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Application of Universal Distribution Factors for Real-Time Complex Power Flow Calculation

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***Abstract*—**Complex power flow distribution factors, which relate line complex power flows to the bus injected complex powers, have been widely used in various power system planning and analysis studies. In particular, AC distribution factors have been used extensively in the recent power and energy pricing studies in free electricity market field. As was demonstrated in the existing literature, many of the electricity market related costing studies rely on the use of the distribution factors. These known distribution factors, whether the injection shift factors (ISF's) or power transfer distribution factors (PTDF's), are linear approximations of the first order sensitivities of the active power flows with respect to various variables. This paper presents a novel model for evaluating the universal distribution factors (UDF's), which are appropriate for an extensive range of power systems analysis and free electricity market studies. These distribution factors are used for the calculations of lines complex power flows and its independent of bus power injections, they are compact matrix-form expressions with total flexibility in determining the position on the line at which line flows are measured. The proposed approach was tested on IEEE 9-Bus system. Numerical results demonstrate that the proposed approach is very accurate compared with exact method.

***Keywords*—**Distribution Factors, Power System, Sensitivity Factors, Electricity Market.

I. INTRODUCTION

AC distribution factors play an important role in power systems and have been applied extensively in the recent power and energy pricing studies in free electricity market field. In the open-market energy pricing studies, more flexibility is required in the derivation and use of complex power distribution factors in order to allow market participants to base their calculated complex line flows on a reference (point of calculation) of their market contractual choice.

The AC universal distribution factors (UDF's) was discussed in [1], the universal distribution factors formulation to calculate the line complex power flows was also presented. The author proposed a novel model for evaluating universal distribution factors and he demonstrated the practical calculation of the universal distribution factors as well as their sensitivities with respect to the line voltage profile.

The author of [2] presented the power transfer distribution factors are insensitive to the operating point for fixed topology. He also analyzed a power to current distribution factors that more closely relates to thermal constraints. In [3], a novel approach of applications of sensitivity analysis and power transfer distribution factors has been described for the determination of available transfer capabilities. The authors of [4] proposed an algorithm by utilizing the formulation of power transfer distribution factors and line outage distribution factors. The generation shifts have been calculated so that the power flow on transmission lines that violate security limits because of single or multiple-line outage events are set on its security limits.

A generalized generation distribution factors (GGDF's) was developed and discussed in [5] to replace the generation shift distribution factors, the line flows can be calculated from GGDF's directly without running load flow when total system generation changes. The proposed method for solving the problem of active power transmission loss allocation has been mentioned in [6]. By using sensitivities technique and AC power flows equations, the author calculated the loss factors for generators and loads. A new network sensitivity factor, named Jacobian-based distribution factor (JBDF's) for line complex power flows calculations was proposed in [7]. The JBDF's was tested on IEEE 14-Bus and 30-Bus systems, the results were nearly the same as those using the exact method.

This paper presents a novel model for evaluating the universal distribution factors (UDF's), which are appropriate for an extensive range of power systems analysis and free electricity market studies. The proposed approach was tested on IEEE 9-Bus system and the numerical results were compared with the exact method to demonstrate the effectiveness of the proposed approach.

II. UNIVERSAL DISTRIBUTION FACTORS FORMULATION

A. Problem Formulation

Let n_B be the number of buses in the power network, where $n_B = n_L + n_G$, n_L and n_G are the number of load andgenerator buses, respectively. Also, in the network modelused, n_T the number of transmission branches (lines and transformers). Also, let

V_i : complex voltage at bus i, i= 1,2, … , n_B
V : n-vector of complex voltages $\{V_i\}$
I_i : complex current (injected) at bus i, i= 1,2, .. , n_B
I : n-vector of complex currents $\{I_i\}$
I_{ij} : current in line ij
S_{ij} : complex power of line ij at bus i
S_{ji} : complex power of line ji at bus j

Now;

$$S_{ij}^* = V_i^* I_{ij} \tag{1}$$

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and

$$S_{ji}^* = V_j^* I_{ji} \tag{2}$$

We define the complex power associated with line t joining buses i and j as

$$S_i^* = [\lambda V_i^* + (1-\lambda)V_j^*]I_t = \varphi_t I_t \tag{3}$$

where λ denotes the position on line t at which S_t is evaluated. For example, λ = 1 indicates that S_t is evaluated at bus i, λ = 0 indicates that S_t is evaluated at bus j, λ = 0.5 indicates that S_t is evaluated at the mid-point of line t, that is;

$$S_i^* = [(V_i^* + V_j^*)/2]I_t \tag{4}$$

We now define the diagonal matrix φ as

$$\varphi_t = \text{diagonal } \{\varphi_1, \varphi_2, \ldots, \varphi_{nT}\}$$

then

$$S_T^* = \varphi_T I_T \tag{5}$$

where S_T^* is the n_T-vector of complex conjugate line powers $\{S_t\}$ and I_T the n_T-vector of complex line currents $\{I_T\}$.

Denoting by Y the bus admittance matrix of the network, then

$$V = Y^{-1} I \tag{6}$$

Now, let $A = (n_T \times n_B)$ be the bus incidence matrix (branch-to-node incidence matrix) representing the connectivity pattern between buses and lines. The entries of A are either 0, 1 or -1. Therefore, an element $A_{bt} = 1$ if bus b is feeding a transmission branch t; $A_{bt} = -1$ if bus b is fed from a branch t, otherwise $A_{bt} = 0$ if bus b is not connected to branch t. We note that for practical large-scale networks, the matrix A is extremely sparse.

Then

$$V_T = A\,V \tag{7}$$

hence

$$I_T = Y^P V_T = Y^P A\,(Y^{-1} I) \tag{8}$$

where Y^P is the $(n_T \times n_T)$ primitive admittance matrix in which the diagonal elements represent line self-admittances and off-diagonal elements represent mutual line admittances. In the absence of mutual coupling between lines, the Y^P is a diagonal matrix.

