

# Application of Strong Optical Feedback to Enhance the Modulation Bandwidth of Semiconductor Lasers to the Millimeter-Wave Band

Moustafa Ahmed, Ahmed Bakry, Fumio Koyama

**Abstract**—We report on the use of strong external optical feedback to enhance the modulation response of semiconductor lasers over a frequency passband around modulation frequencies higher than 60 GHz. We show that this modulation enhancement is a type of photon-photon resonance (PPR) of oscillating modes in the external cavity formed between the laser and the external reflector. The study is based on a time-delay rate equation model that takes into account both the strong feedback and multiple reflections in the external cavity. We examine the harmonic and intermodulation distortions associated with single and two-tone modulations in the mm-wave band of the resonant modulation. We show that compared with solitary lasers modulated around the carrier-photon resonance frequency, the present mm-wave modulated signal has lower distortions.

**Keywords**—Distortion, intensity modulation, optical feedback, semiconductor laser.

## I. INTRODUCTION

SEMICONDUCTOR lasers are externally subjected to an amount of optical feedback (OFB) in most of their applications. The OFB is generated due to reflection of laser radiation by an external reflector and re-injection into the laser cavity. Due to this OFB, both the threshold and phase conditions of the semiconductor laser change, depending on the strength and phase of the injected light, which then induces various behaviors in the laser dynamics [1]. It has been observed that even a small amount of OFB can affect the laser behavior [2]. Although the OFB may cause strong instabilities in the laser operation in forms of chaos [3], coherence collapse [4], and bistability [5], it has been used for line width narrowing [6], [7], mode stabilization [8], and reduction of the modulation-induced frequency chirp [9]. Under strong feedback, the laser happens to show dramatic changes of the output power, lasing spectrum, and the laser dynamics [10]-[13].

As a result of the complexity occurring in the nonlinear dynamic behavior of the semiconductor laser under optical feedback, the modulation characteristics of the laser change under OFB. The modulation bandwidth frequency of the

semiconductor laser can be either deteriorated or improved depending on whether OFB limits or extends the resonance between the carriers and photons in the laser cavity [14], [15]. Recently attention has been paid to use the optical feedback to boost the modulation bandwidth of the laser to the millimeter-wave band of the electromagnetic spectrum to apply the laser in broadband communication systems, such as the radio over fiber (RoF) networks [15], [16]. This RoF technology inherently combines the advantage of enormous bandwidth of optical fiber and the flexibility of wireless access technologies, to deliver wireless RF signals directly from the central station to simplified base stations. The mm-wave bands are utilized to meet the demand for higher signal bandwidth and to overcome the frequency jamming in the RoF-based wireless networks [17].

External OFB is regarded as a cost-effective technique to increase the modulation bandwidth of semiconductor lasers compared with other techniques such as the optical injection [18]. Narrow-band high-frequency modulation over 40 GHz has been achieved in quantum well lasers under OFB [19]. Most recently the author's group [20] has newly reported on using strong OFB to boost the modulation frequencies over an ultra-high frequency passband exceeding 50 GHz and has shown improvement of the gain of a corresponding RoF link by about 20 dB. Such enhancement in the intensity modulation (IM) response over an ultra-high frequency passband was attributed as a type of photon-photon resonance due to coupling of oscillating modes in the coupled cavity [14], [21], [22]. It is obtained when the non-modulated laser keeps operation in CW under strong OFB. The authors pointed out also that the noise factor of such mm-wave RoF links improves nearly by 20 dB in the regime of small-signal modulation and 10 dB under large-signal modulation [23].

In this paper, we introduce comprehensive investigation on the mm-wave single- and two-tone modulation characteristics of semiconductor lasers with a short-external cavity and strong OFB. Because the signal distortion is a critical issue in the modulation of semiconductor lasers and the optical analog links [24], [25], we also examine the harmonic distortions in the mm-wave modulated laser signal. We consider modulation of the laser with single mm-frequency and study the associated second-order harmonic distortion (2HD) and third-order harmonic distortion (3HD). We study also the modulation performance of the laser modulation using two adjacent mm-frequencies and evaluate the associated third-order intermodulation distortion (IMD3). The present study is based

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on applying a strong OFB rate equation model, in which OFB is treated as time delay of OFB with round trips (multiple reflections) in an external cavity [4]. We compare the obtained findings with those of a solitary laser when modulated at the carrier-photon resonance (relaxation) frequency, which is the most practical frequency regime of the solitary laser at which the IM is most enhanced. We apply the model to a high-speed DFB laser with a modulation bandwidth of about 25 GHz [26]. We show that the strong optical feedback induces resonant modulation response due to PPR over a frequency passband around  $\sim 62$  GHz when the external cavity length is 0.22 cm. Compared with the solitary laser modulated at the relaxation frequency, the present modulated signal was shown to have 5 dB lower harmonic distortions.

