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# Appling Eyring's Accelerated Life Testing Model to "Times to Breakdown" of Insulating Fluid: A Combined Approach of an Accelerated and a Sequential Life Testing

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***Abstract*—**In this paper, the test purpose will be to assess whether or not the accelerated model proposed by Eyring will be able to translate results for the shape and scale parameters of an underlying Weibull model, obtained under two accelerating using conditions, to expected normal using condition results for these parameters. The product being analyzed is a new type of insulate fluid, and the accelerating factor is the voltage stresses applied to the fluid at two different levels (30KV and 40KV). The normal operating voltage is 25KV. In this case, it was possible to test the insulate fluid at normal voltage using condition. Both results for the two parameters of the Weibull model, obtained under normal using condition and translated from accelerated using conditions to normal conditions, will be compared to each other to assess the accuracy of the Eyring model when the accelerating factor is only the voltage stress.

***Keywords*—**Eyring Accelerated Model, Sequential Life Testing, Two-Parameter Weibull Distribution, Voltage Stresses.

## I. Introduction

In situations when stresses other than temperature are involved, the Eyring Model offers a general solution to the problem of additional stresses. It has a theoretical derivation based on chemical reaction rate theory and quantum mechanics. The Eyring model is given by:

$$R_{rate} = T^{\alpha} \; e^{-(E/K)T_n + C} \; \exp\left[DS_1\right]$$

Here, $R_{rate}$ is the rate of reaction, E represents the energy of activation of the reaction, K the gas constant (1.986 calories per mole), $T_n$ the temperature in degrees Kelvin (273.16 plus the degrees Centigrade) at normal condition of use, $S_1$ is a second stress, C and D are constants. From (1) we can notice how the first term, which models the effect of temperature, compares to the Arrhenius model. Except for the $T^{\alpha}$ factor, this term is the same as the Arrhenius. Therefore, the Arrhenius model is successful because it is a useful simplification of the theoretically derived Eyring model.

The accelerating factor $AF_{2/1}$ for the Eyring model (or the ratio of the specific rates of reaction $R_2/R_1$), at two different stress temperatures, $T_2$ and $T_1$, and at two different stress voltages, $V_1$ and $V_2$, will be given by:

$$AF_{2/1} = \frac{A T_2^{\alpha} \, e^{-(E/K)T_2 + C} \quad e^{-(E/K)V_2 + D}}{A T_1^{\alpha} \, e^{-(E/K)T_1 + C} \quad e^{-(E/K)V_1 + D}}, \text{ or yet:}$$

$$AF_{2/1} = \left(\frac{T_2}{T_1}\right)^{\alpha} \exp\left[\frac{E}{K}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right] \exp\left[\frac{E}{K}\left(\frac{1}{V_1} - \frac{1}{V_2}\right)\right] \quad (2)$$

Since the testing temperatures at the two accelerating testing conditions are the same and the only different accelerating stress is the voltage, (2) becomes:

$$AF_{2/1} = \exp\left[\frac{E}{K}\left(\frac{1}{V_1} - \frac{1}{V_2}\right)\right] \quad (3)$$

Applying natural logarithm to both sides of (3) and after some algebraic manipulation, we will obtain:

$$\ln\left(AF_{\;2/1}\right) = \frac{E}{K}\left(\frac{1}{V_1} - \frac{1}{V_2}\right) \quad (4)$$

From (4) we can estimate the term E/K by testing at two different stress voltages and computing the acceleration factor on the basis of the fitted distributions. Then;

$$\frac{E}{K} = \frac{\ln\left(AF_{2/1}\right)}{\left(\frac{1}{V_1} - \frac{1}{V_2}\right)} \quad (5)$$

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The acceleration factor $AF_{2/1}$ will be given by the relationship $\theta_1/\theta_2$, with $\theta_i$ representing a scale parameter or a percentile at a stress level corresponding to $V_i$. Once the term E/K is determined, the acceleration factor $AF_{2/n}$ to be applied at the normal stress voltage is obtained from (4) by replacing the stress voltage $V_1$ with the stress voltage at normal condition of use $V_n$. Then:

$$AF_{2/n} = \exp\left[\frac{E}{K}\left(\frac{1}{V_n} - \frac{1}{V_2}\right)\right] \tag{6}$$

