International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:14, No:11, 2020

Application of Differential Transformation Method for Solving Dynamical Transmission of Lassa Fever Model

M. A. Omoloye, M. I. Yusuff, O. K. S. Emiola

Abstract**—**The use of mathematical models for solving biological problems varies from simple to complex analyses, depending on the nature of the research problems and applicability of the models. The method is more common nowadays. Many complex models become impractical when transmitted analytically. However, alternative approach such as numerical method can be employed. It appropriateness in solving linear and non-linear model equation in Differential Transformation Method (DTM) which depends on Taylor series make it applicable. Hence this study investigates the application of DTM to solve dynamic transmission of Lassa fever model in a population. The mathematical model was formulated using first order differential equation. Firstly, existence and uniqueness of the solution was determined to establish that the model is mathematically well posed for the application of DTM. Numerically, simulations were conducted to compare the results obtained by DTM and that of fourth-order Runge-Kutta method. As shown, DTM is very effective in predicting the solution of epidemics of Lassa fever model.

Keywords**—**Differential Transform Method, Existence and uniqueness, Lassa fever, Runge-Kutta Method.

I. Introduction

NOWADAYS, the use of mathematical models for solving biological problems is common and varies from simple to complex analyses, depending on the nature of the research problems and applicability of the models [7]. However, many complex models become impractical when transmitted analytically. An alternative approach such as numerical method can be employed. It is always appropriate in solving linear and non-linear model equation especially, when DTM is employed [7]. It is a semi-analytical numerical technique which depends on Taylor series and has application in many areas including Science, Engineering, Mathematics and Biomathematics. Hence, this works was embarked on based on its appropriateness and applicability in many areas of science and technology.

Omoloye Musibau Abayomi (Ph.D. in view) is with the Department of Mathematics/Statistic, School of Applied Science and Technology, Federal Polytechnics, Offa, Nigeria (phone: 08034890497; email: omoloye2011@gmail.com

Yusuff Idowu Moshood (M, 2020), (Ph.D. in view) is with the Department of Science Technology, School of Applied Science and Technology, Federal Polytechnics Offa, Nigeria (phone: 08060854041; e-mail:idmos_nig@yahoo.com).

Emiola Steve Kolapo Olawale (PhD in Operation Research) is with the Department of Mathematics/Statistic, School of Applied Science and Technology, Federal Polytechnics, Offa, Nigeria (phone: 07030289857; email: stevemiola@fedpoffaonline.edu.ng

Lassa fever is a zoonotic disease, usually transmitted from an infected animal to a human, and can also be transmitted from human-to-human [2]-[4]. The animal host of Lassa virus is rat specie known as mastomy (mastomynatalensis in particular) [6]. Lassa fever is an endemic in West African countries such as Guinea, Liberia, Sierra Leone and Nigeria [8]. Studies have showb that about 500,000 cases of Lassa fever occur per year in West Africa with approximately 5,000 deaths [12]. The animal host of the Lassa virus is rodents [5].

Health workers are at high risk especially when caring for patient infected with Lassa fever without proper prevention and control practices [13], [14]. The symptoms of Lassa fever begin to show after being infected between one to three weeks with signs and symptoms including cough, vomiting, muscular fatigue, meningitis, facial and hypertension. Neurological problems such as loss of hearing (transient or permanent, tremors and encephalitis) may also be the results of Lassa virus [9]-[11]. Finding a solutions to its transmission saves health workers from the risk of been infected and prevent its transmission. Therefore, DTM was employed for this work.

The DTM adopted for this work is a computational method that can be used to solve linear or nonlinear ordinary differential equations with their initial conditions respectively [10]. As proposed, the concepts of differential transformation are the image of a transformed function computed by differential operations. This method becomes a numerical-analytical technique that formalizes the Taylor series in a different manner.

Computationally, DTM was employed to solve the system of differential equations described by [1], the Lassa virus model and approximate the solutions in a sequence of time interval but, fourth order Runge-Kutta method was also compared in order to establish the accuracy of the method.

II. Model Formulation

Model was formulated by dividing the total human population $N(t)$ into five sub-populations groups as; Susceptible $S(t)$, Latently infected $L(t)$, Infectious $I(t)$, Isolated $J(t)$, and Recovered $R(t)$.

