

# Application of Homotopy Perturbation Method to Solve Steady Flow of Walter B Fluid A Vertical Channel In Porous Media

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**Abstract**—In this article, a simulation method called the Homotopy Perturbation Method (HPM) is employed in the steady flow of a Walter's B' fluid in a vertical channel with porous wall. We employed Homotopy Perturbation Method to derive solution of a nonlinear form of equation obtained from exerting similarity transforming to the ordinary differential equation gained from continuity and momentum equations of this kind of flow. The results obtained from the Homotopy Perturbation Method are then compared with those from the Runge–Kutta method in order to verify the accuracy of the proposed method. The results show that the Homotopy Perturbation Method can achieve good results in predicting the solution of such problems. Ultimately we use this solution to obtain the other terms of velocities and physical discussion about it.

**Keywords**—Steady flow; Walter's B' Fluid; vertical channel; porous media; Homotopy Perturbation Method (HPM); Numerical Solution (NS)

## I. INTRODUCTION

THE flow of Newtonian and non-Newtonian fluids in a porous surface channel has always attracted the interest of many investigators in view of its applications in engineering practice. Examples of these are the cases of boundary layer control, transpiration cooling, gaseous diffusion, add reactants (in chemical processes) and prevent corrosion and reduce drag. i.e. in transpiration cooling, the walls of a channel carrying a hot fluid are made of a porous material through which fluid is injected to form a protective layer of cooler fluid near the wall. In view of its importance, the flow of Newtonian and non-Newtonian fluids through porous channels has been investigated by numerous authors. The case of a two-dimensional, incompressible, steady, laminar suction flow of a Newtonian fluid in a parallel-walled porous channel was studied by Berman [1]. He solved the Navier-Stokes equations by using a perturbation method for very low cross-flow Reynolds numbers. After his pioneering work, this problem has been studied by many researchers considering various variations in the problem, e.g., Cox [2] and Choi et al. [3]. A literature survey clearly indicates that little attention has been paid to the flows of non-Newtonian fluids in a vertical channel. This scientific problem is modeled by ordinary or partial differential equation. In most cases, analytical solutions cannot be applied to this problem, so this equation should be solved using special techniques. According to this argument many investigators are finding

analytical methods to solve these nonlinear forms of equations. The Homotopy perturbation method (HPM) is well-known method to solve the nonlinear equations. This method is introduced by He [4–5] for the first time. This method has been used by many authors such as Ganji in [6-7] and the references therein to handle a wide variety of scientific and engineering applications such as linear and nonlinear, homogeneous and inhomogeneous as well, because these methods continuously deform a difficult problem into a simple one, which is easy to solve. They were shown by many authors that these methods provide improvements over existing numerical techniques. With the rapid development of nonlinear science, many different methods were proposed to solve various boundary–value problems (BVP) [8] and fractional order [9], such as Homotopy perturbation method (HPM). These methods give successive approximations of high accuracy of the solution.

### A. Description of the problem

We consider the steady Flow of a Walter's B' Fluid in a Vertical Channel with porous wall. A fluid is injected through a vertical porous plate at  $y = d$  with uniform velocity  $U$ . The fluid strikes another vertical impermeable plate at  $y = 0$ . It flows out through the opening of the plates, due to the action of gravity along the  $z$ -axis. We have further assumed the distance between the walls,  $d$ , is small compared to the dimensions of the plates, i.e.  $L \gg B \gg d$ . Due to this assumption the edge effects can be ignored and the isobars are parallel to the  $z$ -axis. The basic equations of the problem are the following:

Continuity equation [10]:

$$\nabla \cdot \mathbf{v} = 0, \quad (1)$$

Equations of motion

$$\rho(\mathbf{v} \cdot \nabla \mathbf{v}) = \nabla T + \rho \mathbf{g}, \quad (2)$$

Where  $\rho$  is the density,  $\mathbf{g}$  the gravitational acceleration vector. The assumptions made in the above equations are as follows: (a) The flow is steady and laminar; (b) The fluid is incompressible; (c) The body force per unit mass is taken to be equal to the gravitational acceleration; (d) the physical properties of the fluid remain invariable throughout the fluid; (e) The effects of viscous dissipation are negligible.

$$\rho(\mathbf{v} \cdot \nabla \mathbf{v}) = -\nabla p + \rho \mathbf{g} + \eta_0 \nabla^2 \mathbf{v} - 2k_0 \mathbf{v} \cdot \nabla \nabla^2 \mathbf{v} + k_0 \nabla^2 (\mathbf{v} \cdot \nabla \mathbf{v}) \quad (3)$$

The boundary conditions for the velocity field are:

$$f(0) = 0, f(1) = 1, f'(0) = 0, f'(1) = 0, \quad (4)$$

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After utilizing some variable and modification the final equation is

$$F'''' + \text{Re}(F'''F - F'F'') + \text{Re}S(F F'''' - F'F''') = 0. \quad (5)$$

**B. Application of HPM**

In this section, we will apply the HPM to nonlinear ordinary differential equation (6). According to HPM, we can construct homotopy of equation (6) as follows:

$$(1-p)(F'''(y) - F_0'''(y)) + p(F''' + \text{Re}(F''F - F'F'') + \text{Re}S(F F'''' - F'F''')) = 0, \quad (6)$$