## II. THEORETICAL MODEL

The dynamics of semiconductor under IM and OFB are described by the following time-delay rate equations of the carrier number  $N(t)$ , photon number  $S(t)$  and optical phase  $\theta(t)$  [20]

$$\frac{dN}{dt} = \frac{1}{e} I(t) - \frac{N}{\tau_s} - \frac{av_g}{V} \frac{N - N_g}{1 + \varepsilon S} S \quad (1)$$

$$\frac{dS}{dt} = \left[ \Gamma \frac{av_g}{V} \frac{N - N_g}{1 + \varepsilon S} - G_{th} \right] S + C \frac{N}{\tau_s} \quad (2)$$

$$\frac{d\theta}{dt} = \frac{1}{2} \left( \alpha \Gamma \frac{av_g}{V} (N - N_{th}) - \frac{v_g}{L_D} \varphi \right) \quad (3)$$

where  $G_{th}$  is the threshold gain under OFB and is determined by the photon lifetime  $\tau_p$  in the laser cavity of length  $L_D$  and refractive index  $n_D$ ,

$$G_{th} = \frac{1}{\tau_p} - \frac{v_g}{L_D} \ln|U(t - \tau)| \quad (4)$$

In the above equations,  $a$  is the differential gain,  $v_g$  is the group velocity in the active layer of length  $L_D$ ,  $\Gamma$  is the confinement factor,  $\alpha$  is the linewidth enhancement factor,  $\tau_s$  is the spontaneous emission lifetime,  $\varepsilon$  is coefficient of gain suppression,  $N_g$  is the electron number at transparency, and  $N_{th}$  is the electron number at threshold. In (4),  $U(t - \tau)$  is an OFB function describing the time delay of laser light due to round trips (i.e., multiple reflections) in the external cavity (of length  $L_{ex}$  and refractive index  $n_{ex}$ ) formed between the laser facet (of reflectivity  $R_f$ ) and the external mirror ( $R_{ex}$ ) [12], [13],

$$\begin{aligned} U(t - \tau) &= |U(t - \tau)| e^{-j\varphi} \\ &= 1 - \frac{1 - R_f}{R_f} \sum_{p=1}^{\infty} (R_f R_{ex})^{p/2} e^{-jp\omega\tau} \\ &\quad \times \sqrt{\frac{S(t - p\tau)}{S(t)}} \frac{e^{j\theta(t - p\tau)}}{e^{j\theta(t)}} \end{aligned} \quad (5)$$

$$\varphi = -\tan^{-1} \frac{\text{Im}\{U(t - \tau)\}}{\text{Re}\{U(t - \tau)\}} + n\pi \quad n: \text{integer} \quad (6)$$

with  $\omega$  being the angular frequency of the laser emission and  $\tau = 2n_{ex}L_{ex}/c$  as the round trip time. The strength of OFB is measured by the coupling coefficient  $K_{ex}$ , which is determined by the ratio between  $R_{ex}$  and  $R_f$  [12], [13],

$$K_{ex} = (1 - R_f) \sqrt{\eta \frac{R_{ex}}{R_f}} \quad (7)$$

where  $\eta$  is the external coupling efficiency of the injected light into the laser cavity. In (6)  $n$  is an integer and is chosen to vary continuously for time evolution, because the solution of arc tangent is limited in the range of  $-\pi/2$  to  $+\pi/2$  in the computer work. At a given time  $t$ , the phase difference between the time-delayed (externally injected) field and the field inside the laser cavity is given by  $\theta(t - m\tau) - \theta(t)$ , which is equal to zero or  $\pi$  in the cases of in-phase and out-of-phase conditions.

Under single-tone modulation, the injection current  $I(t)$  is composed of a bias component  $I_b$ , and a sinusoidal component of amplitude  $I_m$  and frequency  $f_m$

$$I(t) = I_b + I_m \sin(2\pi f_m t) \quad (8)$$

The modulation depth is given as  $m = I_m / I_b$ . In the case of modulation with two mm-frequencies  $f_{m1}$  and  $f_{m2}$  with  $f_{m1} < f_{m2}$ , the injection current  $I(t)$  is represented by

$$I(t) = I_b + I_m [\sin(2\pi f_{m1} t) + \sin(2\pi f_{m2} t)] \quad (9)$$

$f_{m1}$  and  $f_{m2}$  are closely spaced carrier frequencies.

