# Applications of Rough Set Decompositions in Information Retrieval 

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#### Abstract

This paper proposes rough set models with three different level knowledge granules in incomplete information system under tolerance relation by similarity between objects according to their attribute values. Through introducing dominance relation on the discourse to decompose similarity classes into three subclasses: little better subclass, little worse subclass and vague subclass, it dismantles lower and upper approximations into three components. By using these components, retrieving information to find naturally hierarchical expansions to queries and constructing answers to elaborative queries can be effective. It illustrates the approach in applying rough set models in the design of information retrieval system to access different granular expanded documents. The proposed method enhances rough set model application in the flexibility of expansions and elaborative queries in information retrieval.


Keywords-Incomplete information system, Rough set model, tolerance relation, dominance relation, approximation, decomposition, elaborative query.

## I. INTRODUCTION

INFORMATION retrieval (IR) is one of the four modules: information retrieval (IR), entity recognition (ER), information extraction (IE), and text mining (TM) in a text mining system. IR retrieves documents relevant to certain interested topics. ER identifies concepts for further information extraction and text mining. IE extracts facts involving two or more entities. TM performs either traditional text mining tasks including text clustering, text classification, topic detection, and multi-document summary, or more advanced hypothesis generation and discovery. Due to the complexity of text mining, many new approaches such as probabilistic, Bayesian, or Markov models have been introduced $[1,8,9]$.

Rough set theory [7,11] has been recently applied in Intelligent Systems, Machine Learning, Pattern Recognition, Data Mining, Decision Making and etc[1,3,5,6,9]. In an incomplete information system, an indiscernibility relation between objects according to definitive attribute values of them may no longer be formed because of existing null values(not non-applicable) to some attributes of certain objects. How to deal with it properly? Two approaches are always introduced by researchers [3,4,5,7,9]. One is to filling them out with high

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frequently occurring values by statistics. The other is to extend original RST model to facilitate incomplete systems to be processed directly.

Because it is hard to find the real occurrence of values in the null positions in filling out by statistics or other high computation, directly handling becomes an hot topic. For example, Kryszkiewicz M. generalized equivalence relation into tolerance relation possessing reflexivity and symmetry [6].Similarity relation, valued tolerance relation, and etc. are also introduced by other experts[4,7,11]. In information retrieval, approaches based on rough set model are proposed [2]. But a lack of retrieval diversity or a limitation of completeness becomes restricts.

Implementing diverse information retrieval in text mining in incomplete information system is more sophisticated than that in complete one because null values have to be processed and tolerance but not equivalence relation may only be created. At first, similarity measure between numerical or non numerical attribute values of two objects is set up. Then the similarity measure between objects is formed. Thirdly, a tolerance relation based on object similarity is established. A tolerance relation is not always an equivalence relation which at least has transitivity, so a partition might not be formed, but a collection of similar classes can be constructed. With dominance relation referring to attribute values of objects, similar classes can be further decomposed into three subclasses: little better sub-classes, little worse subclasses and vague subclasses. A query naturally includes a term subset $X$. Requirements of queries differ from each other according to their semantics. For certain queries, the decomposed lower or upper approximation by dominance relation can meet the needs more similar or precise. We show how to get expansions and to retrieve information by decompositions for elaborative queries.

The paper is organized as follows. Section 2 defines a similarity measurement for determining similarity relationship between two objects through attribute values. Section 3 reviews concepts of document-term matrix representation. Section4 suggests three granules using maximally compatible classes as primitives. Section 5 introduces ideas and principles of new approximations and decompositions under dominance relation based on the granules. Section 6 demonstrates expansions at different levels for a given query and implements elaborative queries through different granules proposed by us. Section 7 concludes our study.

## II. BASIC CONCEPTS

As in [6], an incomplete information system (IIS for short) is a quadruple $S=(U, A T, V, f)$, and a tolerance relation derived by $A \subseteq A T$, named $\operatorname{SIM}(A)$, is defined as in [6]. The tolerance class $S_{A}(x)$ for $x \in U$ is probably the biggest subset in which each element has tolerance relation $\operatorname{SIM}(A)$ with $x$ and can not be distinguished with $x$. All elements in $S_{A}(x)$ are tolerant with $x$, but anyone can not make sure whether or not two elements in $S_{A}(x)$ are compatible except one of them is $x$.

Different from traditional document representation approach, a given set of text documents is represented by an incomplete information system. Each document in a corpus is expressed by a term vector in vector space model (VSM). Traditionally, weights of those terms not occurring in a document are set zero. But some potentially relevant information would be lost due to such an unreasonable representation. To complement this shortage and not to degrade retrieval performance, incomplete information system is preferred to be used to describe data set. From the prospective of information system, such a consideration is natural and rational.

Instead of assigning zero to weights of those terms absent in a document, their weights are considered missing. Based on this idea, an approach of representing documents as incomplete term vectors is proposed. In this way, an incomplete information system with some missing values is constructed. Information loss can be avoided and therefore retrieval quality can be promoted. Suppose we have $m$ documents and $n$ different terms in a corpus. Each document is expressed by an $n$-dimensional term vector. Every different term occurred in the corpus is viewed as a dimension in term vector.

In order to apply rough set model in general, some real-valued weights have to be discretized. The most often used method is using " 0 " and " 1 " to represent the weights. But, the term vectors cannot reflect the extent of how frequent it is for each term and thus the computed similarity measure of documents can not well reflect the actual similarity between documents. To overcome this kind of information loss, the representation of term vectors here is not discretized in binary formation but somewhat in fuzzy form. RST models are then used to extract hidden and associative terms which convey an idea for text retrieval in the incomplete information system. The missing weights of extracted terms of a document do not need filled. Documents in the original text dataset are expanded and the ordinary smoothing work is also got rid of.

