# Ant System with Acoustic Communication 

S. Bougrine, S. Ouchraa, B. Ahiod, A. A. El Imrani


#### Abstract

Ant colony optimization is an ant algorithm framework that took inspiration from foraging behavior of ant colonies. Indeed, ACO algorithms use a chemical communication, represented by pheromone trails, to build good solutions. However, ants involve different communication channels to interact. Thus, this paper introduces the acoustic communication between ants while they are foraging. This process allows fine and local exploration of search space and permits optimal solution to be improved.


Keywords-Acoustic Communication, Ant Colony Optimization, Local Search, Traveling Salesman Problem.

## I. INTRODUCTION

OPTIMIZATION problems consist on finding the best solution for continuous or discrete problems. Therefore, many algorithms were performed to solve these problems and they are mostly inspired from the nature. For instance, there is particle swarm optimization [1] that is inspired from collective behavior of beings such as Ant Algorithms. Indeed, Ant algorithms are inspired by real ant colonies' behavior such as foraging, division of labor, brood sorting and cooperative transport [2]. As a matter of fact, these algorithms seek for solving difficult discrete optimization problems. Particularly, ant colony optimization (ACO) is a framework of ant algorithms, including ant system (AS) [4], ant colony system (ACS) [5], max-min ant system (MMAS) [6] that are inspired by foraging behavior of ant colonies [3]. On one hand, in order to find the shortest path between food sources and their nest, real ants use a chemical communication. In fact, ants deposit pheromones on the ground. These pheromones are accessible by others influencing their moves. On the other hand, the main difference between these algorithms lies in the update of pheromone.

However, the basic ants' models do not introduce a good mechanism for local search. Several algorithms has been developed to enhance this mechanism such as ACS hybridized with a new local search for the sequential ordering problem [12], MMAS and local search for the TSP [6], ACS with local search for Markov random field image segmentation[13].

The purpose of this paper is to introduce a new localized search based on the acoustic communication between ants called AS-AC. Indeed, AS-AC is based on the AS enhanced with a local search that took inspiration from the biological behavior of ants while they are foraging using the acoustic communication.
S. Bougrine, S. Ouchraa, and A. A. El Imrani are with the Conception and Systems Laboratory, Physics Department, Faculty of Sciences, Mohammed V - Agdal University, Rabat, Morocco (e-mail: saad.bougrine@gmail.com, salma.ouchraa@gmail.com, elimrani@fsr.ac.ma).
B. Ahiod is with the Computer Science Department, Mohammed V Agdal University, Rabat, Morocco (e-mail: bahiod@hotmail.com).

In this model, we focus on the AS algorithm proposed by Dorigo et al. in 1991 [4]. In brief, AS is applied to the traveling salesman problem (TSP) where ants build solutions by moving from one city to another. In each step, the choice of the next city is made through a probability rule policy. This policy is set up using the pheromone rate deposited in each node which is proportional to the quality of solutions they built. Nevertheless, pheromones represent a means of indirect communication, called stigmergy, requiring frequent updating. Besides, they are long distance information.

Furthermore, knowing that ants stridulate to attract its nestmates at a given range, acoustic communication completes the chemical communication by inserting a direct short-distance communication [7], [8].

This paper is organized as follows. Section II introduces some of ACO algorithms. Section III explains the biological aspects of acoustic communication related to foraging behavior of ant colonies. Section IV presents the AS-AC algorithm and its application to the TSP. In Section V, the experimental results are given and analyzed. Section VI presents conclusions about the proposed method.

## II. Ant Colony Optimization

ACO is a meta-heuristic set of ant-algorithms defined for discrete optimization problems. In other words, these problems can be presented as a fully connected graph $G=$ $(V, E)$ where $V$ and $E$ are respectively set of vertices and edges. The goal is to find optimal solution traversing graph $G$ visiting each vertex once.

## A. Ant System

Ant system is the first algorithm among ACO algorithms and it is applied to the TSP. TSP can be defined as the problem to find minimal solution performing the tour of all cities visiting each one once. In this context, at each iteration, ants build a greedy solution and all of them update the pheromone amount on each edge they visited.