A. Description of Model Formulations

The susceptible population increases by the recruitment rate π and decreases by infection following effective contact with infectious and susceptible and natural death (at the rate μ).

Thus; $\frac{dS}{dt} = \pi - \beta SI - \mu S$ ·

Latently infected population increases through the infected individuals following effective contact with infectious and susceptible. Decreases through the following models obtained with progression rate γ, isolation rate θ_1 and natural death μ are:

$$\frac{dL}{dt} = \beta SI - (\gamma + \theta_1 + \mu)L \cdot$$

Population of infectious individual increases by progression from latently class γ and decline due to isolation rate θ_1, disease death rate δ and natural death rate μ. Thus; $\frac{dI}{dt} = \gamma L - (\theta_2 + \delta + \mu)I \cdot$

Isolated individual by population also increases by rate of infected but isolated at θ_1 and infectious but isolated at rate θ_2. The population decreases due to recovery rate α and natural death rate.

$$\frac{dJ}{dt} = \theta_1 L + \theta_2 I - (\alpha + \mu)J$$

The population of recovered class increases through the recovery rate of isolated individual and decreases by natural death rate μ. Hence; $\frac{dR}{dt} = \alpha J - \mu R$ Concisely, from the above assumptions and formulations, the following system of differential equations were obtained (both variable and parameters used are defined in Table I).

$$\begin{aligned}
\frac{dS}{dt} &= \pi - \beta SI - \mu S \\
\frac{dL}{dt} &= \beta SI - (\gamma + \theta_1 + \mu)L \\
\frac{dI}{dt} &= \gamma L - (\theta_2 + \delta + \mu)I \\
\frac{dJ}{dt} &= \theta_1 L + \theta_2 I - (\alpha + \mu)J \\
\frac{dR}{dt} &= \alpha J - \mu R
\end{aligned} \tag{1}$$

III. Existence and Uniqueness of Solution

Theorem1. Let

$$\begin{aligned}
x_1' &= f_1(x_1, x_2, ..., x_n, t), x_1(t_0) = x_{10} \\
x_2' &= f_2(x_1, x_2, ..., x_n, t), x_2(t_0) = x_{20} \\
x_3' &= f_3(x_1, x_2, ..., x_n, t), x_3(t_0) = x_{30} \\
&\vdots \\
x_n' &= f_n(x_1, x_2, ..., x_n, t), x_n(t_0) = x_{n0}
\end{aligned} \tag{2}$$

TABLE I
Description of Model Variables and Parameters

Variables	Description
S	Susceptible individual
L	Latently infected individual
I	Infectious individual
J	Isolated individual
R	Recovered individual
Parameters	**Interpretation**
π	Recruitment rate
β	Contact rate
θ_1	Isolated rate of latently infected individual
θ_2	Isolated rate of infectious individual
μ	Natural death rate
δ	Disease death rate
α	Recovery rate from isolated individual
γ	Progression rate from latently

Suppose D is the region in $(n+1)$-dimensional space (one dimension for t and n dimensions for the vector x). If the partial derivatives $\frac{\partial f_i}{\partial x_j}$ where $i, j = 1,2,...n$ are continuous in $D = \{(x,t) : |t - t_0| \le a, |x - x_0| \le b\}$, then there is a constant $\delta > 0$ such that there exists a unique continuous vector solution $x = [x_1(t), x_2(t), x_3(t), ...x_n(t)]$ in the interval $|t - t_0| \le \delta$.

Let:

$$\begin{aligned}
f_1 &= \frac{dS}{dt} = \pi - \beta SI - \mu S \\
f_2 &= \frac{dL}{dt} = \beta SI - (\gamma + \theta_1 + \mu)L \\
f_3 &= \frac{dI}{dt} = \gamma L - (\theta_2 + \delta + \mu)I \\
f_4 &= \frac{dJ}{dt} = \theta_1 L + \theta_2 I - (\alpha + \mu)J \\
f_5 &= \frac{dR}{dt} = \alpha J - \mu R
\end{aligned} \tag{3}$$

$$D = \{(S, L, I, J, R) : |S - S_0| \le a, |L - L_0| \le b, |I - I_0| \le c, |J - J_0| \le d, |R - R_0| \le e\}$$