## III. Similarity Measurement

Let $a \in A \subseteq A T$ be an attribute of the system. $R_{a} \subseteq V_{a}^{2}$. If for any $v \in V_{a},(v, v) \in R_{a}$, then $R_{a}$ is reflexive. For
any $v_{i} \in V_{a}$ and $v_{j} \in V_{a}$, if $\left(v_{i}, v_{j}\right) \in R_{a}$ and $\left(v_{j}, v_{i}\right) \in R_{a}$, then $R_{a}$ is called symmetric. Objects $x, y$ are called similar with respect to $a$, if $(a(x), a(y)) \in R_{a} \cdot q_{a}: V_{a} \times V_{a} \rightarrow[0,1]$ is a measurement function to compute similarity between two attribute values $v_{i}$ and $v_{j}$ in $V_{a} . q_{a}$ is symmetry, i.e., $q_{a}\left(v_{i}, v_{j}\right)=q_{a}\left(v_{j}, v_{i}\right)$, and is capable of processing null values, i.e., $v_{i}$ or $v_{j}$ may be $*$. If one of them is *, then $q_{a}\left(v_{i}, v_{j}\right)=1$. Assume that $\lambda_{a} \in[0,1]$ is a threshold for $a$, which is given by domain experts and supposed to be a minimal value for similarity between $v_{i}$ and $v_{j}$. If $q_{a}\left(v_{i}, v_{j}\right) \geq \lambda_{a}$, then $v_{i}$ and $v_{j}$ are said to be similar, otherwise dissimilar.
(1) If $a \in A$ is continuous and numerical, and any one of $v_{i}, v_{j}$ is not ${ }^{*}$, then $q_{a}\left(v_{i}, v_{j}\right)=1-\left|v_{i}-v_{j}\right| /$ $\left(\sup \left(V_{a}\right)-\inf \left(V_{a}\right)\right)$, where $\sup \left(V_{a}\right)$ is the superior value of $V_{a}$ and $\inf \left(V_{a}\right)$ is the inferior value of $V_{a}$.
(2) If $a$ is discrete, numerical, finite and ordered, and anyone of $v_{s}, v_{t}$ is not ${ }^{*}$, then $q_{a}\left(v_{s}\right.$, $\left.v_{t}\right)=1-|s-t| /(k-1)$, where $\quad V_{a}=\left\{v_{1,} \quad v_{2}\right.$, $\left.\ldots, v_{s}, \ldots, v_{t}, \ldots, v_{k}\right\}, \quad\left(k=\operatorname{card}\left(V_{a}\right)\right.$ and $v_{1}<v_{2}<$ $\ldots<v_{s}<\ldots<v_{t}<\ldots<v_{k}$.
(3) If $v_{s}$ is * or $v_{t}$ is * or both of them are $*$ for $a \in A$ is numerical or not, then $q_{a}\left(v_{s}, v_{t}\right)$ is 1 .
(4) If $a \in A$ is non-numerical but symbolic, then $q_{a}\left(v_{s}, v_{t}\right)$ is given by domain experts according to different semantics of attributes. Referring to [4], an aggregative similarity measure function $q_{A}$ as similarity degree between two objects on attribute subset can also be defined.
Let $A \subseteq A T$. The similarity degree between $x$ and $y$ is defined by $\quad q_{A}(x, y)=f\left(A, q_{a}(a(x), a(y))\right)$. If $q_{A}(x, y)=1$, then $x$ is similar to $y$, otherwise, they are dissimilar. $f$ depends on domain knowledge or domain experts. If all attributes in $A$ are numerical attributes, then one of following formula can be used.
$q_{A}(x, y)=\left\{\begin{array}{c}1, \forall a \in A, q_{a}(a(x), a(y)) \geq \lambda_{a} \\ 0, \text { otherwise }\end{array}\right.$
$q_{A}(x, y)=\left\{\begin{array}{c}1, \forall a \in B, q_{a}(a(x), a(y)) \geq \lambda_{a}, B \subseteq A, \\ |B| /|A| \geq 1 / 2 \\ 0, \text { otherwise }\end{array}\right.$
$q_{A}(x, y)=\left\{\begin{array}{c}1, \sum_{a \in A} q_{a}(a(x), a(y)) \geq \Delta_{A} \\ 0, \text { otherwise }\end{array}\right.$
where $\lambda_{a}$ is a threshold for attribute a and $\Delta_{A}$ is an
aggregating threshold for attribute subset A. After gaining a definition of similarity degree between objects, a similarity relation on $U$ can be obtained:

$$
\operatorname{SIM}_{A}=\left\{(x, y) \mid q_{A}(x, y)=1\right\}
$$

Dominance relation is defined by $D_{A}$ $=\left\{(x, y) \mid x P_{A} y\right\}$, where $x P_{A} y$ if, and only if $x P_{a} y$ for each $a$ in $A$. If $a$ is numerical attribute, then $x P_{a} y$ if and only if $a(x)$ dominates $a(y)$, or both $a(x)$ and $a(y)$ are not * or any or both of $a(x)$ and $a(y)$ are *. If $a$ is a non-numerical attribute, then $x P_{a} y$ is determined by domain knowledge or experts. If any of them is *, then dominance relation always holds between them.

If a partially ordered relation exists in a non-numerical attribute value space, then a dominance relation can be easily established. For example, $\leq, \geq$ may be a dominance relation on numerical value attribute, while $\supseteq, \subseteq$ may be a dominance relation in any collection of subsets on a set. A dominance relation has reflexivity, transitivity but not necessarily symmetry. We will build a dominance relation by attribute value comparison not by object priority to combine with a tolerance relation to make decompositions in our paper in section 5.