In fact, to build a solution, ant executes $n$ steps moving from a city to another. These moves are ruled by a stochastic mechanism biased by pheromone values. Thus, an ant klying in the city $i$ chooses to go to the city jusing the probability given by:

$$
p_{i j}^{k}=\left\{\begin{array}{l}
\frac{\tau_{i j}^{\alpha} \cdot \mu_{i j}^{\beta}}{\sum_{l \in N_{i}} \tau_{i l}^{\alpha} \cdot \mu_{i l}^{\beta}} \text { ifj } \in N_{i},  \tag{1}\\
0 \quad \text { otherwise },
\end{array}\right.
$$

where $N_{i}$ is the set of cities not visited yet by the ant $k, \alpha$ and $\beta$ are two parameters, $\tau_{i j}$ is the amount of pheromone trail on edge $(i, j)$ and $\mu_{i j}$ is the heuristic value for switching from the
city $i$ to city $j$ defined as follows:

$$
\begin{equation*}
\mu_{i j}=\frac{1}{d_{i j}} \tag{2}
\end{equation*}
$$

where $d_{i j}$ is the distance between the city $i$ and city j .
When the $m$ ants finish their tour, pheromone trails are updated using (3).

$$
\begin{equation*}
\tau_{i j}=(1-\rho) \cdot \tau_{i j}+\sum_{k=1}^{m} \Delta \tau_{i j}^{k} \tag{3}
\end{equation*}
$$

where $\rho \in[0,1]$ is the evaporation rate, and $\Delta \tau_{i j}^{k}$ is the quantity of pheromone deposited by the ant $k$ on the edge $(i, j)$ defined as follows:

$$
\Delta \tau_{i j}^{k}= \begin{cases}Q / L_{k} & \text { if ant } k \text { used }(i, j)  \tag{4}\\ 0 & \text { otherwise },\end{cases}
$$

where $Q$ is a constant, and $L_{k}$ is the quality of solution constructed by ant $k$.

## B. Ant Colony System

Ant colony system is also an ACO algorithm applied to the TSP. However, the difference between ACS and AS lies at the state transition rule and the pheromone update. In fact, on one hand, ACS uses the state transition rule called pseudo-randomproportional rule given by:

$$
j=\left\{\begin{array}{l}
\underset{l \in N_{i}}{\operatorname{argmax}} \tau_{i l}^{\alpha} \cdot \mu_{i l}^{\beta} \text { if } q \leq q_{0}  \tag{5}\\
S \quad \text { otherwise }
\end{array}\right.
$$

where $q$ is a random number uniformly distributed over $[0,1]$, $q_{0} \in[0,1]$ is a parameter, and S is a random variable given by (1).

On the other hand, ACS uses two updating rules: the global updating rule, and the local updating rule. Indeed, the global one concerns only the best global solution found by ant $g b$ which is defined as follows:

$$
\begin{equation*}
\tau_{i j}^{g b}=(1-\alpha) \cdot \tau_{i j}^{g b}+\alpha \cdot \Delta \tau_{i j}^{g b} \tag{6}
\end{equation*}
$$

where $\alpha \in[0,1]$ is the pheromone decay coefficient, and $\Delta \tau_{i j}^{g b}$ is given by:

$$
\Delta \tau_{i j}^{g b}= \begin{cases}1 / L_{g b} & \text { if ant gb used }(i, j),  \tag{7}\\ 0 & \text { otherwise },\end{cases}
$$

where $L_{g b}$ is the length of the best solution found by ant $g b$.
Furthermore, the local updating rule is applied while ants build a solution which is defined as follows:

$$
\begin{equation*}
\tau_{i j}=(1-\rho) \cdot \tau_{i j}+\rho \cdot \tau_{0} \tag{8}
\end{equation*}
$$

where $\tau_{0}$ is the initial value of pheromone.

## C.Max-Min Ant System

In turn, max-min ant system performs the global updating
rule applied for the globally or locally best ant or both using (9). Nevertheless, the pheromone value is bounded between [ $\tau_{\text {min }}, \tau_{\text {max }}$ ].

$$
\begin{equation*}
\tau_{i j}=\left[(1-\rho) \cdot \tau_{i j}+\Delta \tau_{i j}^{b e s t}\right]_{\tau_{\min }}^{\tau_{\max }} \tag{9}
\end{equation*}
$$

where the operator $[x]_{a}^{b}$ is defined as follows:

$$
[x]_{a}^{b}=\left\{\begin{array}{l}
a \text { if } x>a  \tag{10}\\
b \text { if } x<b, \\
x \text { otherwise }
\end{array}\right.
$$

$\operatorname{and} \Delta \tau_{i j}^{\text {best }}$ is similar to (1) using $\mathrm{Q}=1$ and $\mathrm{k}=$ best.