We consider  $F$  as follows:

$$F(\eta) = F_0(\eta) + F_1(\eta) + F_2(\eta) + F_3(\eta) + \dots = \sum_{i=0}^n F_i(\eta), \quad (7)$$

By substituting  $F$  from Eq. (7) into Eq. (6) and after some simplifications and rearranging based on powers of  $p$ -terms, we have:

$$p^0 : F_0''' = 0, \quad F_0'(0) = 0, \quad F_0(0) = 0, \quad F_0'''(0) = a, \quad F_0''(0) = b. \quad (8)$$

$$p^1 : F_1''' + \text{Re}(F_0'''F_0 - F_0'F_0'') + \text{Re}S(F_0F_0'''' - F_0'F_0''') = 0, \quad (9)$$

$$F_1''(0) = 0, \quad F_1(0) = 0, \quad F_1'''(0) = 0, \quad F_1'(0) = 0,$$

$$p^2 : F_2''' + \text{Re}(F_0'''F_1 + F_1'''F_0 - (F_1'F_0'' + F_0'F_1'')) + \text{Re}S(F_1F_0'''' + F_0F_1'''' - (F_1'F_0''' + F_0'F_1''')) = 0, \quad (10)$$

$$F_2''(0) = 0, \quad F_2(0) = 0, \quad F_2'''(0) = 0, \quad F_2'(0) = 0,$$

Solving Eqs. (8)–(10) with boundary conditions, we have:

$$F_0(\eta) = \frac{1}{6}a\eta^3 + \frac{1}{2}b\eta^2,$$

$$F_1(\eta) = \frac{\text{Re}a^2}{2520}\eta^7 + \frac{\text{Re}ab}{360}\eta^6 + \frac{\text{Re}b^2}{120}\eta^5, \quad (11)$$

$$F_2(\eta) = -\frac{a^3\text{Re}^2}{2494800}\eta^{11} - \frac{\text{Re}^2a^2b}{226800}\eta^{10} - \frac{\text{Re}^2ab^2}{90720}\eta^9 + \dots \quad (12)$$

where  $a = F'''(0)$  and  $b = F''(0)$  to are determined from the boundary conditions. Solutions  $F_3(\eta)$  to  $F_5(\eta)$  were too long to be mentioned here; therefore, they are shown graphically. The solution of this equation, when  $p \rightarrow 1$ , will be as follows:

$$F(\eta) = F_1(\eta) + F_2(\eta) + F_3(\eta) + \dots + F_5(\eta) \quad (13)$$

**II. RESULTS AND DISCUSSION**

Table (1) shows the errors involved Homotopy Perturbation Method, along with the Numerical Solution obtained by the fourth-order Runge–Kutta method. Figures (1) and (2) exhibit the approximate solution of  $F(\eta)$  and  $F'(\eta)$  obtained for various Reynolds number (Re) when  $S = 0.1$  by using HPM. Figures (3) and (4)

show effect of various Elastic number ( $S$ ) on  $F(y)$  and  $F'(y)$  by HPM when  $\text{Re} = 1$  respectively

**III. CONCLUSION**

In this study, we have presented the operation of Homotopy Perturbation Method (HPM) on steady flow of a Walter's B' fluid in a vertical channel with porous wall. The results show that this scheme provides excellent approximations to the solution of this nonlinear equation with high accuracy. Finally, it has been attempted to show the capabilities and wide-range applications of the Homotopy Perturbation Method (HPM) in comparison with the numerical solution of steady flow of a Walter's B' fluid in a vertical channel with porous wall problems.

TABLE I  
COMPARISON OF THE HOMOTOPY PERTURBATION METHOD (HPM) WITH NUMERICAL SOLUTIONS (NS) FOR  $F(\eta)$  WHEN  $\text{Re} = 1$  AND  $S = 0$ .

$\eta$	HPM	NS	Errors
0	0.0000000000	0.0000000000	0.0000000000
0.1	0.0298802351	0.0298802345	0.0000000006
0.2	0.1100264507	0.1100264491	0.0000000016
0.3	0.2264689211	0.2264689181	0.0000000030
0.4	0.3657361663	0.3657361622	0.0000000041
0.5	0.5149401506	0.5149401457	0.0000000049
0.6	0.6617922354	0.6617922307	0.0000000047
0.7	0.7945849545	0.7945849512	0.0000000033
0.8	0.9021711100	0.9021711083	0.0000000017
0.9	0.9739668800	0.9739668817	0.0000000017
1	1.0000000000	1.0000000000	0.0000000000

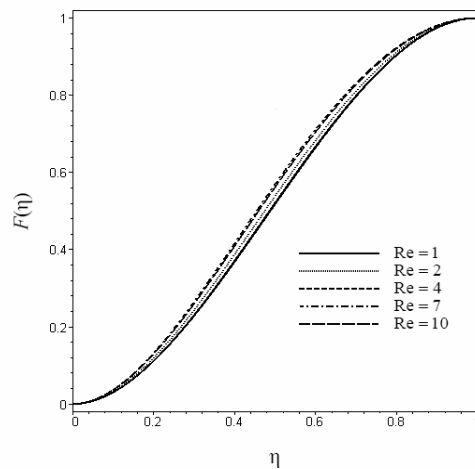


Fig. 1 Effect of various Reynolds number (Re) on  $F(\eta)$  by HPM when  $S = 0.1$