## IV. Primitive and Non-primitive Granules

Since tolerance relation $S I M_{A}$ derived by $A \subseteq A T$ is also a compatible relation actually, we use another symbol $\operatorname{CSIM}_{A}$ to substitute for $S I M_{A}$ i.e. $\operatorname{CSIM}_{A}=\operatorname{SIM}_{A}$. However, its complete covering, denoted by $U / \operatorname{CSIM}_{A}$, is different from $U / S I M_{A} . U / C S I M_{A}$ is defined in the following:

$$
\begin{aligned}
& U / \operatorname{CSIM}_{A}=\left\{X \subseteq U: X \times X \subseteq \operatorname{CSIM}_{A}\right. \\
& \left.\forall x\left(x \in U \wedge x \notin X \rightarrow(X \cup\{x\})^{2} \nsubseteq C S I M_{A}\right)\right\} .
\end{aligned}
$$

The difference between them is that $U / \operatorname{CSIM}_{A}$ is made up from maximal compatible classes whereas $U / S I M_{A}$ is consisted of tolerance classes. $U /$ CSIM $_{A}$ and $U / \operatorname{SIM}_{A}$ may not form partitions on $U$.Two objects in a maximal compatible class always have compatibility with $\operatorname{CSIM}_{A}$. Maximal compatible class is a compatible class which could not be included in another else compatible class. If adding an extra object to a maximal compatible class, the compatibility of its maximum will be broken. Elements of $U / \operatorname{CSIM}_{A}$ are considered to be primitive granules by us. Two other kinds of granules for $x \in U$ are formed from $U / \operatorname{CSIM}_{A}$ :

$$
\begin{aligned}
S U_{A}(x) & =\cup X\left(X \in U / C S I M_{A} \wedge x \in X\right) ; \\
S L_{A}(x) & =\cap X\left(X \in U / C \operatorname{CSI}_{A} \wedge x \in X\right) .
\end{aligned}
$$

Two arbitrary elements in $S L_{A}(x)$ are still compatible, but it is not true in $S U_{A}(x) .\left\{S U_{A}(x) \mid x \in U\right\},\left\{S L_{A}(x) \mid\right.$ $x \in U\}$ and $U /$ CISIM $_{A}$ are three knowledge expression systems $[3,11]$ at different levels. Three lower and upper approximations may also be respectively obtained:

$$
\begin{aligned}
& \overline{S U}_{A}(X)=\left\{x \in U: S U_{A}(X) \cap X \neq \varnothing\right\} ; \\
& \overline{S L}_{A}(X)=\left\{x \in U: S L_{A}(X) \cap X \neq \varnothing\right\} ; \\
& \overline{S C}_{A}(X)=\left\{x \in U: Z \cap X \neq \varnothing, Z \in U / \text { CISIM }_{A}\right\} ; \\
& \underline{S U}_{A}(X)=\left\{x \in U: S U_{A}(X) \subseteq X\right\} ; \\
& \underline{S L}_{A}(X)=\left\{x \in U: S L_{A}(X) \subseteq X\right\} ; \\
& \underline{S C}_{A}(X)=\left\{x \in U: x \in Z, Z \subseteq X, Z \in U / \text { CISIM }_{A}\right\}
\end{aligned}
$$

## V. Decompositions and Approximation under Dominance Relation

By dominance relation, documents corpus $D$ can be divided into two subsets, denoted by $D_{\mathrm{p}}{ }^{+}\left(d_{i}\right)$ and $D_{p}^{-}\left(d_{i}\right)$ respectively with respect to $d_{i} . D_{\mathrm{p}}^{+}\left(d_{i}\right)$ includes all elements dominating $d_{i}$ and $D_{p}^{-}\left(d_{i}\right)$ includes all elements dominated by $d_{i}$. According to a theorem 3 [2], we have $S_{A}\left(d_{i}\right)=S U_{A}\left(d_{i}\right)$. We can use $S_{A}\left(d_{i}\right)$ or $S U_{A}\left(d_{i}\right)$ freely and equivalently. Similar class $S_{A}\left(d_{i}\right)$ may be decomposed into three subclasses[4]: positively similar subclass $S_{A}^{+}\left(d_{i}\right)$, negatively similar subclass $S_{A}^{-}\left(d_{i}\right)$, and purely similar subclass $S_{A}^{0}\left(d_{i}\right) . S_{A}^{+}\left(d_{i}\right)=S_{A}\left(d_{i}\right) \cap D_{P}^{+}\left(d_{i}\right)$ represents a collection of elements similar to and a little bit better than $d_{i} . S_{A}^{-}\left(d_{i}\right)=S_{A}\left(d_{i}\right) \cap D_{P}^{-}\left(d_{i}\right)$ denotes a collection of elements similar to and a little bit worse than $d_{i}$. $\left.S_{A}^{0}\left(d_{i}\right) £ S_{A}\left(d_{i}\right)-\mathrm{D}_{\mathrm{P}}^{+}\left(d_{i}\right)-\mathrm{D}_{\mathrm{P}}^{-}\left(d_{i}\right)\right) \cup\left\{d_{i}\right\} \quad$ expresses $\quad$ a collection of elements purely similar to $d_{i}$. So $S_{A}\left(d_{i}\right)=S_{A}^{+}\left(d_{i}\right) \cup S_{A}^{-}\left(d_{i}\right) \cup S_{A}^{0}\left(d_{i}\right)$.
In this way, $S L_{A}\left(d_{i}\right)$ may also be divided into three overlapped subclasses: $S L_{A}^{+}\left(d_{i}\right), S L_{A}^{-}\left(d_{i}\right), S L_{A}^{0}\left(d_{i}\right)$, and $S C_{A}\left(d_{i}\right)$ into three overlapped subclasses: $S C_{A}^{+}\left(d_{i}\right), S C_{A}^{-}\left(d_{i}\right)$, $S C_{A}^{0}\left(d_{i}\right)$. We also have:
$S C_{A}\left(d_{i}\right)=S C_{A}^{+}\left(d_{i}\right) \cup S C_{A}^{-}\left(d_{i}\right) \cup S C_{A}^{0}\left(d_{i}\right) ;$
$S L_{A}\left(d_{i}\right)=S L_{A}^{+}\left(d_{i}\right) \cup S L_{A}^{-}\left(d_{i}\right) \cup S L_{A}^{0}\left(d_{i}\right)$.
According to the three subclasses in the decomposition of $S_{A}\left(d_{i}\right)$, we may obtain three groups of upper approximation, lower approximation and boundary subsets for any subset $X \subseteq D$.