Then (1) has a unique solution. Taking the partial derivatives to obtain the following;

$$\left|\frac{\partial f_1}{\partial S}\right| = |-(\mu + \beta I)| < \infty, \left|\frac{\partial f_1}{\partial L}\right| = 0 < \infty, \left|\frac{\partial f_1}{\partial I}\right| = |-\beta S| < \infty,$$

$$\left|\frac{\partial f_1}{\partial J}\right| = 0 < \infty, \left|\frac{\partial f_1}{\partial R}\right| = 0 < \infty, , \left|\frac{\partial f_2}{\partial S}\right| = |\beta I| < \infty,$$

$$\left|\frac{\partial f_2}{\partial L}\right| = |-(\gamma + \mu + \theta_1)| < \infty, \left|\frac{\partial f_2}{\partial I}\right| = |\beta S| < \infty, \left|\frac{\partial f_2}{\partial J}\right| = 0 < \infty,$$

$$\left|\frac{\partial f_2}{\partial R}\right| = 0 < \infty, \left|\frac{\partial f_3}{\partial S}\right| = 0 < \infty, \left|\frac{\partial f_3}{\partial L}\right| = |\gamma| < \infty,$$

$$\left|\frac{\partial f_3}{\partial I}\right| = \left|-(\mu+\delta+\theta_2)\right| < \infty, \left|\frac{\partial f_3}{\partial J}\right| = 0 < \infty, \left|\frac{\partial f_3}{\partial R}\right| = 0 < \infty,$$
$$\left|\frac{\partial f_4}{\partial S}\right| = 0 < \infty, \left|\frac{\partial f_4}{\partial L}\right| = |\theta_1| < \infty, \left|\frac{\partial f_4}{\partial I}\right| = |\theta_2| < \infty,$$
$$\left|\frac{\partial f_4}{\partial J}\right| = \left|-(\mu+\alpha)\right| < \infty, \left|\frac{\partial f_4}{\partial R}\right| = 0 < \infty,, \left|\frac{\partial f_5}{\partial S}\right| = 0 < \infty,$$
$$\left|\frac{\partial f_5}{\partial L}\right| = 0 < \infty, \left|\frac{\partial f_5}{\partial I}\right| = 0 < \infty, \left|\frac{\partial f_5}{\partial J}\right| = |\alpha| < \infty, \left|\frac{\partial f_5}{\partial R}\right| = |-\mu| < \infty$$

Since, the partial derivative exists, continuous and bounded as shown; therefore, by Theorem1, the model has unique solution.

IV. Solution of the Model Applying DTM

This section depicts the steps followed in applying DTM using operational properties in Table I to obtain the following system of transformed equations:

$$S(k+1) = \frac{1}{k+1}\left[\pi - \beta\left(\sum_{l=0}^{k} S(l)I(k-l)\right) - \mu S(k)\right]$$
$$L(k+1) = \frac{1}{k+1}\left[\beta\left(\sum_{l=0}^{k} S(l)I(k-l)\right) - k_1 L(k)\right]$$
$$I(k+1) = \frac{1}{k+1}\left[\gamma L(k) - k_2 I(k)\right]$$
$$J(k+1) = \frac{1}{k+1}\left[\theta_1 L(k) + \theta_2 I(k) - k_3 J(k)\right]$$
$$R(k+1) = \frac{1}{k+1}\left[\alpha J(k) - \mu R(k)\right]$$

where $k_1 = \gamma + \mu + \theta_1, k_2 = \delta + \mu + \theta_2, k_3 = \mu + \alpha$ subject to the initial conditions: $S(0) = 200, L(0) = 150, I(0) = 100,$ $J(0) = 90, R(0) = 70$ and parameters value $\pi = 0.15,$ $\beta = 0.05, \theta_1 = 0.5, \theta_2 = 0.6, \mu = 0.02, \delta = 0.3, \alpha = 0.6.$