## III. NUMERICAL CALCULATIONS

We solve (1)–(3) numerically by the 4<sup>th</sup> order Runge-Kutta method using a time integration step as short as 0.2 ps. Five round trips,  $p=1 \rightarrow 5$ , are counted in the calculations. We use the numerical values listed in Table I, which correspond to single-mode quantum-well DFB laser [24]. This laser has a threshold current of  $I_{th}=10$  mA. The laser is assumed to be biased above threshold,  $I_b = 5I_{th}$ . We changed the length of the external cavity to be  $n_{ex}L_{ex} = 0.22, 0.25$  and  $0.3$  cm, which correspond to an external-cavity resonance frequency spacing of 68, 60 and 50 GHz. The fast Fourier transform (FFT) is used to simulate the frequency content of the modulated laser signal [27]. The IM response is calculated numerically as

$$\text{IM - response} = a_1(f_m) / a_1(f_m \rightarrow 0) \quad (10)$$

where  $a_1(f_m)$  is the fundamental amplitude of the FFT power spectrum of the photon number  $S(t)$  at the modulation frequency  $f_m$ .

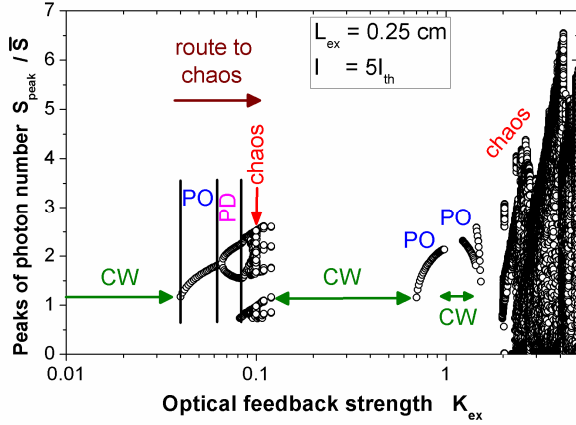
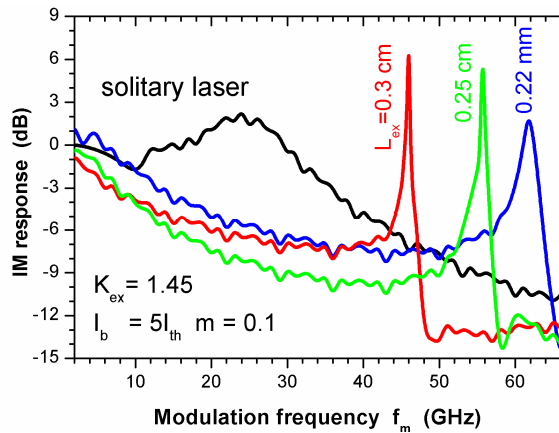


Fig. 1 Bifurcation diagram of the laser output under OFB

Fig. 2 The IM responses of the laser under OFB with  $K_{ex} = 1.45$  when  $L_{ex} = 0.25$  and  $0.30$  cm with  $m = 0.1$ . The IM response of the solitary laser is included for comparisonTABLE I  
DEFINITION AND NUMERICAL VALUES OF THE SOLITARY HIGH-SPEED LASER PARAMETERS

Symbol	Quantity	Value
$\lambda$	Wavelength	1.55 $\mu\text{m}$
$V$	Active layer volume	$3 \times 10^{-17} \text{ m}^3$
$v_g$	Group velocity	$8.33 \times 10^7 \text{ m/s}$
$L_D$	Active layer length	120 $\mu\text{m}$
$A$	Differential gain coefficient	$8.25 \times 10^{-12} \text{ m}^2$
$N_g$	Carrier number at transparency	$3.69 \times 10^7 \text{ m}^{-3}$
$\alpha$	Linewidth enhancement factor	3.5
$\Gamma$	Confinement factor	0.15
$\tau_p$	Photon lifetime	1.69 ps
$\tau_s$	Spontaneous emission lifetime	776 ps
$R_f$	Front facet reflectivity	0.2
$R_b$	Back facet reflectivity	0.6
$\beta_{sp}$	Spontaneous emission factor	$3 \times 10^{-5}$
$\varepsilon$	Nonlinear gain suppression factor	$2.77 \times 10^{-23} \text{ m}^3$