For $S U_{A}^{+}\left(d_{i}\right)$, we define

$$
\begin{aligned}
& \underline{S U}_{A}^{+}(X)=\cup S U_{A}^{+}\left(d_{i}\right)\left(S U_{A}^{+}\left(d_{i}\right) \subseteq X\right) ; \\
& \overline{S U}_{A}^{+}(X)=\cup S U_{A}^{+}\left(d_{i}\right)\left(S U_{A}^{+}\left(d_{i}\right) \cap X \neq \varnothing\right) ; \\
& B n d S U_{A}^{+}(X)=\overline{S U}_{A}^{+}(X)-\underline{S U}_{A}^{+}(X) .
\end{aligned}
$$

Similarly, for $S U_{A}^{-}\left(d_{i}\right)$ and $S U_{A}^{0}\left(d_{i}\right)$ we can also define $\underline{S U}_{A}{ }^{-}(X), \quad \overline{S U}_{A}{ }^{-}(X), \quad B n d S U_{A}{ }^{-}(X)$ and $\underline{S U}_{A}{ }^{0}(X)$, $\overline{S U}_{A}{ }^{0}(X), B n d S U_{A}{ }^{0}(X)$. According to [5], we have:

$$
\begin{gathered}
\underline{S U}_{A}(X)=\underline{S U}_{A}^{+}(X) \cap \underline{S U}_{A}^{-}(X) \cap \underline{S U}_{A}^{0}(X) ; \\
\overline{S U}_{A}(X)=\overline{S U}_{A}^{+}(X) \cup \overline{S U}_{A}{ }^{( }(X) \cup \overline{S U}_{A}{ }^{0}(X) .
\end{gathered}
$$

For $S C_{A}\left(d_{i}\right)$ and $S L_{A}\left(d_{i}\right)$, we can also give definitions of the following denotations: $\underline{S C}_{A}^{+}(X), \overline{S C}_{A}^{+}(X), B n d S C_{A}^{+}(X)$, $\underline{S C}_{A}^{-}(X), \overline{S C}_{A}^{-}(X), B n d S C_{A}^{-}(X), S C_{A}^{0}(X), \overline{S C}_{A}^{0}(X)$, $B n d S C_{A}^{0}(X) \quad \underline{S L_{A}^{+}}(X), \overline{S L}_{A}^{+}(X), B n d S L_{A}^{+}(X), \underline{S L}_{A}^{-}(X)$, $\overline{S L}_{A}^{-}(X), \quad B n d S L_{A}^{-}(X), \underline{S L}_{A}^{0}(X), \overline{S L}_{A}^{0}(X), B n d S L_{A}^{0}(X)$. Similarly, it is not difficult to prove the following formula:

$$
\begin{aligned}
& \underline{S C}_{A}(X)=\underline{S C}_{A}^{+}(X) \cap \underline{S C}_{A}^{-}(X) \cap \text { C }_{A}^{0}(X) ; \\
& \overline{S C}_{A}(X)=\overline{S C}_{A}^{+}(X) \cup \overline{S C}_{A}^{-}(X) \cup \overline{S C}_{A}^{0}(X) ; \\
& \underline{S L}_{A}(X)=\underline{S L}_{A}^{+}(X) \cap \underline{S L}_{A}^{-}(X) \cap \underline{S L}_{A}^{0}(X) ; \\
& \overline{S L}_{A}(X)=\overline{S L}_{A}^{+}(X) \cup \overline{S L}_{A}^{-}(X) \cup \overline{S L}_{A}^{0}(X) .
\end{aligned}
$$

## VI. Similarity Expansions and Elaborative Queries Using Decompositions Measurement

A document corpus can be represented by $S=(U, A T, V, f)$, where $U=D$ is the set of documents; each document is an object in $D ; A T=T M \cup\{$ topic $\}$ and $T M$ is a set of total terms occurring in documents; and topic is the decision attribute, i.e., class labels of documents. $V, f$ are defined similarly in a information system. So $S=(D, T M \cup\{$ topic $\}, V, f)$. As an example, let $U=D$ $=\left\{d_{1}, d_{2}, \ldots, d_{13}\right\}$ be a corpus, $T M=\{a, b, c, d\}$, topic $=\varnothing$. The term-documentary matrix are discretized as in Table 1. "*" represents zero or an ambiguity. Integers represent the regularized tf -idf weights or occurrence times.

| $D$ | a | b | c | d | $D$ | a | b | c | d |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| d1 | 12 | 22 | 13 | t | d 8 | 2 | 10 | $*$ | m |
| d 2 | 14 | 22 | $*$ | t | d 9 | $*$ | $*$ | 4 | m |
| $d_{3}$ | 15 | $*$ | 13 | $*$ | $d_{10}$ | 5 | $*$ | 4 | $m$ |
| $d_{4}$ | $*$ | 20 | 16 | $t$ | $d_{11}$ | 6 | 10 | $*$ | $b$ |
| $d_{5}$ | 18 | $*$ | 18 | $s$ | $d_{12}$ | 16 | 22 | $*$ | $h$ |
| d 6 | 18 | 27 | 18 | s | d 13 | 16 | $*$ | 20 | $*$ |
| d 7 | 2 | 10 | 4 | $*$ |  |  |  |  |  |