Hence

$$\varphi_T I_T = \varphi_T(Y^P A\,Y^{-1})\,I \tag{9}$$

We now write the n_B-vector $S^*$of complex conjugate bus powers (injected) as

$$S^* = E^* I \tag{10}$$

or

$$I = E^{*-1} S^* \tag{11}$$

where E is a diagonal matrix of bus voltages, that is

$$E = \text{diagonal } \{V_1, V_2, \ldots, V_{nB}\}$$

Therefore

$$S_T^* = \varphi_T I_T = [\varphi_T(Y^P A\,Y^{-1})\,E^{*-1}]S^* = D_F^* S^* \tag{12}$$

The universal distribution factors $(n_T \times n_B)$ matrix D_F relates the n_T-vector of line complex power flows S_T to the n_B-vector of bus injected complex powers S, with n_B and n_T denoting, respectively, number of buses and number of lines in the system, as follows

$$S_T = D_F\,S \tag{13}$$

where the distribution factors matrix DF is given by

$$D_F = \varphi_T^*(Y^{P*} A\,Y^{*-1})E^{-1} \tag{14}$$

B. Universal Distribution Factors Evaluation

In order to investigate the validity of the calculated universal distribution factors and the effectiveness of the calculated universal distribution factors utilizations under bus injected powers variations of the loads and generation buses, the system will be study in a various scenarios including change in loads and generation with different percentages as well transactions between different buses. This variations will applied to the bus-injected powers in load and generation buses and the line complex power flows will be calculate as in (15) in different scenarios.

$$P_{L\,new} = DF\;P_{B\,new} \tag{15}$$

where;

$$P_{L\,new} = P_{L_0} + \Delta P_L \tag{16}$$

and

$$P_{B\,new} = P_{B_0} + \Delta P_B \tag{17}$$

which ΔP_L and ΔP_B represent the change in bus-injected powers and the change in the lines complex power-flow.

The line complex power-flow calculated from universal distribution factors compared with line complex power-flow calculated from Power world simulator by using Newton-Raphson method should have a zero mismatches. The evaluation of the universal distribution factors in a different scenarios including changes in bus injected powers will gives a validity range for the calculated universal distribution factors utilizations and the need of the recalculate this universal distribution factors if there is a mismatches.

III. ILLUSTRATIVE EXAMPLE: THE IEEE 9-BUS SYSTEM

This section presents an illustrative example of the distribution factors applications. The implementation of the proposed approach is applied to the IEEE 9-bus system.

A. Test Case Used in Illustrative Example

The test case used in this paper is IEEE 9-bus system. This base case is built in Power world simulator 17.0 within a given parameter to solve the load flow. The single line diagram for the test case is built in power world simulator as shown in Fig. 1.

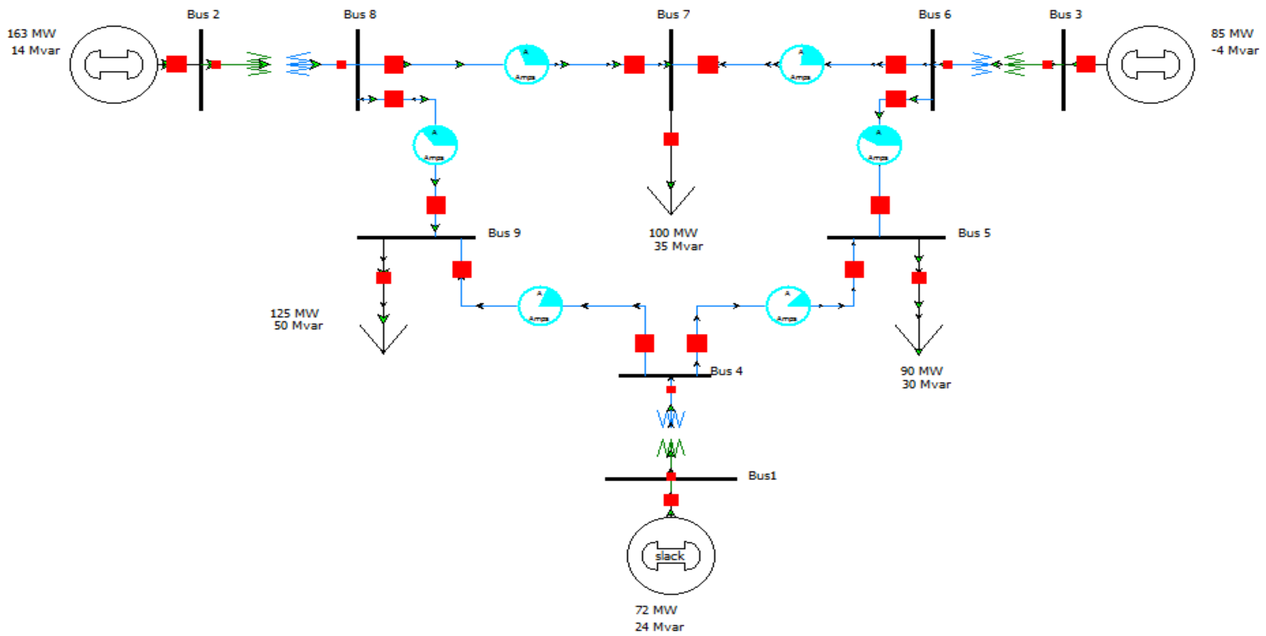


Fig. 1 IEEE 9-Bus test case system

The IEEE 9-Bus system has three generators and three loads. Bus number 1 is slack bus, bus number 2 and 3 are PV buses (generation buses) and bus number 4, 5, 6, 7, 8 and 9 are PQ buses (load buses). The network has nine branches, the branches number 1, 2 and 3 are transformers and branches number 4, 5, 6, 7, 8 and 9 are transmission lines.