## III. Acoustic Communication

Real ants are commonly known for their high organization. In fact, they live and work together in societies called colonies using different channels of communication. The channels are of importance to ants as they for instance, allow them to interact with each other to report a danger or to recruit its nestmates. For this aim, ants mainly use chemical communication through pheromone's deposits. Though, ants also communicate using the acoustic channel by stridulating.

According to Markl, ants are sensitive to vibrations detected through their legs from the substrate [9]. In this work, we took inspiration from Atta Cephalotes workers that feed from leafs [7].
As a matter of fact, when a worker finds a food source, it cuts leaf using their mandibles, thus transmitting vibrations through the leaf itself with a given intensity.

However, these vibrations have a limited scope in space depending on the leaf's quality. In fact, the higher the leaf quality is, the greater the impact ray becomes. Therefore, only ants lying on a given range will perceive these stridulatory vibrations. After that, some of them orient themselves toward the source of the vibrations and start the leaf cutting in turn.
The proposed approach implements this concept as a local search mechanism. In this model, the stridulation intensity represents the quality of the solution.

## IV. Ant Colony Optimization with Acoustic Communication

Fig. 1 represents the flowchart of the proposed model.


Fig. 1 Model of Ant Colony Optimization with Acoustic Communication

At each iteration, AS-AC algorithm has three phases. First, each ant builds a solution by following the pheromone trail. After that, ants that have a good solution send a signal to orient the near nest-mates toward the signal source improving their solutions. Finally, the pheromone trail is updated.

## A. Chemical Communication

This step is similar to the first phase of ACO. In fact, an ant starts from an empty partial solution. Then, at each step of construction, a solution component is added, taking into account of the pheromone trail, till the solution is completed [3].

## B. Acoustic Communication

This phase simulates the acoustic communication mechanism. In the real world, ants' stridulations affect its neighborhoods. Thus, to implement this concept for a given optimization problem, a neighborhood strategy should be defined.

The r-neighborhood of an ant A is defined as a set of ants whom the distance between each one of them and A is less than $r$, where $r$ is a ray calculated using the solution quality of A that corresponds to the stridulation intensity. As result, a mathematical distance functionmust be defined verifying the following properties:

$$
\begin{align*}
& \forall\left(s_{1}, s_{2}\right) \in P^{2}, d\left(s_{1}, s_{2}\right)=d\left(s_{2}, s_{1}\right)  \tag{11}\\
& \forall\left(s_{1}, s_{2}\right) \in P^{2}, d\left(s_{1}, s_{2}\right)=0 \Rightarrow s_{1}=s_{2}  \tag{12}\\
& \forall\left(s_{1}, s_{2}, s_{3}\right) \in P^{3}, d\left(s_{1}, s_{3}\right) \leq d\left(s_{1}, s_{2}\right)+d\left(s_{2}, s_{3}\right) \tag{13}
\end{align*}
$$

where $P$ is the solutions population.
After obtaining a solution s from the previous phase "Chemical communication", its neighborhood is generated using the defined neighborhood strategy. In order to exploit these neighbor solutions, it is necessary to define an operator or a local search mechanism. We call it acoustic follow (ACFL). This operator helps an ant to guide its neighborhood to find better results.

## C.Pheromone Update

In this step, after all ants have built and improved their solutions, some of them update the pheromone trail depending on parameters problem.

## V.APPLICATION TO TSP

## A. Model's Operators

In this section, the TSP is used to test the performance of the proposed approach. For this problem, some operators should be defined.

Initially, distance between two permutations and the $A C F L$ operator are defined to describe how a permutation can be improved by another.