Attribute $d$ expresses a fuzzy result. Its role is to deliberatively explain how to implement a non-binary discretization and to achieve the target of document
expansions. An associative matrix is gained based on similarity measure. It is essential to find out primitives. But here it is omitted.
table il class $S U_{A}\left(d_{i}\right)$ and its decompositions

| $D$ | $S U_{A}$ | $S U_{A}^{+}$ | $S U_{A}^{-}$ | $S U_{A}^{0}$ |
| :--- | :--- | :--- | :--- | :--- |
| d 1 | $\mathrm{~d} 1-\mathrm{d} 4$ | $\mathrm{~d} 1-\mathrm{d} 3$ | d 1 | $\mathrm{~d} 1, \mathrm{~d} 4$ |
| d 2 | $\mathrm{~d} 1-\mathrm{d} 5, \mathrm{~d} 13$ | $\mathrm{d} 2, \mathrm{~d} 3$, <br> $\mathrm{d} 5, \mathrm{~d} 13$ | $\mathrm{d} 2, \mathrm{~d} 1, \mathrm{~d}$ <br> 4 | d 2 |
| d 3 | $\mathrm{~d} 1-\mathrm{d} 4, \mathrm{~d} 12$ | $\mathrm{d} 3, \mathrm{~d} 4$, <br> d 12 | $\mathrm{~d} 1-\mathrm{d} 3$ | d 3 |
| d 4 | $\mathrm{~d} 1-\mathrm{d} 5, \mathrm{~d} 13$ | $\mathrm{d} 2, \mathrm{~d} 4$, <br> $\mathrm{d} 5, \mathrm{~d} 13$ | $\mathrm{~d} 3, \mathrm{~d} 4$ | $\mathrm{~d} 4, \mathrm{~d} 1$ |
| d 5 | $\mathrm{d} 2, \mathrm{~d} 4-\mathrm{d} 6$, <br> d 13 | d 5 | $\mathrm{d} 2, \mathrm{~d} 4, \mathrm{~d}$ <br> 5 | $\mathrm{~d} 5, \mathrm{~d} 6, \mathrm{~d} 13$ |
| d 6 | $\mathrm{~d} 6, \mathrm{~d} 5, \mathrm{~d} 13$ | d 6 | d 6 | $\mathrm{~d} 5, \mathrm{~d} 6, \mathrm{~d} 13$ |
| d 7 | $\mathrm{~d} 7-\mathrm{d} 11$ | $\mathrm{~d} 7, \mathrm{~d} 10, \mathrm{~d} 11$ | d 7 | $\mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9$ |
| d 8 | $\mathrm{~d} 7-\mathrm{d} 10$ | $\mathrm{~d} 8, \mathrm{~d} 10$ | d 8 | $\mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9$ |
| d 9 | $\mathrm{~d} 7-\mathrm{d} 10$ | d 9 | d 9 | $\mathrm{~d} 7-\mathrm{d} 10$ |
| d 10 | $\mathrm{~d} 7-\mathrm{d} 10$ | $\mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 10$ | $\mathrm{d} 7, \mathrm{~d} 8$, <br> d 10 | $\mathrm{~d} 9, \mathrm{~d} 10$ |
| d 11 | $\mathrm{~d} 7, \mathrm{~d} 11$ | d 11 | $\mathrm{~d} 11, \mathrm{~d} 7$ | d 11 |
| d 12 | $\mathrm{~d} 12, \mathrm{~d} 3, \mathrm{~d} 13$ | d 12 | $\mathrm{~d} 12, \mathrm{~d} 3$ | $\mathrm{~d} 12, \mathrm{~d} 13$ |
| d 13 | $\mathrm{d} 13, \mathrm{~d} 2, \mathrm{~d} 4-\mathrm{d}$ <br> 6,d12 | d 13 | $\mathrm{d} 13, \mathrm{~d} 2$, <br> d 4 | $\mathrm{d} 5, \mathrm{~d} 6$, <br> $\mathrm{d} 12, \mathrm{~d} 13$ |

Similarity measure as in Section 3 is calculated as follows. Let $\quad \lambda_{a}=0.9, \quad \lambda_{b}=0.85, \quad \lambda_{c} \quad=0.8, \quad$ and $q_{d}(" t ", " s ")=1, q_{d}(" b ", " h ")=1$, and $q_{d}(v, v)=1$ for $v$ in $\{" t ", " s ", " m ", " b ", " h "\}$, and $\mathrm{q}_{A}\left(v_{1}, v_{2}\right)=0$ for other pairs $\left(v_{1}, v_{2}\right)$, where $v_{1}, v_{2}$ are all in $\{" t ", " s ", " m ", " b ", " h "\}$. "t" stands for "tiny", "s" for "small", " m " for "medium", "b" for "big" and "h" for "huge". " $t$ " $<" s$ " < " $m$ " $<" b "<" h "$ is supposed to be a dominance relation on Vd. $\lambda_{d}=1 . \Delta_{A}=3.6$ for $\mathrm{A}=\{\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d}\}$. SIMA $=$ $\left\{(x, y) \mid q_{A}(x, y)=1\right\}$.
Three different granules $S U_{A}\left(d_{i}\right), S C_{A}\left(d_{i}\right), S L_{A}\left(d_{i}\right)$ are counted at three levels. By natural dominance relation, three decompositions of granules are shown in Table 2, Table 3 and Table 4 respectively. Table 3 supplies multiple components.