B. Algorithm

The determination of the universal distribution factors of the network can be calculated as follows:

i) Compute the required data from the system (base case) such as, bus admittance matrix, primitive admittance matrix, bus incidence matrix, complex bus voltages, φ matrix and bus-injected powers.

ii) Determine the universal distribution factors and calculate lines complex power flow.

iii) Evaluate the universal distribution factors by subtracting the lines complex power flow obtained from universal distribution factors from load flow results and the mismatches should be zero.

iv) Finally evaluate the universal distribution factors by comparing the lines complex power flow obtained from universal distribution factors and those obtained from load flow calculations in two scenarios; first when applying changes in bus-injected powers at different buses, second when assuming bilateral transactions.

C. Results and Discussion

For the same network and injected bus powers S, the universal distribution factors $(n_T \times n_B)$ matrix D_F and the associated n_T vector of line complex power flows S_T are calculated at three different values of parameter λ = 0 (S_T is evaluated based on voltages at the receiving bus j), λ = 0.5 (S_T is evaluated based on voltages at the mid-point of line t) and λ = 1 (St is evaluated based on voltages at the sending bus i).

The universal distribution factor at λ = 1 where the line complex power flows S_T evaluated based on voltages at the sending bus i is

$$\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.4376-j0.0203 & 0.0776-j0.0182 & -0.1744+j0.0331 & 0.4438-j0.0019 & -0.4361-j0.0146 & -0.1752+j0.0244 & -0.0276+j0.0052 & 0.0794-j0.0102 & 0.3333+j0.0166 \\
0.4407-j0.0175 & -0.1963+0.0403 & 0.0537-j0.0216 & 0.4469+j0.0009 & 0.3191+j0.0090 & 0.0545-j0.0188 & -0.0959-j0.0022 & -0.2002+j0.0199 & -0.4520-j0.0225 \\
-0.2567-j0.0031 & 0.1201-j0.0088 & 0.3751-j0.0242 & -0.2597-j0.0140 & -0.4005-j0.0376 & 0.3745-j0.0058 & 0.2316+j0.0073 & 0.1208+j0.0035 & -0.1340-j0.0155 \\
0.0416-j0.0106 & -0.33003+j0.0328 & 0.4344-j0.0175 & 0.0425-j0.0090 & 0.1862+j0.0092 & 0.4332+j0.0037 & -0.4396-j0.0157 & -0.3329-j0.0012 & -0.0903-j0.0158 \\
0.0968+j0.0146 & 0.4562-j0.0404 & -0.3006-j0.0036 & 0.0973+j0.0189 & -0.0437+j0.0003 & -0.2991-j0.0182 & -0.4483-j0.0246 & 0.4597+j0.0066 & 0.2314+j0.0348 \\
-0.2719-j0.0234 & 0.3789-j0.0404 & 0.1284+j0.0103 & -0.2742-j0.0352 & -0.1364-j0.0223 & 0.1273+j0.0165 & 0.2812+j0.0202 & 0.3825-j0.0012 & -0.4090-j0.0568 \\
0.0531-j0.0017 & 0.0586-j0.0108 & 0.0595-j0.0056 & 0.0539+j0.0005 & 0.0577+j0.027 & 0.0595-j0.0027 & 0.0610-j0.0014 & 0.0596-j0.0048 & 0.0585+j0.0029 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0592-j0.0019 & 0.0652-j0.0121 & 0.0663-j0.0063 & 0.0600+j0.0005 & 0.0643+j0.0030 & 0.0663-j0.0030 & 0.0679-j0.0016 & 0.0664-j0.0053 & 0.0652+j0.0032 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.1375+j0.0087 & 0.1341-j0.0152 & 0.1231-j0.0047 & 0.1388+j0.0147 & 0.137+j0.0181 & 0.1228+j0.0013 & 0.1329+j0.0054 & 0.1355-j0.0014 & 0.1434+j0.0200 \\
0.0802+j0.0051 & 0.0783-j0.0089 & 0.0719-j0.0027 & 0.0810+j0.0086 & 0.0799+j0.0106 & 0.0717+j0.0007 & 0.0776+j0.0031 & 0.0791-j0.0008 & 0.0837+j0.0116 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.0564+j0.0052 & 0.0506-j0.0047 & 0.0557+j0.0002 & 0.0569+j0.0077 & 0.0579+j0.0096 & 0.0555+j0.0029 & 0.0539+j0.0037 & 0.0510+j0.0004 & 0.0571+j0.0096 \\
0.1160+j0.0108 & 0.10403-j0.0097 & 0.1145+j0.0005 & 0.1169+j0.0159 & 0.1190+j0.0197 & 0.1140+j0.0060 & 0.1108+j0.0078 & 0.1048+j0.0010 & 0.1172+j0.0197 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