## II. The Accelerating Condition

Reference [1] has shown that under a linear acceleration assumption, if a three-parameter Weibull model represents the life distribution at one stress level, a three-parameter Weibull model also represents the life distribution at any other stress level. The same reasoning applies to the two-parameter Weibull model. We will be assuming a linear acceleration condition. In general, the scale parameter can be estimated by using two different stress levels (temperature or voltages or cycles or miles, etc.), and their ratios will provide the desired value for the acceleration factor $AF_\theta$. So, we will have:

$$AF_\theta = \frac{\theta_n}{\theta_a} \tag{7}$$

Again, according to [1] for the two-parameter Weibull model the cumulative distribution function at normal testing condition $F_n(t_n)$ for a certain testing time $t = t_n$, will be given by:

$$F_n(t_n) = F_a\left(\frac{t_n}{AF}\right) = 1 - \exp\left[-\left(\frac{t_n}{\theta_a\, AF}\right)^{\beta_a}\right] \tag{8}$$

Equation (8) tells us that, under a linear acceleration assumption, if a two-parameter Weibull model represents the life distribution at one stress level, a two-parameter Weibull model also represents the life distribution at any other stress level. The shape parameter remains the same while the accelerated scale parameter is multiplied by the acceleration factor. The equal shape parameter is a necessary mathematical consequence of the other two assumptions, that is, assuming a linear acceleration model and assuming a two-parameter Weibull sampling distribution. If different stress levels yield data with very different shape parameters, then either the two-parameter Weibull sampling distribution is the wrong model for the data or we do not have a linear acceleration condition.

## III. Maximum Likelihood Estimation for the Weibull Model for Censored Type II Data (Failure Censored)

The likelihood function for the shape and scale parameters of a Weibull sampling distribution for censored Type II data (failure censored) will be given by:

$$L(\beta;\theta) = k!\left[\prod_{i=1}^{r} f(t_i)\right]\left[1 - F(t_r)\right]^{n-r}$$, or yet:

$$L(\beta;\theta) = k!\left[\prod_{i=1}^{r} f(t_i)\right]\left[R(t_r)\right]^{n-r};\ t>0 \tag{9}$$

With $f(t) = \frac{\beta}{\theta^\beta}(t_i)^{\beta-1} e^{-(t/\theta)^\beta}$ and $R(t_r) = e^{-(t_r/\theta)^\beta}$, we will have:

$$L(\beta;\theta) = k!\,\frac{\beta^r}{\theta^{\beta r}}\left[\prod_{i=1}^{r} t_i\right]^{\beta-1} e^{-\sum_{i=1}^{r}(t_i/\theta)^\beta} \times \left[e^{-(t_r/\theta)^\beta}\right]^{n-r} \tag{10}$$

The log likelihood function will be given by:

$$\mathrm{L} = \ln[L(\beta;\theta)] = \ln(k) + r\ln(\beta) - r\beta\ln(\theta) + $$
$$+ (\beta-1)\sum_{i=1}^{r}\ln(t_i) - \sum_{i=1}^{r}\left(\frac{t_i}{\theta}\right)^\beta - (n-r)\left(\frac{t_r}{\theta}\right)^\beta \tag{11}$$

To find the values of θ and β that maximizes the log likelihood function, we take the θ and β derivatives and make them equal to zero. Then, we will have:

$$\frac{dL}{d\theta} = -\frac{r\beta}{\theta} \frac{\beta \times \sum_{i=1}^{r}(t_i)^\beta}{\theta^{\beta+1}} + \frac{\beta(n-r)(t_r)^\beta}{\theta^{\beta+1}} = 0 \tag{12}$$

$$\frac{dL}{d\beta} = \frac{r}{\beta} - r\ln(\theta) + \sum_{i=1}^{r}\ln(t_i) - $$
$$-\sum_{i=1}^{r}\left(\frac{t_i}{\theta}\right)^\beta \times \ln\left(\frac{t_i}{\theta}\right) - (n-r)\left(\frac{t_r}{\theta}\right)^\beta \ln\left(\frac{t_r}{\theta}\right) = 0 \tag{13}$$