1) Different expansions to a given query

Similar to neighborhood system, three different upper and lower approximations of documents are viewed as different neighborhoods. But they are naturally generated from incomplete information systems, so they are not very theoretical. The topic discussed here can be regarded as an application of neighborhood system. For each $d_{i},\left\{d_{i}\right\} \subseteq$ $S L_{A}\left(d_{i}\right) \subseteq S C_{A}\left(d_{i}\right) \subseteq \quad S U_{A}\left(d_{i}\right) \quad$ and $\quad d_{i} \in H \in$ $\left\{S L_{A}^{+}\left(d_{i}\right), S L_{A}^{-}\left(d_{i}\right), S L_{A}^{0}\left(d_{i}\right), S C_{A}^{+}\left(d_{i}\right), S C_{A}^{-}\left(d_{i}\right)\right.$, $\left.S C_{A}^{0}\left(d_{i}\right), S U_{A}^{+}\left(d_{i}\right), S U_{A}^{-}\left(d_{i}\right), S U_{A}^{0}\left(d_{i}\right)\right\}$.
(1) Use $S L_{A}\left(d_{i}\right), S L_{A}^{+}\left(d_{i}\right), S L_{A}^{-}\left(d_{i}\right), S L_{A}^{0}\left(d_{i}\right)$ to reach expansions. Let the query be: retrieve documents that satisfy: ' $((\mathrm{a}=18) \&(\mathrm{~d}=" \mathrm{~s} "))$ or $((\mathrm{b}=10) \&(\mathrm{~d}=" \mathrm{~m} "))$ '. The initial retrieval result is $\mathrm{X}=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\}$ under inner condition. Then, expansions to the senses of $S L_{A}\left(d_{i}\right), S L_{A}^{+}\left(d_{i}\right)$, $S L_{A}^{-}\left(d_{i}\right), S L_{A}^{0}\left(d_{i}\right)$ are the same:

$$
\begin{aligned}
& \cup_{l=5}^{10} S L_{A}\left(d_{l}\right) \cup S L_{A}\left(d_{13}\right)=\cup_{l=5}^{10} S L_{A}^{+}\left(d_{l}\right) \cup S L_{A}^{+}\left(d_{13}\right) \\
= & \cup_{l=5}^{10} S L_{A}^{-}\left(d_{l}\right) \cup S L_{A}^{-}\left(d_{13}\right)=\bigcup_{l=5}^{10} S L_{A}^{0}\left(d_{l}\right) \cup S L_{A}^{0}\left(d_{13}\right) \\
= & \{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\} .
\end{aligned}
$$

(2) Apply $S C_{A}\left(d_{i}\right)$ or its decompositions $S C_{A}^{+}\left(d_{i}\right), S C_{A}^{-}\left(d_{i}\right), S C_{A}^{0}\left(d_{i}\right)$ to gain expansions to the same query. The initial retrieval result is still $X$. Because many choices exist, expansions can be obtained alternatively. For $S C_{A}\left(d_{5}\right)$, two options denoted by $S C_{A}^{\prime}\left(d_{5}\right)=\{\mathrm{d} 2, \mathrm{~d} 4, \mathrm{~d} 5, \mathrm{~d} 13\}$ and $S C_{A}^{\prime \prime}\left(d_{5}\right)=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 13\}$ respectively. Two alternatives for $d_{7}$ are:

$$
S C_{A}^{\prime}\left(d_{7}\right)=\{\mathrm{d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10\}, S C_{A}^{\prime \prime}\left(d_{7}\right)=\{\mathrm{d} 7, \mathrm{~d} 11\} .
$$

Meanwhile, there are three candidates for d13 to choose for $S C_{A}\left(d_{13}\right):$
$S C_{A}^{\prime}\left(d_{13}\right)=\{\mathrm{d} 2, \mathrm{~d} 4, \mathrm{~d} 5, \mathrm{~d} 13\}, S C_{A}^{\prime \prime}\left(d_{13}\right)=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 13\}$,
$S C_{A}^{\prime \prime \prime}\left(d_{13}\right)=\{\mathrm{d} 12, \mathrm{~d} 13\}$.
So many expansions are available. Here some are given as examples:
$\{\mathrm{d} 2, \mathrm{~d} 4, \mathrm{~d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\} ;$
$\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\} ;$
$\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 12, \mathrm{~d} 13\} ;$
\{d2,d4,d5,d6,d7,d8,d9,d10,d12,d13\};
\{d2,d4,d5,d6,d7,d8,d9,d10,d11,d13\};
$\{\mathrm{d} 2, \mathrm{~d} 4, \mathrm{~d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 11, \mathrm{~d} 12, \mathrm{~d} 13\}$.

Two expansions to the sense of $S C_{A}^{+}\left(d_{i}\right)$ are:

$$
\begin{aligned}
& \cup_{l=5}^{10} S C_{A}^{+}\left(d_{l}\right) \cup S C_{A}^{+}\left(d_{7}\right) \cup S C_{A}^{+}\left(d_{13}\right) \\
= & \{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\} ; \\
& \cup_{l=5}^{10} S C_{A}^{+}\left(d_{l}\right) \cup S C_{A}^{+"}\left(d_{7}\right) \cup S C_{A}^{+}\left(d_{13}\right)
\end{aligned}
$$

$=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 11, \mathrm{~d} 13\}$.
Two expansions to the sense of $S C_{A}^{-}\left(d_{i}\right)$ are:
\{d2,d4,d5, d6, d7,d8,d9,d10,d13\};
\{d5, d6, d7, d8, d9, d10, d13\}.
Two expansions to the sense of $S C_{A}^{0}\left(d_{i}\right)$ are:
\{d5,d6,d7,d8,d9,d10,d13\};
\{d5, d6, d7, d8, d9, d10, d12, d13\}.
These reflect the diversity of expansions.
(3) Make $S U_{A}\left(d_{i}\right)$ and $S U_{A}^{+}\left(d_{i}\right), S U_{A}^{-}\left(d_{i}\right), S U_{A}^{0}\left(d_{i}\right)$ get expansions to the query. $\mathrm{X}=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\}$.