The universal distribution factor at λ = 0.5 where the line complex power flows S_T evaluated based on voltages at the mid-point of line t is

$$\begin{pmatrix}
0.9930-j0.0207 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.9954-j0.0511 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.0011-j0.0245 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.4346-j0.0262 & 0.0769-j0.0192 & -0.1729+j0.0353 & 0.4411-j0.0081 & -0.4336-j0.0084 & -01738+j0.0268 & -0.0274+j0.0056 & 0.0787-j0.0112 & 0.3315+j0.0119 \\
0.4338-j0.0246 & -0.1927+j0.0429 & 0.0526-j0.0221 & 0.4401-j0.0064 & 0.3144+j0.0036 & 0.0534-j0.0195 & -0.0945-j0.0006 & -0.1968+j0.0229 & -0.4455-j0.0147 \\
-0.2526+j0.0098 & 0.1177-j0.0147 & 0.3676-j0.0427 & -0.2561-j0.0007 & -0.3958-j0.0167 & 0.3680-j0.0246 & 0.2282-j0.0044 & 0.1190-j0.0026 & -0.1325-j0.0084 \\
0.0411-j0.0110 & -0.3266+j0.0362 & 0.4302-j0.0222 & 0.0420-j0.0094 & 0.1846+j0.0070 & 0.4293-j0.0011 & -0.4358-j0.0106 & -0.3299+j0.0024 & -0.0897-j0.0146 \\
0.0966+j0.0119 & 0.4523-j0.0526 & -0.2989+j0.0045 & 0.0973+j0.0162 & -0.0435+j0.0015 & -0.2978-j0.0100 & -0.4463-j0.0123 & 0.4571-j0.0059 & 0.2309+j0.0282 \\
-0.2669-j0.0043 & 0.3670-j0.0652 & 0.1259+j0.0013 & -0.2700-j0.0157 & -0.1346-j0.0124 & 0.1253+j0.0074 & 0.2758+j0.0005 & 0.3732-j0.0272 & -0.4029-j0.0276 \\
0.0265-j0.0008 & 0.0293-j0.0054 & 0.0297-j0.0028 & 0.0269+j0.0002 & 0.0288+j0.0013 & 0.0297-j0.0013 & 0.0305-j0.0007 & 0.0298-j0.0024 & 0.0292+j0.0014 \\
-0.0278+j0.0014 & -0.0290+j0.0062 & -0.0286+j0.0036 & -0.0282+j0.0002 & -0.0270-j0.0002 & -0.0287+j0.0021 & -0.0300+j0.0015 & -0.0296+j0.032 & -0.0301-j0.0008 \\
0.0296-j0.0009 & 0.0326-j0.0060 & 0.0331-j0.0031 & 0.0300+j0.0002 & 0.0321+j0.0015 & 0.0331-j0.0015 & 0.0339-j0.0008 & 0.0332-j0.0026 & 0.0326+j0.0016 \\
-0.0302+j0.0018 & -0.0307+j0.0068 & -0.0322+j0.0042 & -0.0307+j0.0005 & -0.0324-j0.0005 & -0.0323+j0.0026 & -0.0325+j0.0019 & -0.0313+j0.0036 & -0.0300-j0.0002 \\
-0.0630+j0.0032 & -0.0659+j0.0140 & -0.0650+j0.0081 & -0.0639+j0.0005 & -0.0611-j0.0005 & -0.0651+j0.0049 & -0.0680+j0.0035 & -0.0672+j0.0072 & -0.0683-j0.0019 \\
0.0687+j0.0043 & 0.0670-j.0076 & 0.0615-j0.0023 & 0.0694+j0.0073 & 0.0685+j0.0090 & 0.0614+j0.0006 & 0.0664+j0.0027 & 0.0677-j0.0007 & 0.0717+j0.0100 \\
0.0401+j0.0025 & 0.0391-j0.0044 & 0.0359-j0.0013 & 0.0405+j0.0043 & 0.0399+j0.0053 & 0.0358+j0.0003 & 0.0388+j0.0015 & 0.0395-j0.0004 & 0.0418+j0.0058 \\
-0.0397-j0.0015 & -0.0366+j0.0052 & -0.0375+j0.0020 & -0.0401-j0.0032 & -0.0403-j0.0044 & -0.0374+j0.0001 & -0.0358-j0.0003 & -0.0371+j0.0014 & -0.0407-j0.0046 \\
-0.0283-j0.0011 & -0.0261-j0.0037 & -0.0267+j0.0014 & -0.0286-j0.0023 & -0.0287-j0.0031 & -0.0267+j0.0001 & -0.0255-j0.0002 & -0.0264+j0.0010 & -0.0290-j0.0033 \\
0.0282+j0.0026 & 0.0253-j0.0023 & 0.0278+j0.0001 & 0.0284+j0.0038 & 0.0289+j0.0048 & 0.0277+j0.0014 & 0.0269+j0.0018 & 0.0255+j0.0002 & 0.0285+j0.0048 \\
0.0580+j0.0054 & 0.0520-j0.0048 & 0.0572+j0.0002 & 0.0584+j0.0079 & 0.0595+j0.0098 & 0.0570+j0.0030 & 0.0554+j0.0039 & 0.0524+j0.0005 & 0.0586+j0.0098 \\
-0.0526+j0.0031 & -0.0534+j0.0119 & -0.0560+j0.0073 & -0.0534+j0.0009 & -0.0563-j0.0009 & -0.0561+j0.0045 & -0.0566+j0.0034 & -0.0545+j0.0064 & -0.0522-j0.0004
\end{pmatrix}$$

The universal distribution factor at λ = 0 where the line complex power flows S_T evaluated based on voltages at the receiving bus j is