From (12) we obtain:

$$\theta = \left(\left[\sum_{i=1}^{r}(t_1)^\beta + (n-r)(t_r)^\beta\right]\Big/ r\right)^{1/\beta} \tag{14}$$

Notice that, when β=1, (14) reduces to the maximum likelihood estimator for the exponential distribution. Using (14) for θ in (13) and applying some algebra, (13) reduces to:

$$\frac{r}{\beta} + \sum_{i=1}^{r} \ln(t_i) - \qquad (15)$$

$$- \frac{r \times \left[ \sum_{i=1}^{r} (t_i)^{\beta} \ln(t_i) + (n-r)(t_r)^{\beta} \ln(t_r) \right]}{\sum_{i=1}^{r} (t_i)^{\beta} + (n-r)(t_r)^{\beta}} = 0$$

Equation (15) must be solved iteratively.

## IV. THE SEQUENTIAL LIFE TESTING

The two-parameter Weibull distribution has a shape parameter β which specifies the shape of the distribution, and a scale parameter θ which represents the characteristic life of the distribution. Both parameters are positive. The Weibull density function is given by:

$$f(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right]; \quad t \geq 0 \qquad (16)$$

The hypothesis testing situations were given by [2] and [3]:

1. For the scale parameter θ: $H_0$: $\theta \geq \theta_0$; $H_1$: $\theta < \theta_0$

The probability of accepting $H_0$ will be set at (1-α) if $\theta = \theta_0$. Now, if $\theta = \theta_1$ where $\theta_1 < \theta_0$, then the probability of accepting $H_0$ will be set at a low level γ.

2. For the shape parameter β: $H_0$: $\beta \geq \beta_0$; $H_1$: $\beta < \beta_0$

The probability of accepting $H_0$ will be set again at (1-α) if $\beta = \beta_0$. Now, if $\beta = \beta_1$, where $\beta_1 < \beta_0$, then the probability of accepting $H_0$ will also be set at a low level γ.

The development of a sequential test uses the likelihood ratio (LR) given by the following relationship proposed by [2]: LR = $L_{1;n}/L_{0;n}$.

The sequential probability ratio (SPR) will be given by SPR = $L_{1;n}/L_{0;n}$. According to [4], for the two-parameter Weibull model the (SPR) will be:

$$\text{SPR} = \left( \frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0} \right)^n \prod_{i=1}^{n} \left( t_i \right)^{\beta_1 - \beta_0} \times$$

$$\times \exp\left[ -\sum_{i=1}^{n} \left( \frac{t_i^{\beta_1}}{\theta_1^{\beta_1}} - \frac{t_i^{\beta_0}}{\theta_0^{\beta_0}} \right) \right]$$

The continue region will become A< SPR < B, where A = γ /(1-α) and also B = (1-γ)/α. We will accept the null hypothesis $H_0$ if SPR ≥ B and we will reject $H_0$ if SPR ≤ A. Now, if A<SPR<B, we will take one more observation. Then, after some mathematical manipulation, we will have:

$$n \ln\left( \frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0} \right) - \ln\left[ \frac{(1-\gamma)}{\alpha} \right] < \text{W} <$$

$$< n \ln\left( \frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0} \right) + \ln\left[ \frac{(1-\alpha)}{\gamma} \right] \qquad (17)$$

$$\text{W} = \sum_{i=1}^{n} \left( \frac{t_i^{\beta_1}}{\theta_1^{\beta_1}} - \frac{t_i^{\beta_0}}{\theta_0^{\beta_0}} \right) + (\beta_0 - \beta_1) \sum_{i=1}^{n} \ln(t_i) \qquad (18)$$