Expansions to the senses of $S U_{A}\left(d_{i}\right), S U_{A}^{+}\left(d_{i}\right)$,
$S U_{A}^{-}\left(d_{i}\right), S U_{A}^{0}\left(d_{i}\right)$ are:
\{d2, d4, d5, d6, d7, d8, d9, d10, d11,d12,d13\},
\{d5, d6, d7, d8, d9, d10, d11,d13\},
\{d2,d4, d5, d6,d7, d8,d9,d10,d13\},
and $\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 12, \mathrm{~d} 13\}$ respectively.
2) Realize Elaborative queries
(1) Retrieve documents that certainly satisfy: ( $(\mathrm{a}=18) \&(\mathrm{~d}=" \mathrm{~s} \mathrm{~s}))$ or $\quad((\mathrm{b}=10) \&(\mathrm{~d}=" \mathrm{~m} "))$ '. Owing to $'((\mathrm{a}=18) \&(\mathrm{~d}=" \mathrm{~s} ")) \operatorname{or}((\mathrm{b}=10) \&(\mathrm{~d}=>\mathrm{m} \mathrm{m}))$, we can first retrieve the information system in Table 1(* is said to match any value).
The initial retrieval result is $X=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\}$. The answer to this query is $\underline{S I M}_{A}(X)=$ $\underline{S I M}_{A}^{+}(X) \cap \underline{S I M}_{A}^{-}(X) \cap \underline{S I M}_{A}^{0}(X)$. Because
$S_{S I M_{A}^{+}}(X)=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 11, \mathrm{~d} 13\}$,
$\underline{S I M}_{A}^{-}(X)=\{\mathrm{d} 2, \mathrm{~d} 4, \mathrm{~d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 11, \mathrm{~d} 12, \mathrm{~d} 13\}$, $\underline{S I M}_{A}^{0}(X)=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\}$,
thus, $\underline{S I M}_{A}(X)=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10\}$.

| $D$ | $S C_{A}$ | $S C_{A}^{+}$ | $S C_{A}^{-}$ | $S C_{A}^{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| d1 | d1-d4 | d1-d3 | d1 | d1,d4 |
| d2 | d1-d4 | d2,d3 | d2,d1,d4 | d2 |
|  | $\begin{aligned} & \mathrm{d} 2, \mathrm{~d} 4, \\ & \mathrm{~d} 5, \mathrm{~d} 13 \end{aligned}$ | $\begin{aligned} & \mathrm{d} 2, \mathrm{~d} 5, \\ & \mathrm{~d} 13 \end{aligned}$ | d2,d4 |  |
| d3 | d1-d4 | d3,d4 | d1-d3 | d3 |
|  | d3,d12 | d3,d12 | d3 |  |
| d4 | d1-d4 | d4,d2 | d3,d4 | d4,d1 |
|  | $\begin{aligned} & \mathrm{d} 2, \mathrm{~d} 4, \\ & \mathrm{~d} 5, \mathrm{~d} 13 \end{aligned}$ | $\begin{aligned} & \mathrm{d} 2, \mathrm{~d} 4, \\ & \mathrm{~d} 5, \mathrm{~d} 13 \end{aligned}$ | d4 | d4 |
| d5 | $\begin{aligned} & \mathrm{d} 2, \mathrm{~d} 4, \\ & \mathrm{~d} 5, \mathrm{~d} 13 \end{aligned}$ | d5 | d5,d2,d4 | d5,d13 |
|  | d5,d6,d13 |  | d5 | $\begin{aligned} & \hline \text { d5,d6, } \\ & \text { d13 } \end{aligned}$ |
| d6 | d5,d6, d13 | d6 | d6 | $\begin{aligned} & \text { d5, d6, } \\ & \text { d13 } \\ & \hline \end{aligned}$ |
| d7 | d7-d10 | d7,d10 | d7 | d7-d9 |
|  | d7,d11 | d7,d11 |  | d7 |
| d8 | d7-d10 | d8,d10 | d8 | d7-d9 |
| d9 | d7-d10 | d9 | d9 | d7-d10 |
| d10 | d7-d10 | d10 | d7,d8, d10 | d9, d10 |
| d11 | d7,d11 | d11 | d7,d11 | d11 |
| d12 | d12,d13 | d12 | d12 | d12,d13 |
|  | d3,d12 |  | d3,d12 | d12 |
| d13 | $\begin{aligned} & \mathrm{d} 2, \mathrm{~d} 4, \\ & \mathrm{~d} 5, \mathrm{~d} 13 \end{aligned}$ | d13 | $\begin{aligned} & \mathrm{d} 2, \mathrm{~d} 4, \\ & \mathrm{~d} 13 \\ & \hline \end{aligned}$ | d5,d13 |
|  | d5,d6,d13 |  | d13 | $\begin{aligned} & \hline \mathrm{d} 5, \mathrm{~d} 6, \\ & \mathrm{~d} 13 \end{aligned}$ |
|  | d12,d13 |  |  | d12,d13 |