$$\begin{pmatrix}
0.9861-j0.0414 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.9909-j0.1023 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.0022-j0.0492 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.4317-j0.0321 & 0.0762-j0.0201 & -0.1714+j0.0375 & 0.4384-j0.0142 & -0.4312-j0.0024 & -0.1724+j0.0290 & -0.0272+j0.0060 & 0.0782-j0.0123 & 0.3298+j0.0072 \\
0.4268-j0.0316 & -0.1890+j0.0455 & 0.0514-j0.0227 & 0.4334-j0.0139 & 0.3097-j0.0017 & 0.0523-j0.0201 & -0.0931+j0.0010 & -0.1935+j0.0260 & -0.4390-j0.0069 \\
-0.2485+j0.0228 & 0.1152-j0.0207 & 0.3602-j0.0613 & -0.2526+j0.0126 & -0.3911+j0.0040 & 0.3616-j0.0434 & 0.2247-j0.0163 & 0.1172-j0.0088 & -0.1312-j0.0015 \\
0.0406-j0.0114 & -0.3233+j0.0396 & 0.4262-j0.0270 & 0.0416-j0.0098 & 0.1831+j0.0048 & 0.4255-j0.0061 & -0.4321-j0.0056 & -0.3269+j0.0063 & -0.0891-j0.0135 \\
0.0964+j0.0092 & 0.4485-j0.0648 & -0.2972+j0.0128 & 0.0972+j0.0135 & -0.0432+j0.0028 & -0.2965-j0.0018 & -0.4443-j0.0000 & 0.4545-j0.0185 & 0.2305+j0.0218 \\
-0.2619+j0.0147 & 0.3551-j0.0900 & 0.1236-j0.0076 & -0.2658+j0.0038 & -0.1328-j0.0027 & 0.1234-j0.0016 & 0.2704-j0.0190 & 0.3638-j0.0533 & -0.3970+j0.0016 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0556+j0.0028 & -0.0581+j0.0124 & -0.0574+j0.0072 & -0.0565+j0.0005 & -0.0540-j0.0005 & -0.0575+j0.0044 & -0.0600+j0.0031 & -0.0594+j0.0064 & -0.0603-j0.0017 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0605+j0.0036 & -0.0614+j0.0137 & -0.0645+j0.0084 & -0.0615+j0.0011 & -0.0648-j0.0011 & -0.0646+j0.0053 & -0.0652+j0.0039 & -0.0628+j0.0074 & -0.0601-j0.0005 \\
-0.1261+j0.0064 & -0.1318+j0.0281 & -0.1300+j0.0163 & -0.1280+j0.0011 & -0.1224-j0.0011 & -0.1302+j0.0099 & -0.1360+j0.0071 & -0.1345+j0.0145 & -0.1367-j0.0039 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.0794-j0.0031 & -0.0732+j0.0105 & -0.0751+j0.0041 & -0.0802-j0.0066 & -0.0807-j0.0089 & -0.0749+j0.0004 & -0.0717-j0.0008 & -0.0742+J0.0030 & -0.0816-j0.0093 \\
-0.0566-j0.0022 & -0.0522+j0.0075 & -0.0535+j0.0029 & -0.0572-j0.0047 & -0.0575-j0.063 & -0.0534+j0.0003 & -0.0511-j0.0005 & -0.0529+j0.0021 & -0.0582-j0.0067 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.1053+j0.0063 & -0.1068+j0.0238 & -0.1122+j0.0147 & -0.1069+j0.0020 & -0.1127-j0.0019 & -0.1124+j0.0091 & -0.1133+j0.0068 & -0.1091+j0.0128 & -0.1045-j0.0009
\end{pmatrix}$$

The line complex power flow S_T calculated from the universal distribution factors at three different values of parameter λ = 1, λ = 0.5 and λ = 0 is shown in Table I.

TABLE I
LINE COMPLEX POWER-FLOW S_T CALCULATED FROM THE UNIVERSAL DISTRIBUTION FACTORS

Branch No.	S_T at λ = 1	S_T at λ = 0.5	S_T at λ = 0
1	71.955+j24.069	71.955+j22.413	71.954+j20.756
2	163+j14.46	163+j6.053	163.01-j2.352
3	85-j3.649	85.005-j5.742	85.01-j7.835
4	30.731-j0.597	30.643+j6.539	30.554+j13.677
5	41.223+j21.354	41.091+j28.539	40.96+j35.724
6	60.903-j12.4	60.174+j1.959	59.446+j16.319
7	24.107+j4.555	24.095+j14.483	24.01+j24.41
8	76.503+j0.207	76.247+j5.395	75.99+j10.583
9	86.503-j2.557	85.272+j5.858	84.04+j14.273

To evaluate the universal distribution factors we compare the results of the line complex power flow S_T calculated from universal distribution factors (UDF's) at two different values of parameter λ = 1 and λ = 0 and from load flow results by using Power World Simulator(PWS) as shown in Table II. The mismatches in MVA and percentages error between UDF's and PWS are also calculated. Fig. 2 shows that the mismatches are zero.

TABLE II
THE LINE COMPLEX POWER FLOWS S_T CALCULATED AT Λ = 1 AND Λ = 0 FROM UDF'S AND PWS

Branch No.	Line Complex Power Flow S_T at λ = 1		Line Complex Power Flow S_T at λ = 0	
	From UDF (MVA)	From PWS (MVA)	From UDF (MVA)	From PWS (MVA)
1	$75.874 \angle 18.49$	$75.874 \angle 18.49$	$74.888 \angle 16.09$	$74.888 \angle 16.09$
2	$163.64 \angle 5.06$	$163.64 \angle 5.06$	$163.02 \angle -0.8$	$163.02 \angle -0.8$
3	$85.078 \angle -2.45$	$85.078 \angle -2.45$	$85.37 \angle -5.26$	$85.37 \angle -5.30$
4	$30.73 \angle -1.11$	$30.73 \angle -1.09$	$33.48 \angle 24.12$	$33.48 \angle 24.13$
5	$46.42 \angle 27.38$	$46.42 \angle 27.37$	$54.35 \angle 41.09$	$54.35 \angle 41.09$
6	$62.15 \angle -11.51$	$62.15 \angle -11.53$	$61.64 \angle 15.35$	$61.64 \angle 15.34$
7	$24.53 \angle 10.70$	$24.53 \angle 10.66$	$34.23 \angle 45.47$	$34.23 \angle 45.46$
8	$76.50 \angle 0.16$	$76.50 \angle 0.19$	$76.72 \angle 7.93$	$76.72 \angle 7.94$
9	$86.54 \angle -1.69$	$86.54 \angle -1.68$	$85.24 \angle 9.64$	$85.24 \angle 9.64$