#### IV. RESULTS AND DISCUSSION

##### A. Laser Output under Optical Feedback

The laser dynamics of the laser are examined by means of the bifurcation diagram, which is constructed by plotting the

peaks of the photon number at each strength  $K_{ex}$  of OFB. The simulated bifurcation diagram when  $L_{ex} = 0.25$  cm is given in Fig. 1. The diagram shows the period-doubling (PD) route-to-chaos that characterizes the case of the short-external cavity (the external-cavity frequency spacing  $f_{ex} > f_r$ ). The diagram can be understood as follows. Under very weak OFB, the solution of the rate equations is still stationary and the laser operates in continuous wave (CW) for which no points are plotted in the figure. With the increase in OFB strength,  $K_{ex}$ , this stationary solution bifurcates first into a stable limiting cycle characterizing periodic oscillation (undamped relaxation oscillation), which is represented by a single point in the diagram. The starting point of the bifurcation is called a Hopf-bifurcation point. With further increase in OFB, the figure shows that the solution of the rate equations bifurcates into a PD route to chaos in which periodic oscillation bifurcates first into two branches, where the trajectory of  $S(t)$  has two peaks of different heights in every two successive periods. With the increase in  $K_{ex}$ , the oscillation period is multiplied to more than twice and the laser is attracted to transition to chaos. CW operation is also obtained in the region of strong OFB around  $K_{ex} = 1$ . The frequency of the periodic oscillation (PO) was found to increase with the increase in  $K_{ex}$ ; it approaches  $f_{ex}$  in the region of strong OFB.

##### B. Single-Tone Modulation Characteristics

In Fig. 2, we plot examples of the numerical IM responses of the laser under strong OFB that are characterized by resonance enhancement over a mm-waveband. The shown IM responses are simulated for three short-external cavities with lengths of  $L_{ex} = 0.22$ ,  $0.25$  and  $0.30$  cm, which correspond to external-cavity resonance frequencies of 68, 60 and 50 GHz, respectively. The modulation depth is  $m = 0.1$  and the OFB is as strong as  $K_{ex} = 1.45$ . This OFB strength corresponds to CW operation of the non-modulated laser diode under OFB, as shown in Fig. 1. In this case, the injected delay light is nearly in phase with the optical field in the laser cavity. The IM response of the solitary laser is also plotted for comparison. The IM response of the solitary laser has a maximum around the relaxation frequency  $f_r \sim 15$  GHz, and has a 3dB-modulation bandwidth of  $f_{3dB} = 25$  GHz. Fig. 2 shows that the IM response under strong OFB drops under the  $-3$ dB level at the frequencies of  $f_m = 13$ ,  $8$  and  $6$  GHz when  $L_{ex} = 0.22$ ,  $0.25$  and  $0.30$  mm, respectively, which are much lower than  $f_{3dB}$  of the solitary laser. In the high-frequency regime the IM response is enhanced over the mm-wave bands of (59.5 and 63 GHz), (54.4 and 56.5 GHz) and (45 and 46.6 GHz) centered at the frequencies  $f_m = 62$ ,  $55.8$  and  $46$  GHz when  $L_{ex} = 0.22$ ,  $0.25$  and  $0.30$  cm, respectively. These frequency bands are much higher than  $f_{3dB}$  of the solitary laser. The IM enhancement over the IM response of the solitary laser is as large as 2, 5.4 dB and 6.5 dB, respectively, which may be due to higher degree of phase-matching between the coupling OFB and the optical field in the laser cavity. Similar behavior of the narrow-band enhancement of the IM was reported by Troppenz et al. [28] around 40 GHz. This mm-narrow band enhancement of the modulation response can be attributed to

coupling between the resonance modes of the external cavities because of the carrier pulsation in the laser cavity at the beating frequency  $f_{ex}$ . This carrier pulsation is induced by the modulating current signal  $I(t)$  as indicated by (8) and (1) of the injected carrier number  $N(t)$ . This resonance is induced by optical modes and is different from the conventional carrier-photon resonance, which occurs around the relaxation frequency of the laser. Therefore, this response is referred to as “photon-photon resonance (PPR)” [21]. Similar effect is observed in vertical-cavity surface-emitting lasers (VCSELs) coupled to a transverse cavity [16], in which the photon-photon resonance is induced by transverse oscillating modes.