## V. EXPECTED SAMPLE SIZE OF A SEQUENTIAL LIFE TESTING FOR TRUNCATION PURPOSE

According to [5], an approximate expression for the expected sample size E(n) of a sequential life testing for truncation purpose will be given by:

$$E(n) = \frac{E(W_n^*)}{E(w)} \qquad (19)$$

$$\text{w} = \ln \frac{f(t; \theta_1, \beta_1)}{f(t; \theta_o, \beta_o)} \qquad (20)$$

$$E(W_n^*) \cong P(\theta,\beta) \ln A + [1 - P(\theta,\beta)] \ln B \qquad (21)$$

For two-parameter Weibull sampling distribution, we will have:

$$E(w) = \ln\left( \frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0} \right) + (\beta_1 - \beta_0) E[\ln(t)] -$$

$$- \frac{1}{\theta_1^{\beta_1}} E\left(t^{\beta_1}\right) + \frac{1}{\theta_0^{\beta_0}} E\left(t^{\beta_0}\right) \qquad (22)$$

$$E[\ln(t)] = \ln(\theta) + \frac{1}{\beta} \times \frac{g}{3} \times$$

$$\times \left\{ \sum_{i=1}^{n+1} \left[ \ln(U_i) e^{-U_i} \times (1, 2\, or\, 4) \right] \right\}; \quad \text{U} = \left(\frac{t}{\theta}\right)^{\beta} \qquad (23)$$

To find the E[ln(t)] some numerical integration procedure (Simpson's 1/3 rule in this work) will have to be used. The solution of each component of (22) can be found in [4].

## VI. EXAMPLE

The data is from [6]. Table 1 shows data on time to breakdown of an insulating fluid recorded at three different voltages (40KV, 30KV and 25 KV) on 12 transformers. The operating normal voltage is 25KV. The test purpose will have two objectives: The first purpose will be to assess whether or not the accelerated model proposed by Eyring will be able to translate results for the shape and scale parameters of an underlying Weibull model, obtained under two accelerating using conditions, to expected normal using condition results for these two parameters The second purpose will be to verify if times to breakdown of insulating fluid between electrodes recorded at three different voltages have an exponential distribution as predicted by theory. The test was truncated at 30KV and 25KV. The sample size of each voltage was 12.

TABLE I
TIMES TO BREAKDOWN OF INSULATING FLUID WITH CENSORING
TIME TO BREAKDOWN = SECONDS; KV = KILOVOLTS

| Voltages | 40KV | 30KV | 25KV |
|---|---|---|---|
| Time/sec | 1.5 | 50. | 2,500. |
| Time/sec | 1.5 | 134. | 4,056. |
| Time/sec | 2. | 187. | 12,553. |
| Time/sec | 3. | 882. | 40,290. |
| Time/sec | 12. | 1,448. | * |
| Time/sec | 25. | 1,468. | * |
| Time/sec | 46. | 2,290. | * |
| Time/sec | 56. | 2,932. | * |
| Time/sec | 68. | 4,138. | * |
| Time/sec | 109. | 15,750. | * |
| Time/sec | 323. | * | * |
| Time/sec | 417. | * | * |

Using the maximum likelihood estimator approach for the scale and shape parameters of the underlying Weibull sampling distribution for censored Type II data (failure censored), we obtain the following values for these parameters:

| Voltage | Shape Parameter β | Scale Parameter θ |
|---|---|---|
| 40KV (12 Items) | 0.573 | 55.9568 |
| 30KV (10 Items) | 0.540 | 4,723.266 |
| 25KV (4 items) | 0.653 | 160,234.4 |