Equation (14) presents the set of solution defined for the TSP.

$$
\begin{equation*}
P=\left\{x=\left.\left(x_{1}, x_{2}, \cdots, x_{n}\right)\right|_{1 \leq i, j \leq n, i \neq j \Rightarrow x_{i} \neq x_{j}, 1 \leq x_{i} \leq n}\right\}(1 \tag{14}
\end{equation*}
$$

To define the acoustic follow operator for TSP solutions, some other operators must be defined such as slide operator (15) taken from [11], the reverse operator (16), and the acoustic step operator (18).

$$
\begin{equation*}
S L(p, k)=(p(k+1) \% n, p(k+2) \% n, \cdots, p(k+n) \% n) \tag{15}
\end{equation*}
$$

wherep $=\left(p_{1}, \cdots, p_{n}\right)$ a permutation and $k$ an integer,

$$
R E V(p, i, j)=\left(p_{1}, \cdots, p_{i-1}, p_{j}, p_{j-1}, \cdots, p_{i}, p_{j+1}, \cdots p_{n}\right)(16)
$$

wherep $=\left(p_{1}, \cdots, p_{n}\right)$ a permutation, $i=1, \cdots, n, j=1, \cdots, n$ and $\mathrm{i}<j$.
$\operatorname{ACSP}(X, Y)=\min _{f(z)}\left\{\begin{array}{l|c}Z \in P \left\lvert\, \begin{array}{c}\exists l=1, \cdots, n, Z=\operatorname{REV}(X, l, m) \\ \mid \exists!m=1, \cdots, n, X_{m}=Y_{l}\end{array}\right.\end{array}\right\}(17)$
whereX $=\left(x_{1}, \cdots, x_{n}\right)$ and $Y=\left(y_{1}, \cdots, y_{n}\right)$ are two permutations.
$A C F L_{f c}(X, Y)= \begin{cases}\operatorname{ACSP}\left(X_{f c-1}, S L(Y,(f c-1) \% m)\right) & \text { if } f c \neq 0, \\ 0 & \text { othewise },\end{cases}$
where $X=\left(x_{1}, \cdots, x_{n}\right)$ and $Y=\left(y_{1}, \cdots, y_{n}\right)$ are two permutations.

The acoustic follow $A C F L_{f c}$ is a sequence that for all $i>j A C F L_{i} \leq A C F L_{j}$. This shows that this operator allows a solution X to moves in its neighborhood using the information from another solution $Y$ by applying (15) and (17).

Many metrics of permutations calculates the distance between two permutations such as hamming distance, adjacency based distance and Position based distance [10].

Among these metrics, mathematical properties of distance verify only the adjacency based distance if we consider the fact that a solution s is equivalent of $S L(s, i)$ for allinteger $i$. Thus, we use in this proposed method the adjacency based distance with a small modification as metric. The mathematical definition of this distance is:

$$
\begin{equation*}
d\left(p, p^{\prime}\right)=n-\#\left\{1 \leq i \leq n \| \operatorname{pos}_{p^{\prime}}\left(p_{i}\right)-\operatorname{pos}_{p \prime}\left(p_{(i+1) \% n}\right) \mid=1\right\} \tag{19}
\end{equation*}
$$

wheren is the number of cities, $p$ and $p^{\prime}$ are two permutations, $\operatorname{pos}_{p^{\prime}}\left(p_{i}\right)$ represents the position of city $p_{i}$ in permutation $p$ 'and $\ddagger$ is the cardinality of the set otherwise the number of elements of the set.

In the other hand, we define the r-neighborhood of the best solution $S$ at time $t$ by:

$$
\begin{equation*}
r=\left(1-\frac{f-f_{b}}{f_{w t}-f_{b}}\right) \times\left(r_{\operatorname{maxt}}-r_{\operatorname{mint}}\right)+r_{\operatorname{mint}} \tag{20}
\end{equation*}
$$

where $f$ is the quality of $\mathrm{S}, f_{w t}$ is the quality of the worst solution found at time $\mathrm{t}, f_{b}$ is the quality of the best solution found until time t and $r_{\text {maxt }}\left(r_{\text {mixt }}\right)$ is the maximum(minimum) ray between S and other solutions at time t .

# International Journal of Information, Control and Computer Sciences <br> ISSN: 2517-9942 <br> Vol:8, No:4, 2014 

## B. AS-AC Algorithm

At each iteration, the AS-AC works with $m$ ants. Each ant is positioned in a city chosen randomly from the set of cities. In each step of the construction, ant chooses the next city that is not yet visited by it using the transition rule (1). Once all ants have built their solution, ants that are included in the rneighborhood of $\mathrm{S}_{\mathrm{bt}}$ follow $\mathrm{S}_{\mathrm{bt}}$ using the Acoustic-Follow operator (18). Each of them replaces its solution Sc by $A C F L_{f c}\left(S c, S_{b t}\right)$.