The initial conditions with model parameters values are used to obtain the following series solutions for each variable given in Table I. Let $k = 6$ be the solution to the model in the closed form and obtained as:

$$S(t) = \sum_{n=0}^{k} S(n)t^n = 200 - 1003.85t + 2304.7385t^2$$
$$-4251.68659t^3 + 10689.22037t^4$$
$$-29187.64434t^5 + 61055.21983t^6 \ldots$$
$$L(t) = \sum_{n=0}^{k} L(n)t^n = 150 + 787t - 2853.395t^2$$
$$+5586.978633t^3 + 8684.547019t^4$$
$$+26678.50611t^5 - 12409.73246t^6 \ldots$$
$$I(t) = \sum_{n=0}^{k} I(n)t^n = 100 + 43t + 334.37t^2$$
$$-958.5586333t^3 + 1477.538678t^4$$
$$+1291.351347t^5 + 3803.76871t^6 \ldots$$
$$J(t) = \sum_{n=0}^{k} J(n)t^n = 90 + 79.2t + 185.098t^2$$
$$-446.94542t^3 + 623.8650742t^4$$
$$+968.4000741t^5 + 2252.27597t^6 \ldots$$
$$R(t) = \sum_{n=0}^{k} R(n)t^n = 70 + 52.6t + 23.234t^2$$
$$+36.86470667t^3 - 67.22613653t^4$$
$$+75.13271345t^5 + 96.58956503t^6 \ldots$$

Fig. 1-5 show the results obtained by DTM and Runge-Kutta method of order four (RK4) with the same initial conditions and parameters value respectively. As depicted (Figs. 1-5), the variations show the rate in days and relationships between DTM and Dynamical Transmission rates of Lassa fever based on the proposed model.

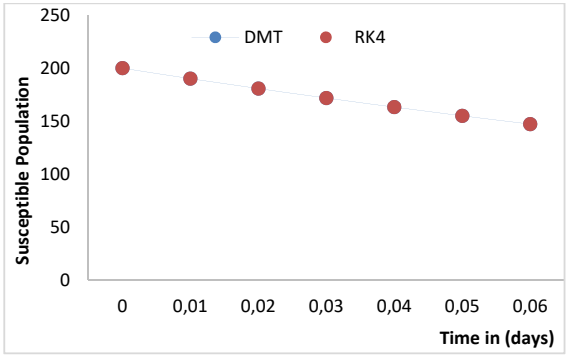

Fig. 1 Solution of Susceptible Population by DTM and RK4

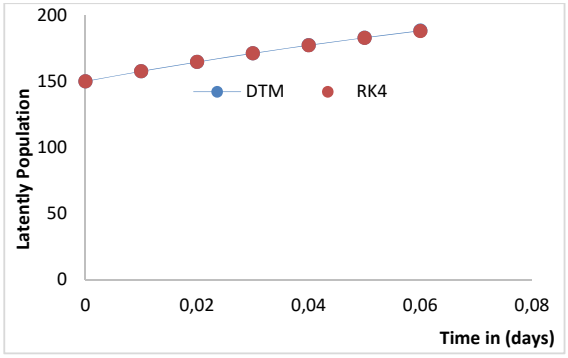

Fig. 2 Solution of Latently Population by DTM and RK4

V. Discussion

The solutions obtained using Differential Transform Method with initial conditions given agreed with the solution obtained by classical fourth-order Runge-Kuta method. Hence, their solutions follow the same pattern of behavior. This

shows that DTM is suitable and efficient to conduct analysis of Lassa models as proven by this work.