Since the values of the shape parameters for the three voltages are relatively close (0.573 for 40KV with the analysis of twelve failure times; 0.540 for 30KV with the observation of ten failure times and 0.653 for 25KV with the inspection of only four failure times), we can assume a linear acceleration condition. Now, using the results of the two shape and scale parameters of the underlying Weibull model obtained at 40KV and 30KV stresses, we will estimate the values of these two parameters under normal voltage using condition (25KV). Then, we will compare these estimate results with the ones obtained at normal testing conditions. We want to assess whether or not the accelerated model proposed by Eyring will be able to translate estimated results for the shape and scale parameters of an underlying Weibull model to expected normal using condition results for the insulating fluid being analyzed. Therefore, using (1) to (7), we will have the acceleration factor for the scale parameter $AF\theta_{2/1}$. Utilizing (7), we will obtain:

$$AF\theta_{2/1} = \theta_1/\theta_2 = 4{,}723.266/55.9568 = 84.409$$

Using now (5), we can estimate the term E/K. Then:

$$\frac{E}{K} = \frac{\ln\left(AF_{2/1}\right)}{\left(\frac{1}{V_1} - \frac{1}{V_2}\right)} = \frac{\ln(84.409)}{[(1/30)-(1/40)]} = 532.281$$

Applying (6), the acceleration factor for the scale parameter to be applied at the normal stress voltage $AF\theta_{2/n}$ will be:

$$AF_{2/n} = \exp\left[\frac{E}{K}\left(\frac{1}{V_n} - \frac{1}{V_2}\right)\right] = \exp\left[532.281\left(\frac{1}{25} - \frac{1}{40}\right)\right]$$

$$AF_{2/n} = 2{,}934.273$$

Therefore, the scale parameter of the component at normal operating stress voltage is estimated to be:

$$\theta_n = AF_{2/n} \times \theta_2$$

$$\theta_n = 2{,}934.273 \times 55.9568 = 164{,}192.53 \text{ seconds}$$

The percentage difference between the estimated value of θ (164,192.53 seconds) and the calculated value of θ obtained with the inspection of only four failure times (160,234.4 seconds) will be:

$$\%\ \text{Difference } (\theta \text{ estimated}/\theta \text{ calculated}) = \frac{164{,}192.52}{160{,}234.4} = 2.47\%$$

Then, we can see that the accelerated model proposed by Eyring will be able to translate with a certain degree of precision, results for the shape and scale parameters of an underlying Weibull model, obtained under two accelerating using conditions, to expected normal using condition results for these two parameters.

To evaluate the accuracy (significance) of the two-parameter values estimated under normal conditions for the underlying Weibull model we will employ, to the expected normal failure times, a sequential life testing using a truncation mechanism developed by [4]. These expected normal failure times will be acquired by multiplying the twelve failure times obtained under accelerated testing conditions at 40 KV given by Table 1, by the accelerating factor AF of 2,934.273.

Table 2 shows these expected normal failure times.

TABLE II
TIMES TO BREAKDOWN OF INSULATING FLUID WITH CENSORING
TIME TO BREAKDOWN = SECONDS; KV = KILOVOLTS

| Voltages | 40KV | Expected Normal |
|---|---|---|
| Time/sec | 1.5 | 4,401.4 |
| Time/sec | 1.5 | 4,401.4 |
| Time/sec | 2. | 5,868.5 |
| Time/sec | 3. | 8,802.8 |
| Time/sec | 12. | 35,211.3 |
| Time/sec | 25. | 73,356.8 |
| Time/sec | 46. | 134,976.6 |
| Time/sec | 56. | 164,319.3 |
| Time/sec | 68. | 199,530.6 |
| Time/sec | 109. | 319,835.8 |
| Time/sec | 323. | 947,770.2 |
| Time/sec | 417. | 1,223,591.8 |