The complexity of AS-AC is $O\left(\right.$ iter $\left.\times n^{2} \times m\right)$ where iter is the maximum number of iterations.

## C. Results and Discussion

To test the performance of the proposed model AS-AC, a number of benchmark instances of TSP are used to prove its efficiency. The parameter setting of AS-AC are $\mathrm{fc}=3 * \mathrm{n}, \rho$ $=0.5, \alpha=1, \beta=5, \mathrm{Q}=100, \mathrm{~m}=30$,
$\tau_{i j}=1.0 / N N S * n$ for all $i \neq j$, where n is the number of cities, $m$ is the number of Ants and NNS is the solution quality produced by the nearest neighbor search heuristic [14].

In all TSP instances, AS-AC found the best known solution in a small number of iterations except for D198 and kroa200 (see Table I).

Table II reports the results of comparison between AS-AC, MMAS and ACS. The AS-AC is run for 100 iterations. The results of ACS and MMAS are taken form [6].

The results show that the performance of AS-AC is higher than the others except for D198, ACS found the better result than MMAS and AS-AC, but the average length and standard deviation of AS-AC is better.

## VI. CONCLUSION

Ant colony optimization is suitable for global discrete optimization. However, they are not including local search inspired from the nature. To surmount this limitation, a new approach based on acoustic communication between ants has been proposed.

TABLE I
PERFORMANCE OF AS-AC BASED On 10 Independent Runs of 100

| ITERATIONS |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| TSP | Best | Mean | Stdev | Best Known |
| Eil51 | 426 | 426.6 | 0.6 | 426 |
| Berlin52 | 7542.0 | 7542.0 | 0.0 | 7542.0 |
| stt70 | 675.0 | 676,7 | 1.56 | 675.0 |
| Kroa100 | 21282.0 | 21283.4 | 4.2 | 21282.0 |
| D198 | 15925.0 | 15966.8 | 20.7836 | 15780.0 |
| Kroa200 | 29635.0 | 29864.0 | 117.66 | 29368 |

Best is the best solution found by AS-AC. Mean is the average length of 25 independent runs and Stdev is its standard deviation.

This approach takes the solutions constructed by the ant system and tries to improve them by using acoustic follow operator. This method allows ants to communicate between each other to exploit the information stored in the pheromone matrix.

To validate its efficiency, the proposed approach has been implemented for the TSP, and the results obtained have been compared to ACS and MMAS results.

The experimental results have shown that the proposed model has a better performance than the other algorithms concerning convergence speed.

TABLE II
COMPARISON OF AS-AC wITH MMAS AND ACS

| COMPARISON OF AS-AC WITH MMAS AND ACS |  |  |  |
| :---: | :---: | :---: | :---: |
| TSP | ACS | MMAS | AC-AS |
| Eil51 | 428.06 | 426.7 | 426.6 |
|  | $(2.48)$ | $(0.73)$ | $(0.6)$ |
|  | $[426]$ | $[426]$ | $[426]$ |
| Kroa100 | 21420.0 | 21302.80 | 21283.4 |
|  | $(141.72)$ | $(16.39)$ | $(4.2)$ |
|  | $[21282]$ | $[21282]$ | $[21282]$ |
|  | 16054.0 | 16048.60 | 15966.0 |
| D198 | $(71.15)$ | $(79.72)$ | $(20.7836)$ |
|  | $[15888]$ | $[15963]$ | $[15925]$ |

We report the average tour length for 25 runs, the standard deviation (in parentheses) and the best tour length (in square bracket).
$\mathrm{n}:=$ the number of cities
iter := the number of iterations
$\mathrm{S}:=$ arrays of solutions
$\mathrm{S}_{\mathrm{b}}$ :=the best solution
$\mathrm{S}_{\mathrm{bt}}:=$ the best solution at iteration t
r := ray of the r-neighborhood of $\mathrm{S}_{\mathrm{bt}}$
Begin :
Initialize the parameters
For t from 1 to iter
For j from 1 to n
//jth ant buildssolutionS $\mathrm{S}_{\mathrm{j}}$ using AS