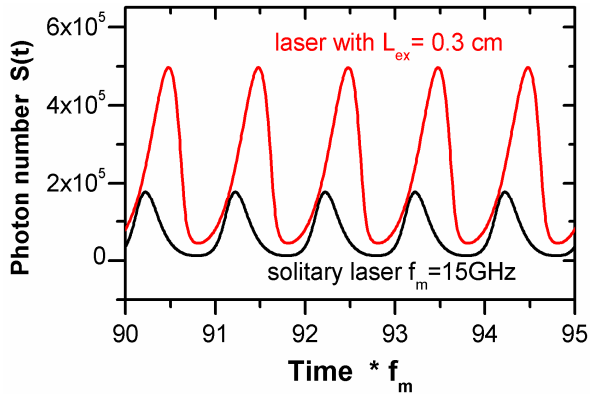


Fig. 3 Modulated waveform  $S(t)$  of the laser under OFB with  $L_{ex} = 0.25$  mm and  $f_m = 55.8$  GHz and the solitary laser with  $f_m = 15$  GHz

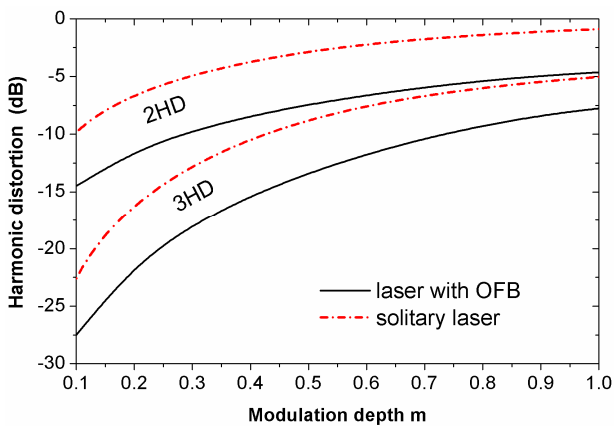


Fig. 4 The distortions 2HD and 3HD associated with intensity modulation as a function of the modulation depth  $m$  for both the ultra-high frequency modulated laser under OFB with  $L_{ex} = 0.25$  mm and the solitary laser with  $f_m = f_r$ .

In Fig. 3, we characterize the waveform of the modulated laser at the peak-frequencies of  $f_m = 55.8$  GHz, which correspond to the external-cavity lengths of 0.30 mm, and compare the results with the waveform of the solitary laser when modulated at the carrier-photon resonance frequency  $f_r$ . The figure indicates that the modulated signals are of the period-1 type oscillation. The oscillation amplitude of the mm-wave modulated signal is much larger than that of the modulated solitary laser. Also, the modulated signals under

OFB are almost sinusoidal, whereas the modulated signal of the solitary laser with  $f_m = f_r$  tends to be clipped. The Fast-Fourier Transform analysis of these signals indicate that the mm-wave modulated signals have 2<sup>nd</sup>-order and 3<sup>rd</sup>-order harmonic distortions of 2HD = -7.4 and 3HD = -13.4 dB, which are lower than those of the solitary laser (-2.87 and -8.8 dB, respectively). Both 2HD and 3HD are calculated as [29]

$$2\text{HD} = 10 \log_{10} \frac{a_2}{a_1} \quad 3\text{HD} = 10 \log_{10} \frac{a_3}{a_1} \quad (11)$$

where  $a_2$  and  $a_3$  are the FFT components at the 2<sup>nd</sup> and 3<sup>rd</sup> harmonics of  $f_m$ , respectively. The comparison of the harmonic distortions between both modulated signals is examined over a wide range of the modulation index  $m$  as given in Fig. 4. The figure shows an increase in the harmonic distortions with the increase in  $m$ , with 2HD being larger than 3HD, for both modulated signals. However, the values of 2HD and 3HD of the modulated signal under OFB are almost 5 dB smaller than those of the modulated signal of the solitary laser. Detailed analysis of the modulated waveforms showed also that the modulated signal of the solitary laser is clipped when  $m \geq 0.9$ , which is not seen in the modulated signals of the laser under strong OFB. This means that the modulation index increases by applying the strong OFB and enhancing the modulation bandwidth.