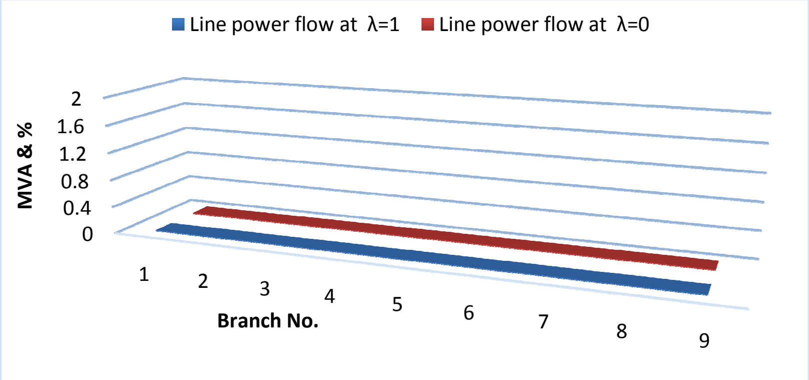


Fig. 2 Mismatch and percentage error of line complex power flows from UDF compared with PWS

D. UDF's Evaluation by Change the System Loads and Generation at Single Bus

To evaluate the calculated universal distribution factors, we have simulated various scenarios. These scenarios are illustrated in Table III with changes the system demand at bus no. 9.

TABLE III
ILLUSTRATE THE DIFFERENT SCENARIOS

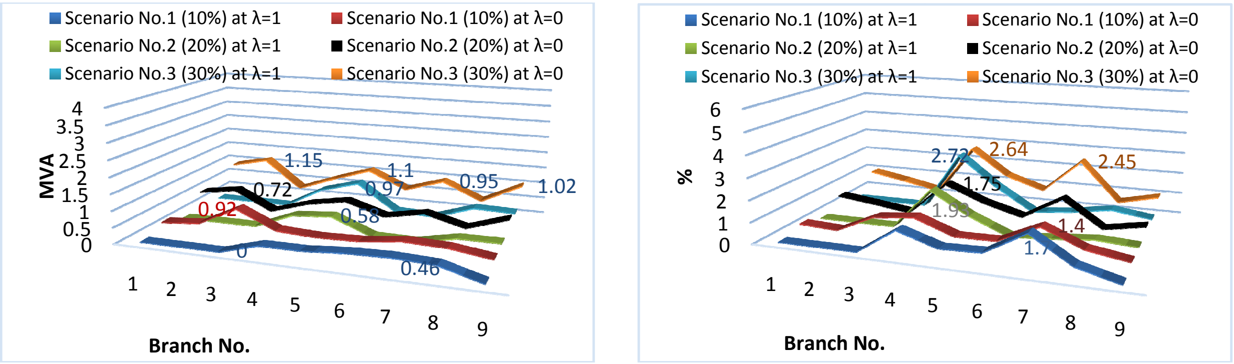
Scenarios	Description of the Scenarios
Scenario 1	System Demand increased by 10% at Bus No. 9
Scenario 2	System Demand increased by 20% at Bus No. 9
Scenario 3	System Demand increased by 30% at Bus No. 9

The numerical results of lines complex power flows (S_T) are calculated from universal distribution factors (UDF's) and load flow results by using Power World Simulator (PWS) at λ = 1 and λ = 0 at scenario 1 (system demand increased by 10% at bus no. 3), scenario 2 (system demand increased by 20% at bus no. 3) and scenario 3 (system demand increased by 30% at bus no. 3) as shown in Table IV.

TABLE IV
S_T AT Λ = 1 AND Λ = 0 AT SCENARIO 1 (10%), SCENARIO 2 (20%) AND SCENARIO 3 (30%) CALCULATED FROM UDF'S AND PWS

Branch No.	Method	S_T at λ = 1 and λ = 0 at Scenario 1 (10%), Scenario 2 (20%) and Scenario 3 (30%) calculated from UDF's and PWS					
		Scenario 1 (10%)		Scenario 2 (20%)		Scenario 3 (30%)	
		at λ = 1	at λ = 0	at λ = 1	at λ = 0	at λ = 1	at λ = 0
1	PWS	82.060 ∠21.086	80.75 ∠18.52	88.505 ∠23.466	86.807 ∠20.733	95.278 ∠25.648	93.134 ∠22.747
	UDF	82.061 ∠21.086	80.99 ∠18.68	88.505 ∠23.466	87.355 ∠21.062	95.278 ∠25.648	94.04 ∠23.245
2	PWS	168.218 ∠6.202	167.234 ∠0.167	172.913 ∠7.399	171.530 ∠1.136	177.693 ∠4.480	175.869 ∠2.108
	UDF	168.22 ∠6.202	167.58 ∠0.305	172.9 ∠7.339	172.25 ∠1.442	177.69 ∠8.48	177.02 ∠2.583
3	PWS	89.255 ∠ − 1.258	89.479 ∠ − 4.248	93.496∠ − 0.094	93.645∠ − 3.230	97.766 ∠1.038	97.825 ∠ − 2.244
	UDF	88.255 ∠ − 1.272	88.558 ∠ − 4.081	93.483 ∠ − 0.094	93.804 ∠ − 2.91	97.766 ∠1.038	98.102 ∠ − 1.77
4	PWS	28.251 ∠ − 1.322	31.25 ∠25.95	25.750 ∠ − 1.593	29.020 ∠28.037	23.47 ∠ − 1.93	26.85 ∠30.418
	UDF	28.593 ∠0.075	31.64 ∠26.17	26.247 ∠1.88	29.53 ∠29.18	24.11 ∠4.30	27.56 ∠32.40
5	PWS	55.014 ∠28.55	62.218 ∠39.58	63.742 ∠29.56	70.148 ∠38.419	72.646 ∠30.432	78.150 ∠37.485
	UDF	54.67 ∠28.29	61.88 ∠39.59	63.156 ∠28.91	69.42 ∠38.45	71.67 ∠29.51	77.05 ∠37.55
6	PWS	64.557 ∠ − 10.459	64.023 ∠14.777	67.01 ∠ − 9.412	66.431 ∠14.25	69.458 ∠ − 8.396	68.836 ∠13.791
	UDF	64.12 ∠ − 10.37	63.66 ∠14.593	67.11 ∠ − 9.14	66.08 ∠13.65	69.63 ∠ − 8.00	68.26 ∠12.79
7	PWS	26.248 ∠11.183	35.67 ∠44.048	27.976 ∠11.718	37.156 ∠42.776	29.714 ∠12.263	38.670 ∠41.634
	UDF	25.78 ∠11.68	35.17 ∠44.20	28.056 ∠12.15	36.57 ∠42.18	29.84 ∠12.86	37.72 ∠40.62
8	PWS	74.844 ∠ − 0.202	75.054 ∠7.808	73.198 ∠ − 0.630	73.380 ∠7.65	71.558 ∠ − 1.093	71.71 ∠7.746
	UDF	75.26 ∠0.307	75.52 ∠7.96	73.53 ∠0.545	73.64 ∠8.14	72.08 ∠0.89	72.13 ∠8.31
9	PWS	92.393 ∠0.466	90.852 ∠9.74	98.393 ∠2.449	96.446 ∠9.796	104.499 ∠4.30	101.99 ∠9.948
	UDF	92.33 ∠0.305	91.15 ∠9.83	98.728 ∠2.113	97.106 ∠9.97	104.99 ∠3.75	103.01 ∠10.08