$$
\mathrm{S}_{\mathrm{j}}:=\text { buildSolution(J) }
$$

End
// update $\mathrm{S}_{\mathrm{bt}}$ the best building solution
$\mathrm{S}_{\mathrm{bt}}=$ bestS();
// calculate the r-neighborhood for $\mathrm{S}_{\mathrm{bt}}$ using (20)
$\mathrm{r}:=$ calculateRay $\left(\mathrm{S}_{\mathrm{bt}}\right)$
For j from 1 to n
If distance $\left(\mathrm{S}_{\mathrm{b} t}, \mathrm{~S}_{\mathrm{j}}\right)<\mathrm{r}$ then
$\mathrm{S}_{\mathrm{j}}=\mathrm{ACFL}_{\mathrm{fc}}\left(\mathrm{S}_{\mathrm{j}}, \mathrm{S}_{\mathrm{bt}}\right)$
End
End
//Update Best solution
$\mathrm{S}_{\mathrm{b}}=$ best()
// update pheromone using update mechanism for AS
For j from 1 to n UpdatePheromone $\left(\mathrm{S}_{\mathrm{j}}\right)$
End
End
return $\mathrm{S}_{\mathrm{b}}$
End

Fig. 2 The AS-AC Algorithm
This newly applied acoustic communication can be used with other variants of ant colony optimization to solve TSP or other optimization problems.

# International Journal of Information, Control and Computer Sciences 

ISSN: 2517-9942
Vol:8, No:4, 2014

## References

[1] J. Kennedy, and R. Eberhart, "Particle swarm optimization," in IEEE International Conference on Neural Networks, Proceedings, vol. 4, pp 1942-1948, 1995.
[2] M. Dorigo, G. Di Caro, and L. M. Gambardella, "Ant algorithms for discrete optimization," Artificial Life, vol. 5, no. 2, pp. 137-172, 1999
[3] M. Dorigo, M. Birattari, et T. Stutzle, "Antcolonyoptimization," IEEE Computational Intelligence Magazine, vol. 1, no 4, pp. 28-39, nov. 2006.
[4] M. Dorigo, V. Maniezzo, and A. Colorni,"Ant system: optimization by a colony of cooperating agents,"IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics, vol. 26, no 1, pp. 29-41, feb. 1996
[5] M. Dorigo, and L. M. Gambardella, "Ant colony system: a cooperative learning approach to the traveling salesman problem," IEEE Transactions on Evolutionary Computation, vol. 1, no 1, pp. 53-66, avr 1997.
[6] T. Stutzle, and H. Hoos, "MAX-MIN Ant System and local search for the traveling salesman problem,"in , IEEE International Conference on Evolutionary Computation, pp. 309-314, 1997
[7] F. Roces, J. Tautz, and B. Hölldobler, "Stridulation in leaf-cutting ants," Naturwissenschaften, vol. 80, no 11, pp. 521-524, nov. 1993.
[8] R. Hickling, and R. L. Brown, "Analysis of acoustic communication by ants," The Journal of the Acoustical Society of America, vol. 108, no 4, pp. 1920-1929, oct. 2000.
[9] Markl H., "Stridulation in Leaf-Cutting Ants," Science, vol. 149, no 3690, pp. 1392-1393, sep. 1965
[10] T. Schiavinotto, and T. Stützle, "A review of metrics on permutations for search landscape analysis,"Computers\&Operations Research, vol. 34, no 10, pp. 3143-3153, oct. 2007.
[11] X. H. Shi, Y. C. Liang, H. P. Lee, C. Lu, and Q. X. Wang, "Particle swarm optimization-based algorithms for TSP and generalized TSP," Information Processing Letters, vol. 103, no 5, pp. 169-176, août 2007.
[12] L. M. Gambardella, and M. Dorigo, "An Ant Colony System Hybridized with a New Local Search for the Sequential Ordering Problem," INFORMS Journal on Computing, vol. 12, no 3, pp. 237-255, août 2000
[13] S. Ouadfel and M. Batouche, "Ant colony system with local search for Markov random field image segmentation," in 2003 International Conference on Image Processing, 2003. ICIP 2003. Proceedings, 2003, vol. 1, pp. I-133-6.
[14] D. J. Rosenkrantz, R. E. Stearns, and P. M. Lewis, II, "An Analysis of Several Heuristics for the Traveling Salesman Problem," SIAM Journal on Computing, vol. 6, no 3, pp. 563-581, sep. 1977.