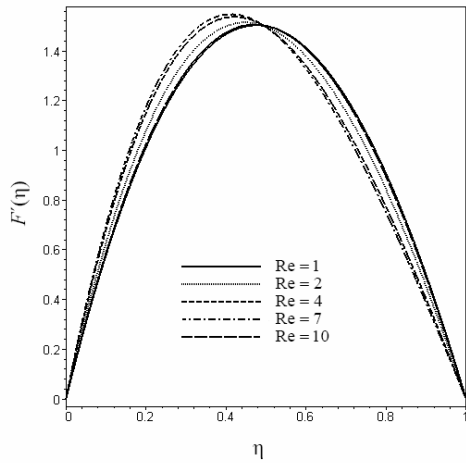


Fig. 2 Effect of various Reynolds number ( $Re$ ) on  $F'(\eta)$  by HPM when  $S = 0.1$

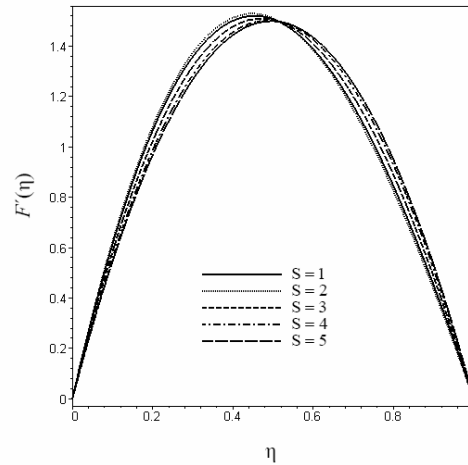


Fig. 4 Effect of various Elastic number ( $S$ ) on  $F'(\eta)$  by HPM when  $Re = 1$

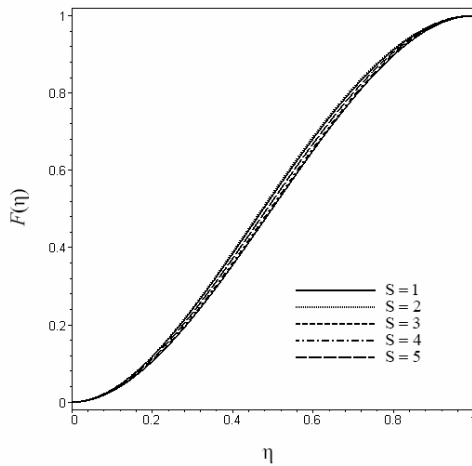


Fig. 3 Effect of various Elastic number ( $S$ ) on  $F(\eta)$  by HPM when  $Re = 1$

REFERENCES

- [1] Berman, A.S. Laminar Flow in Channels with Porous Walls, *J. Appl. Phys.*, 24 (1953) 1232-1235.
- [2] Choi, J.J., Rusak, Z. and Tichy, J.A., Maxwell Fluid Suction Flow in a Channel, *J. Non-Newtonian Fluid Mech.*, 85 (1999) 165-187.
- [3] Cox, S.M., Two Dimensional Flow of a Viscous Fluid in a Channel with Porous Walls, *J. Fluid Mech.* 227 (1991) 1-33.
- [4] J.H. He, Homotopy perturbation technique, *Comp. Meth. App. Mech. Eng.* 178, (1999) 257-262.
- [5] J.H. He, A coupling method of a homotopy technique and a perturbation technique for non-linear problems, *I. J. Non-Lin. Mech.* 35 (2000) 37-43.
- [6] Z.Z. Ganji, D.D. Ganji, Approximate Solutions of Thermal Boundary-layer Problems in a Semi-infinite Flat Plate by using He's Homotopy Perturbation Method, *International Journal of Nonlinear Sciences and Numerical Simulation*, 9(4), (2008) 415-422.
- [7] Z.Z. Ganji, D.D. Ganji, M. Esmailpour, A Study on Nonlinear Jeffery-Hamel Flow by Two He's Semi-Analytic Methods and Comparison with Numerical Results, *Computers and Mathematics with Applications*, 58 (2009) 2107-2116.
- [8] E. Yusufoglu, Homotopy Perturbation Method for Solving a Nonlinear System of Second Order Boundary Value Problems, *Int. J. Non-Lin. Sc. Num. Sim.* 8 (2007) 353-358.
- [9] Z.Z. Ganji, D.D. Ganji, H. Jafari, M. Rostamian, Application of the HPM to Coupled system of PDE, *Topological Methods in Nonlinear Analysis*, 31, (2008) 341-348.
- [10] A. A. Joneidi, G. Domairry, M. Babaelahi, Homotopy Analysis Method to Walter's B fluid in a vertical channel with porous wall, *Meccanica*, DOI: 10.1007/s11012-010-9295-y.