In scenario No. 1, the mismatches and percentages error of UDF's compared with load flow results when the system demand increased by 10% at bus no. 9 are shown in Fig. 3 (a), The maximum complex power flows mismatch of the UDF's at λ = 1 and λ = 0 was 0.92 MVA at λ = 0 in line no. 3, and the maximum percentage error was 1.7% at λ = 1 in line no. 7 as shown in Fig. 3 (b).



(a) (b)

Fig. 3 (a) Mismatch and (b) percentage error of line complex power flow (MVA) from UDF compared with PWS

In scenario No. 2, the mismatches and percentages error of UDF's compared with load flow results when the system demand increased by 20% at bus no. 9 are shown in Fig. 3 (a). The maximum complex power flows mismatch of the UDF's at λ = 1and λ = 0 was 0.72 MVA at λ = 0 in line no. 5, and the maximum percentage error was 1.93% at λ = 1 in line no. 4 as shown in Fig. 3 (b).

In scenario No. 3, the mismatches and percentages error of UDF's compared with load flow results when the system demand increased by 30% at bus no. 3 are shown in Fig. 3 (a). The maximum complex power flows mismatch of the UDF's at λ = 1and λ = 0 was 1.15 MVA at λ = 0 in line no. 2, and the maximum percentage error was 2.72% at λ = 1 in line no. 4 as shown in Fig. 3 (b).

E. UDF's Evaluation by Apply Bilateral Transaction between Different Buses

To evaluate the calculated universal distribution factors, we have simulated various bilateral transactions. These transactions are illustrated in Table V with changes transaction amount in the source bus (seller) and sink bus (buyer).

TABLE V
BILATERAL TRANSACTIONS

Transaction No.	Source Bus (Seller)	Sink Bus (Buyer)	Transaction Amount (MW)
T1	3	9	20
T2	2	7	20

The numerical results of lines complex power flows (S_T) are calculated from universal distribution factors (UDF's) and load flow results by using Power World Simulator (PWS) at λ = 1 and λ = 0 at transaction 1 T1 (system demand increased by 20 MW at bus no. 9 and system generation increased by 20 MW at bus no. 9) and transaction 2 T2 (system demand increased by 20 MW at bus no. 7 and system generation increased by 20 MW at bus no. 2) as shown in Table VI.