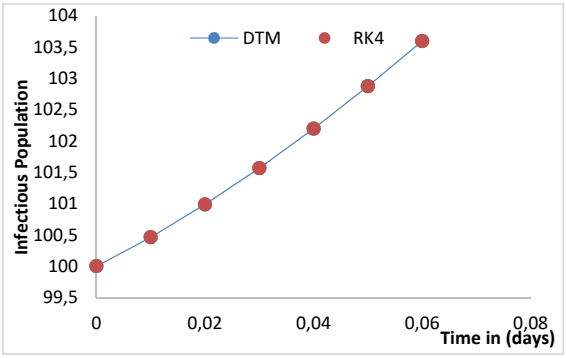

Fig. 3 Solution of Infectious Population by DTM and RK4

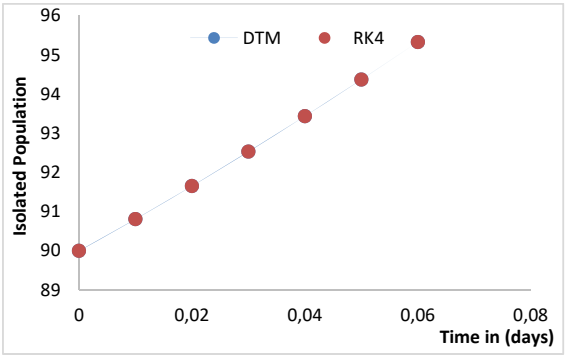

Fig. 4 Solution of Isolated Population by DTM and RK4

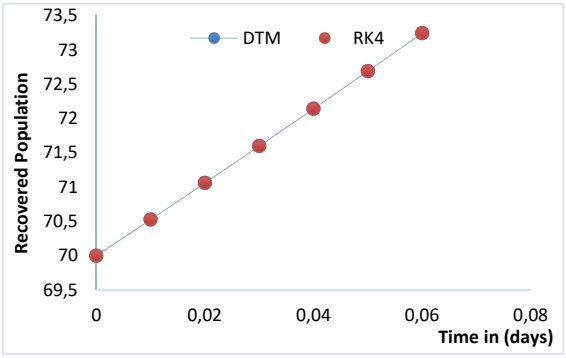

Fig. 5 Solution of Recovered Population by DTM and RK4

VI. Conclusion

Deterministic model on Lassa virus was presented and analyzed using DTM to obtain series solution of the model. Numerical simulations were also employed to compare the results obtained with DTM and fourth-other Ronge-Kuta method as shown graphically (Figs. 1-5). Based on the results of this work, DTM is very effective in predicting the solution of epidemics of Lassa Fever Model.

References

[1] Akinpelu, F.O. and Akinwande, R. (2018). "Mathematical model for Lassa fever and sensitivity analysis". Journal of Scientific and Engineering Research, vol. 5(6): Pp.1-9.

[2] Centers for Disease Control and Prevention (2013). "Lassa fever fact sheet".

[3] Centers for Disease Control and Prevention (2004). "Imported Lassa fever". MMWR Morb . Mortality Wkly Rep., vol. 53(38), Pp. 894-897

[4] Central Intelligence Agency. "World fact book for the year 2014. January, 2015.

[5] Fichet-Calvet E., Becker-Ziaja B., Koivogui, L. and Gunther, S. (2014). "Lassa Serology in Natural Populations of Rodents and Horizontal Transmission". Vector Borne and Zoonotic Diseases. Vol. 14(9): Pp. 665674.

[6] James, T. O., Abdulrahman, S., Akinyemi S. and Akinwade, N. I. (2015). "Dynamics Transmission of Lassa fever Disease". International Journal and Research in Education Science. Vol 2, Issue 1, Pp:2349-5219.

[7] Meksianis Z. Ndii, Nursanti Anggriani & Asep K. Supriatna (2018). "Application of differential transformation method for solving dengue transmission mathematical model". Conference Paper *in* AIP Conference Proceedings · March 2018. DOI: 10.1063/1.5026084. https://www.researchgate.net/publication/323608890

[8] Ogbu, O. E., Ajuluchukwu, C. J. and Uneke, C. J. (2007). "Lassa fever in West Africa sub-region. An Overview". Journal of Vector Borne Diseases.Vol.44, Pp.1-11.

[9] Omilabu, S. A, Badaru, S. O., Okokhere, P., Asogun, D., Drosten, C., and Emmerich, P. (2005). "Lassa fever Nigeria, 2003 and 2004". Emerging Infectious Diseases.*Vol.*11, Pp. 1642–4.

[10] Pukhov, G. E. (1980). "Differential Transformations of Functions and Equations". Naukova Dumka Publication, Kiev.

[11] World Health Organization (2014). "Lassa Fever: Factsheets". Available at http:who.int/mediacenter/factsheets/fs179/en/Accessed on 24/06/2016.

[12] World Health Organization (2005). "Strengthening health security by implementing the international health regulations". Available from: http://www.who.int/ihr/publications/9789241580496/en/.

[13] World Health Organization (2016). "Lassa fever-Sweden". Available from: http://www.who.int/csr/don/8-april-2016-lassa-fever-sweden/en/. Accessed May 21, 2020.

[14] World Fact Book for the year (2017), retrieved Feb 2016. World Health Organization, Disease outbreak news 19 February 2016: http://www.who.int/crs/don/27- may-016-lassa-fever-nigeria/en/.