The computer program is written for determination of body shape and other flow variables. The distribution of P/P_o on the surface is shown in the Fig. 12. The result available in [1] for this case is given in Fig. 13.

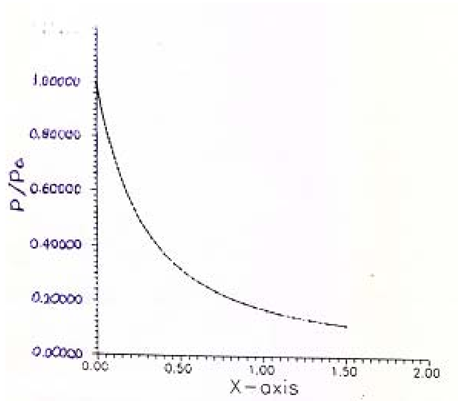

Fig. 12 Pressure / Stagnation pressure on the Body Surface along X-axis Mach No. Infinity

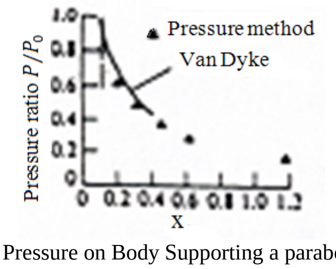

Fig. 13 Surface Pressure on Body Supporting a parabolodial Shock; From Anderson [2] γ= 1.4, M_∞= ∞

VII. CONCLUSION

Contrary to what is found in the literature Maslen's methods does not appear to be suitable for the study of hypersonic flow past blunt bodies. Actually for the cases of parabolic and paraboloidal shocks, the body shapes must be blunt. However for the parabolic shock, Maslen's method does not give a blunt nosed body. For the paraboloidal shock though present calculation based Maslen's methods gives a blunt body, this is not same as given in Maslen's method [1]. This is because the algorithm used for present calculation of the coordinates of body surface for the axisymmetric case is not same as given by Maslen. The reason for the change is that, certain serious difficulties were encountered with Maslen's algorithm. This portion (calculation of coordinates for axiymmetric case) of the work needs closer scrutiny.

ACKNOWLEDGEMENT

The author was supported by scholarship from Government of Islamic Republic of Iran during M.Tech. The support is gratefully acknowledged. I would like also to extend my sincere thanks to my research guide Dr. S.C. Rajan Assistant Prof. Aerospace Dept of IIT Madras in India for his gracious support during the research works in M.Tech.

References

[1] S.H. Maslen, "Inviscid hypersonic flow blunt past smooth Symmetric bodies", AIAA Journal. Vol. 2, 9 No. 6 June 1964, PP 1055-1061.
[2] J.D. Anderson, '"Hypersonic and High temperature Gas Dynamic"' , MacGraw Hill book Co. 1989
[3] C. F.McNeil, "Approximate Viscose Shock Layer Technique for Calculating Hypersonic Flow About blunt-nosed Bodies", Vol. 31, No. 4 July-August 1994, pp. 621-629.
[4] J.R. Christopher, "An approximate method for calculating three-dimensional inviscid hypersonic flow field", NASA Technical paper 3018.
[5] V.D. Milton, "Perfureation Method In Fluid Mechanich", 1964
[6] M.D. Van Dyke, , "The supersonic blunt - body problem - review and extension" , Journal of the Aero/Space Sciences, Vol. 25, August 1958, PP. 485-495
[7] G. Morrti, and M. Abbett, "7 Time dependent computational method for bluntbody flow ",AIAA Journal Vol., No. 12 1966, PP. 20136-2141
[8] J.D. Anderson, "Fundamental of Aerodynamics", 1988