It was decided that the value of $\alpha$ was 0.05 and $\gamma$ was 0.10. In this example, the following values for the alternative and null parameters were chosen: alternative scale parameter $\theta_1$ = 180,000. seconds and alternative shape parameter $\beta_1$ = 0.8; null scale parameter $\theta_0$ = 164,000 seconds and null shape parameter $\beta_0$ = 0.60. Now electing $P(\theta,\beta)$ to be 0.01, we can calculate the expected sample size E(n) for truncation purpose of this sequential life testing under analysis. Applying (19) to (23), we will have:

$$E(w) = \ln\left(\frac{\beta_1}{\theta_1^{\beta_1}} \times \frac{\theta_0^{\beta_0}}{\beta_0}\right) + \left(\beta_1 - \beta_0\right) E[\ln(t)] -$$

$$- \frac{1}{\theta_1^{\beta_1}} \times E\left(t^{\beta_1}\right) + \frac{1}{\theta_0^{\beta_0}} E\left(t^{\beta_0}\right)$$

$$E(w) = -2.1883 + 0.2 \times 13.2025 - 1.1037 + 0.9096 = 0.2581$$

Now, with A = $\gamma/(1-\alpha)$; B = $(1-\gamma)/\alpha$; $\alpha$ = 0.05; $\gamma$ = 0.10 and also $P(\theta,\beta)$ = 0.01, we will have:

$$E\left(W_n^*\right) \cong P(\theta,\beta) \ln A + \left[1 - P(\theta,\beta)\right] \ln B$$

$$E\left(W_n^*\right) \cong -0.01 \times 2.2513 + 0.99 \times 2.8904 = 2.8390$$

$$E(n) = \frac{P(\theta,\beta) \ln A + \left[1 - P(\theta,\beta)\right] \ln B}{E(w)}$$

$$E(n) = \frac{2.8390}{0.2581} = 10.999 \cong 11 \text{ items}$$

So, we could make a decision about accepting or rejecting the null hypothesis $H_0$ after the analysis of observation number 11. Using now (17) and (18) and the twelve failure times obtained under accelerated conditions at 40 KV given by Table 2, multiplied by the accelerating factor AF of 2,934.273, we calculate the sequential life testing limits.

$$n \ln\left(\frac{0.8}{180{,}000^{0.8}} \times \frac{164{,}000^{0.6}}{0.6}\right) - \ln\left[\frac{(1-0.10)}{0.05}\right] =$$

$$= n \times -2.1883 - 2.8904$$

$$n \ln\left(\frac{0.8}{180{,}000^{0.8}} \times \frac{164{,}000^{0.6}}{0.6}\right) + \ln\left[\frac{(1-0.05)}{0.10}\right] =$$

$$= n \times -2.1883 + 2.2513$$

$$W = \sum_{i=1}^{n} \left(\frac{t_i^{0.80}}{180{,}000^{0.80}} - \frac{t_i^{0.6}}{164{,}000^{0.6}}\right) - 0.2 \times \sum_{i=1}^{n} \ln\left(t_i\right)$$

Then, we get:

$$n \times -2.1883 - 2.8904 < W < n \times -2.1883 + 2.2513 \tag{24}$$

The procedure is defined by the following rules:

1. If W ≥ $n \times -2.1883 + 2.2513$, we will accept $H_0$.
2. If W ≤ $n \times -2.1883 - 2.8904$, we will reject $H_0$.
3. If $n \times -2.1883 - 2.8904$< W < $n \times -2.1883 + 2.2513$, we will take one more observation.

Table 3 shows the results of this test for the Weibull model case.

TABLE III
RESULTS FOT THE 32 KV CASE - TWO PARAMETER SAMPLING WEIBULL MODEL

| Unit Number | Lower Limit | Upper Limit | Value of W |
|---|---|---|---|
| 1 | −5.078686 | 0.62977 | −1.740662 |
| 2 | −7.267001 | −2.125337 | −3.481324 |
| 3 | −9.455316 | −4.313652 | −5.287725 |
| 4 | −11.643630 | −6.501967 | −7.187789 |
| 5 | −13.831945 | −8.690281 | −9.407800 |
| 6 | −16.020259 | −10.878596 | −11.777842 |
| 7 | −18.208574 | −13.066910 | −14.235817 |
| 8 | −20.396889 | −15.255225 | −16.709219 |
| 9 | −22.585203 | −17.443540 | −19.188927 |
| 10 | −24.773518 | −19.631854 | −21.633120 |
| 11 | −26.961832 | −21.820169 | −23.473456 |
| 12 | −29.150147 | −24.008483 | −24.983063 |

In this case, even after the observation of 12 times to failure, it was not possible to make the decision to accept or reject the null hypothesis $H_0$. Since we could make a decision about accepting or rejecting the null hypothesis $H_0$ after the analysis of observation number 11, we will introduce a procedure for early truncation.