### C. Two-Tone Modulation Characteristics

The two-tone modulation is important for several applications, such as multi-channel RF-frequency division multiplexed transmission of analog or microwave signals [30]. However, this modulation is often associated with intermodulation distortion, which occurs when the nonlinearity of the laser causes undesired outputs at sum and difference frequencies. The IM3 of two closely spaced carrier frequencies (at  $f_2$  and  $f_2 + \Delta f$ ) is of particular interest [31]. Fig. 5 (a) plots the two-tone modulation response of the laser under OFB when modulated at  $f_{m1} = 55.8$  GHz and  $f_{m1} + \Delta f$  using the frequency spacing  $\Delta f = 10$  MHz. The figure corresponds to the modulation depth  $m = 0.5$ . The figure shows appearance of the 3<sup>rd</sup>-order intermodulation components at  $f_{m1} - \Delta f$  and  $f_{m2} + \Delta f$  in addition to the fundamental harmonics at  $f_{m1}$  and  $f_{m2}$ . IMD3 is defined as the ratio, in dB, of the amplitude of the third-order intermodulation component to that of the fundamental component [30],

$$\text{IMD3} = 10 \log_{10} \frac{a_{f_{m2} + \Delta f}}{a_{f_{m2}}} \quad (12)$$

Fig. 5 (b) plots IMD3 as a function of the modulation depth  $m$ . The figure shows that IMD3 increases with the increase in  $m$ . The slope of such increase is large in the regime of small-signal modulation, and decreases with the increase in  $m$ . The figure indicates that IMD3 ranges between -14.5 and -8 dB. In the figure, we also compare the IMD3 values with those of the solitary laser when modulated at the carrier-photon resonance frequency  $f_{m1} = f_r$ . As shown in the figure, IMD3 of the solitary

laser is little lower than that of the laser under strong OFB up to  $m = 0.2$ . For modulation with larger signals, IMD3 of the solitary laser becomes larger and the differences reaches 3dB when  $m = 1.0$ .

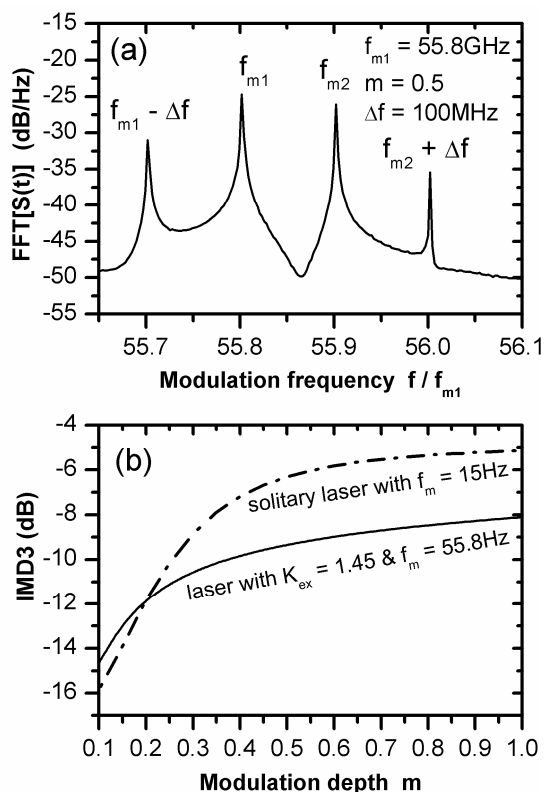


Fig. 5 Characteristics of two-tone modulation with  $f_{m1} = 55.8\text{GHz}$  and  $\Delta f = 100\text{MHz}$ : (a) FFT power spectrum showing intermodulation components at  $f_{m1} - \Delta f$  and  $f_{m2} + \Delta f$ , and (b) influence of modulation depth  $m$  on the IMD3

#### V. CONCLUSION

We presented the modeling of mm-frequency modulation characteristics of semiconductor lasers under strong OFB. The study was based on the theoretical modeling fully handling the strong OFB regime as time delay of laser light due to round-trips in the external cavity. We analyzed the signal distortions and noise associated with both single and two-tone modulations. We show that the enhanced IM response under strong OFB is due to photon-photon resonance resulting from coupling of oscillating modes in the external cavity. When the beating frequency of these coupled modes matches the frequency of the modulating electrical signal, the modulation response reveals resonance over a narrow-frequency band. A key parameter to achieve this mm-wave photon-photon resonance is to modulate the laser when it keeps stable operates in CW under strong OFB, where the injected delay light becomes in phase with the optical field in the laser cavity. Within this mm-frequency passband with enhanced IM response, the laser emits period-1 oscillations with low harmonic distortion. Under modulation with two adjacent mm-

frequencies, IMD3 increases with the increase in the modulation index  $m$ , ranging between -14.5 and -8 dB. Compared with the solitary laser modulated at the relaxation frequency, the present modulated signal was shown to have 5 dB lower harmonic distortions.

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