(2) Retrieve those documents that certainly satisfy: a little bit better than those that satisfy ‘ $((a=18) \&(d=" \mathrm{~s} "))$ or
$((\mathrm{b}=10) \&(\mathrm{~d}=" \mathrm{~m} "))$ ). According to the inner condition, $\mathrm{X}=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\}$ is directly retrieved from Table 1. However, the query may be answered by applying the above decomposition based on dominance relation. In reality, the answer is $\operatorname{SIM}_{A}^{+}(X)=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 11, \mathrm{~d} 13\}$ in which d 11 is a little bit better than the condition ' $(\mathrm{b}=10) \&(\mathrm{~d}=$ " m ")' and there is no other documents a little bit better than ' $(a=18)$ \& (d=" $\mathrm{s} ")$ ' from Table 1.
(3) Retrieve those documents that probably satisfy: 'not worse but not better much than or just only the same as those satisfying ' $((\mathrm{a}=18) \&(\mathrm{~b}=27) \&(\mathrm{c}=18) \&(\mathrm{~d}=>\mathrm{s}$ ")$)$ '. According to the inner condition given by a Boolean expression, we can get $\mathrm{X}=\{\mathrm{d} 5, \mathrm{~d} 6\}$. The answer to the entire query is to find the union of $\overline{\operatorname{SIM}}_{A}^{+}(X)$ and $\overline{S I M}_{A}^{0}(X)$. i.e., $\overline{S I M}_{A}^{+}(X) \cup \overline{S I M}_{A}^{0}(X)$ $=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 13\}$.
(4) A query is to retrieve all documents mutually similar in the corpus satisfying ' $((\mathrm{a}=18) \&(\mathrm{~d}=>\mathrm{s} "))$ or $((\mathrm{b}=10) \&(\mathrm{~d}=\gg \mathrm{m} ")) \cdot \mathrm{t}$ means that we have to find documents to the sense of $S C_{A}\left(d_{i}\right)$. The initial document subset also is $\mathrm{X}=\{\mathrm{d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 13\}$. Then, the expansion of X to $S C_{A}\left(d_{i}\right)$ is:
\{d5,d6, d7,d8, d9, d10,d13\},
\{d2, d4, d5, d6, d7, d8, d9, d10, d13\},
\{d5,d6,d7,d8,d9,d10,d12,d13\},
\{d2, d4, d5, d6, d7, d8, d9, d10,d12, d13\},
$\{\mathrm{d} 2, \mathrm{~d} 4, \mathrm{~d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 11, \mathrm{~d} 13\}$, or $\{\mathrm{d} 2, \mathrm{~d} 4, \mathrm{~d} 5, \mathrm{~d} 6, \mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9, \mathrm{~d} 10, \mathrm{~d} 11, \mathrm{~d} 12, \mathrm{~d} 13\}$.
table iv Class $S L_{A}\left(d_{i}\right)$ and its three decompositions

| $D$ | $S L_{A}$ | $S L_{A}^{+}$ | $S L_{A}^{-}$ | $S L_{A}^{0}$ |
| :--- | :--- | :--- | :--- | :--- |
| d 1 | $\mathrm{~d} 1-\mathrm{d} 4$ | $\mathrm{~d} 1-\mathrm{d} 3$ | d 1 | $\mathrm{~d} 1, \mathrm{~d} 4$ |
| d 2 | $\mathrm{~d} 1-\mathrm{d} 5, \mathrm{~d} 13$ | d 2 | $\mathrm{~d} 2, \mathrm{~d} 4$ | d 2 |
| d 3 | d 3 | d 3 | d 3 | d 3 |
| d 4 | $\mathrm{~d} 2, \mathrm{~d} 4$ | d 4 | $\mathrm{~d} 2, \mathrm{~d} 4$ | d 4 |
| d 5 | $\mathrm{~d} 5, \mathrm{~d} 13$ | d 5 | d 5 | $\mathrm{~d} 5, \mathrm{~d} 13$ |
| d 6 | $\mathrm{~d} 6, \mathrm{~d} 5, \mathrm{~d} 13$ | d 6 | d 6 | $\mathrm{~d} 5, \mathrm{~d} 6, \mathrm{~d} 13$ |
| d 7 | d 7 | d 7 | d 7 | d 7 |
| d 8 | $\mathrm{~d} 7-\mathrm{d} 10$ | $\mathrm{~d} 8, \mathrm{~d} 10$ | d 8 | $\mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 9$ |
| d 9 | $\mathrm{~d} 7-\mathrm{d} 10$ | d 9 | d 9 | d 79 d 10 |
| d 10 | $\mathrm{~d} 7-\mathrm{d} 10$ | d 10 | $\mathrm{~d} 7, \mathrm{~d} 8, \mathrm{~d} 10$ | $\mathrm{~d} 9, \mathrm{~d} 10$ |
| d 11 | $\mathrm{~d} 7, \mathrm{~d} 11$ | d 11 | $\mathrm{~d} 11, \mathrm{~d} 7$ | d 11 |
| d 12 | d 12 | d 12 | d 12 | d 12 |
| d 13 | d 13 | d 13 | d 13 | d 13 |
|  |  |  |  |  |

If a query contains some special requirements such as a little better or a little worse than the condition, answers to the query must be done with a retrieval to the sense of $S C_{A}^{+}\left(d_{i}\right)$
or $S C_{A}^{-}\left(d_{i}\right)$. Queries using $S C_{A}^{+}\left(d_{i}\right), S C_{A}^{-}\left(d_{i}\right), S C_{A}^{0}\left(d_{i}\right)$ and $S L_{A}\left(d_{i}\right), S L_{A}^{+}\left(d_{i}\right), S L_{A}^{-}\left(d_{i}\right), S L_{A}^{0}\left(d_{i}\right)$ are omitted here. Obviously, These expansions enlarge overcalls and may bring diverse responses.

## VII. Conclusions

This paper introduces granules $S U_{A}\left(d_{i}\right), S C_{A}\left(d_{i}\right)$ and $S L_{A}\left(d_{i}\right)$ whose constructing processes are from the view points of using maximal compatible classes as primitive granules. It extends original rough set model based on these granules. Three different kinds of upper and lower approximation decompositions are put forward so as to make document expansions flexible and to give more precise answers to elaborative queries. Through an example, the powers to finish complicated or elaborative queries are demonstrated. It provides us with an important theoretical principle to solve the problem how tolerance rough set model takes effects in the incomplete information system retrieval. It will be our further research targets to construct a prototype incorporating these new ideas with our existed results[4] and implement other expansions such as term expansions in information retrieval or other areas such as medical or biological text mining to realize knowledge acquisition because the approach proposed here is general and scalable.

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