## VII. A PROCEDURE FOR EARLY TRUNCATION

According to [2], when the truncation point is reached, a line partitioning the sequential graph can be drawn as shown in Fig. 1.

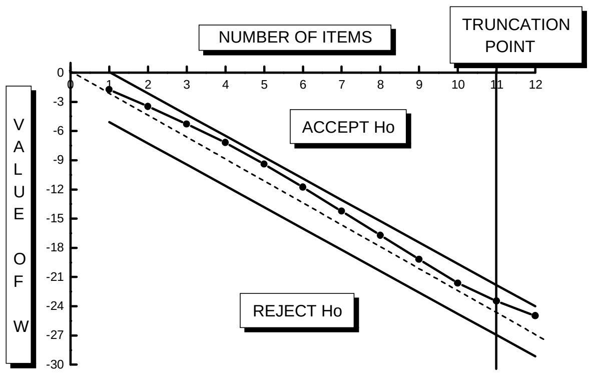


Fig. 1 Sequential test graph for the two-parameter Weibull model

This line is drawn through the origin of the graph parallel to the acceptance and rejection lines. The decision to accept or reject $H_0$ simply depends on which side of the line the final outcome lies. Obviously this procedure changes the levels of $\alpha$ and $\gamma$ of the original test; however, the change is slight if the truncation point is not too small (less than four). Fig. 1 above shows the sequential test graph developed for this example. As we can see in Fig. 1, the null hypothesis $H_0$ should be accepted since the final observation (observation number 11) lies on the side of the line related to the acceptance of $H_0$.

## VIII. CONCLUSION

In this paper, the test purpose was to assess whether or not the accelerated model proposed by Eyring will be able to translate results for the shape and scale parameters of an underlying Weibull model, obtained under two accelerating using conditions, to expected normal using condition results for these two parameters. The product being analyzed is a new type of insulate fluid, and the accelerating factor is the voltage stresses applied to the fluid at two different levels (30KV and 40KV). The normal operating voltage is 25KV. In this case, it was possible to test the insulate fluid at this normal voltage using condition. To estimate the parameters of the Weibull model we used a maximum likelihood approach for censored failure data, with the life-testing terminating at the moment the truncation point was reached. To evaluate the accuracy (significance) of the two-parameter values estimated under normal conditions for the underlying Weibull model we employed, to the expected normal failure times, a sequential life testing using a truncation mechanism developed by [4]. The shape parameter remained the same while the accelerated scale parameter and the accelerated minimum life parameter were multiplied by the acceleration factor. The equal shape parameter is a necessary mathematical consequence of the other two assumptions; that is, assuming a linear acceleration model and a two-parameter Weibull sampling distribution. If different stress levels yield data with very different shape parameters, then either the two-parameter Weibull sampling distribution is the wrong model for the data or we do not have a linear acceleration condition.

Since the obtained values of the shape parameters for the three voltages are relatively close (0.573 for 40KV with the analysis of twelve failure times; 0.540 for 30KV with the observation of ten failure times and 0.653 for 25KV with the inspection of only four failure times), we can assume a linear acceleration condition. Then, we compared the estimated normal using condition results for the two shape and scale parameters with the ones obtained by testing at the normal operating stress voltage (25KV).

The percentage difference between the estimated value of $\theta$ (164,192.53 seconds) and the calculated value of $\theta$ obtained with the inspection of only four failure times (160,234.4 seconds) is only 2.47%. Therefore, we can assume that the accelerated model proposed by Eyring, (when the accelerating factor is only the voltage stress), is able to translate with a certain degree of precision, results for the shape and scale parameters of an underlying Weibull model, obtained under two accelerating using conditions, to expected normal using condition results for these two parameters.

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