TABLE VI
S_T AT Λ = 1 AND Λ = 0 AT TRANSACTION NO.1 AND 2 CALCULATED FROM UDF'S AND PWS

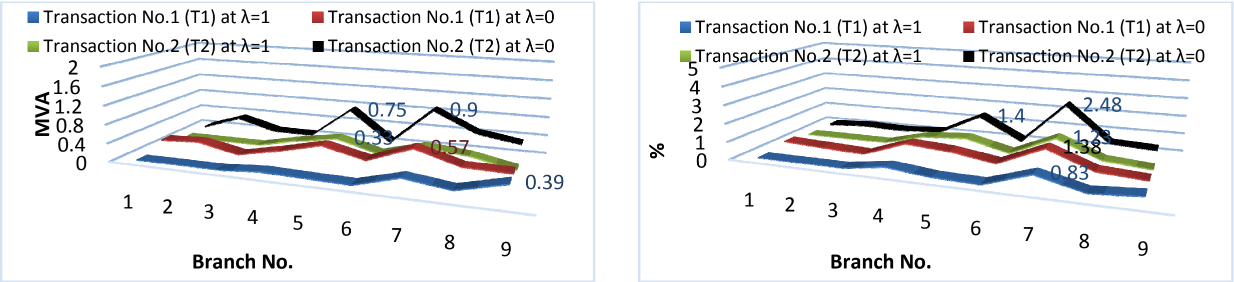
Branch No.	Method	S_T at λ = 1 and λ = 0 at Transaction No.1 and 2 calculated from UDF's and PWS			
		Transaction 1 (T1)		Transaction 2 (T2)	
		λ= 1	λ= 0	λ= 1	λ= 0
1	PWS	$78.437 \angle 21.316$	$77.220 \angle 18.866$	$76.427 \angle 18.981$	$75.40 \angle 16.562$
	UDF	$78.437 \angle 21.316$	$77.418 \angle 18.912$	$76.427 \angle 18.981$	$75.435 \angle 16.577$
2	PWS	$163.846 \angle 5.824$	$163.00 \angle -0.053$	$184.020 \angle 6.036$	$183.009 \angle -0.568$
	UDF	$163.85 \angle 5.824$	$163.23 \angle -0.072$	$184.02 \angle 6.035$	$183.33 \angle 0.139$
3	PWS	$105.014 \angle -0.93$	$105.318 \angle -4.451$	$85.021 \angle -1.272$	$85.220 \angle -4.120$
	UDF	$105.01 \angle -0.93$	$105.37 \angle -3.741$	$85.021 \angle -1.272$	$85.313 \angle -4.079$
4	PWS	$21.417 \angle 3.071$	$26.497 \angle 36.528$	$32.880 \angle -1.453$	$35.276 \angle 22.148$
	UDF	$21.34 \angle 4.75$	$26.265 \angle 36.656$	$33.074 \angle -0.736$	$35.362 \angle 22.097$
5	PWS	$56.911 \angle 24.745$	$63.292 \angle 35.850$	$45.288 \angle 29.538$	$53.694 \angle 43.195$
	UDF	$56.969 \angle 24.17$	$62.846 \angle 35.639$	$44.95 \angle 29.225$	$52.938 \angle 42.613$
6	PWS	$71.705 \angle -9.810$	$70.166 \angle 11.70$	$59.976 \angle -11.951$	$59.711 \angle 16.242$
	UDF	$71.67 \angle -9.71$	$70.4 \angle 11.733$	$59.882 \angle -10.695$	$59.621 \angle 16.261$
7	PWS	$34.581 \angle 6.716$	$41.362 \angle 34.276$	$27.067 \angle 13.451$	$36.552 \angle 44.669$
	UDF	$34.87 \angle 8.56$	$41.936 \angle 34.879$	$26.733 \angle 10.855$	$35.645 \angle 43.02$
8	PWS	$66.203 \angle 0.294$	$66.854 \angle 10.084$	$94.572 \angle 0.662$	$94.231 \angle 5.537$
	UDF	$66.04 \angle 0.493$	$66.546 \angle 9.473$	$94.82 \angle 1.149$	$94.705 \angle 6.554$
9	PWS	$96.799 \angle -0.291$	$94.587 \angle 7.858$	$88.482 \angle -1.883$	$86.871 \angle 8.771$
	UDF	$97.19 \angle -0.45$	$94.872 \angle 8.106$	$88.538 \angle -0.941$	$87.197 \angle 9.342$

In transaction No. 1 (T1), the mismatches and percentages error of UDF's compared with actual flow from load flow calculations when the system demand increased by 20 MW at bus no. 9 and system generation increased by 20 MW at bus no. 3 are shown in Fig. 4 (a). The maximum complex power flows mismatch of the UDF's at λ = 1and λ = 0 was 0.57 MVA at λ = 0 in line no. 7, and the corresponding maximum percentage error was 1.38% as shown in Fig. 4 (b).

In transaction No.2 (T2), the mismatches and percentages error of UDF's compared with actual flow from load flow calculations when the system demand increased by 20 MW at bus no. 7 and system generation increased by 20 MW at bus no. 2 are shown in Fig. 4 (a). The maximum complex power flows mismatch of the UDF's at λ = 1and λ = 0 was 0.9 MVA at λ = 0 in line no. 7, and the corresponding maximum percentage error was 2.48% as shown in Fig. 4 (b).

IV. DISCUSSION

Summing up all the numerical results, the computation mismatches and percentages error of the UDF's compared with actual load flow outputs were zero while they were small in all other scenarios including bilateral transaction scenario, the relation between the line complex power flows calculated from UDF' with the change in the bus-injected powers are linear relation.



(a) (b)

Fig. 4 (a) Mismatch and (b) percentage error of line complex power flow (MVA) from UDF compared with PWS

Although the percentage error in some lines was higher than the percentage error in others lines that have higher mismatch, this was caused by low line complex power flow and the error were amplified because of small deviations between the simulations results of load flow and UDF's approach divided by the results of load flow. For instance, in Section III *D*, the maximum percentage error of the scenario no. 6 was 2.72% at λ = 1 in line no. 4 and its corresponding mismatch was only 0.64 MVA. Consequently, it is doubtless that the mismatches of the UDF's are very small, and they are would not affect the applications of the proposed UDF's approach. We conclude that the accuracy degree of the UDF's is very high.

V. CONCLUSIONS

In this paper, a novel model for evaluating universal distribution factors was evaluated with several scenarios applied on IEEE-9-bus system. The model appropriate for an extensive range of power systems analysis and free electricity market studies. These universal distribution factors (UDF's), which incorporate the exact AC power flow model, are similar, in function to the known distribution factors in the sense that they relate line complex power flows to the bus injections complex power. However, the universal distribution factors are compact matrix-form expressions with total flexibility in determining the position on the line at which line flows are measured.

In real-time applications, it is worth noting that greater errors in lines flows calculations occurs for large changes in system load demand and generation, if the degree of error is unacceptable, these calculated UDF's must recalculated and executed again to ensure an acceptable solution. In this paper, we simulated different changes in the system demands and generation and we applied transactions on IEEE 9-Bus system. The degree of error was acceptable. Using this new method UDF's, line complex power flows can easily be calculated, reflecting changes in bus complex power injection into the line flows. As shown by the numerical results, the complex power flows calculated by the proposed approach is nearly the same as these using the exact method, the proposed approach demonstrate a high degree of accuracy. More testing on large real systems are to be conducted to check how far UDF's can be used with compromising the accuracy of the results.

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