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# Analyzing Multi-Labeled Data Based on the Roll of a Concept against a Semantic Range

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*Abstract*—Classifying data hierarchically is an efficient approach to analyze data. Data is usually classified into multiple categories, or annotated with a set of labels. To analyze multi-labeled data, such data must be specified by giving a set of labels as a semantic range. There are some certain purposes to analyze data. This paper shows which multi-labeled data should be the target to be analyzed for those purposes, and discusses the role of a label against a set of labels by investigating the change when a label is added to the set of labels. These discussions give the methods for the advanced analysis of multi-labeled data, which are based on the role of a label against a semantic range.

*Keywords*—Classification Hierarchies, Data Analysis, Multi-labeled Data, Orders of Sets of Labels

## I. Introduction

FOR the purpose of getting more competitive advantages in economic competition, companies and governments have now needed information systems which can analyze collected data to support their decision making. With rapidly increasing data, including numerical data, texts data, image data, and audio data, it is becoming more important to organize collected data properly. Classifying data hierarchically is one of the efficient approach to organize collected data [1] [8]. Individual data is classified into various categories, or annotated with their categories which are used to specify a set of data to be analyzed.

Data is usually assumed to be classified into one category and annotated with the label of the category [1] [11]. For example, each news document in *Newsgroups* data set is classified into only one category [9]. Although specifying such data is straightforward if the data has only one label, there is data which should be classified into multiple categories. If such data is classified into only one category, it is not specified by the labels of the other classes into which the data should also be classified.

Multiple classifications give much more information for analysis because the data can be specified by several labels. For example, data, which is about the comparison between Japan and U.S.A, can be analyzed as the data related to both categories *Japan* and *U.S.A*, if the data is classified into those two categories. Such data is usually classified into multiple categories and annotated with multiple labels of the categories [5] [6]. For example, the data mentioned above is annotated with multiple labels $\{$*Japan, U.S.A*$\}$.

To analyze multi-labeled data, such data must be specified by giving a set of labels as a semantic range. There are some current researches about classification on data annotated with multiple labels. However, the set of data specified by a set of labels are usually the result of the intersection or union of the data specified by each label of the set of labels [2] [7] [8].

What data is expressed by a set of labels has been discussed in [4] by introducing the orders between the set of labels. The previous researches do not refer to the analysis of the multi-labeled data from such detailed discussions. This paper discusses the semantic ranges of data to be analyzed and the correspondence between the ranges and the orders precisely. And then, the role of a label against a set of labels is shown by investigating the change when a label is added to the set of labels. This discussion gives methods for the advanced analysis of multi-labeled data, which are based on the role of a label against a set of labels.

This paper is organized as follows. The orders for sets of labels in order to express multi-labeled data are introduced in Section 2. Section 3 discusses the semantic ranges of data to be analyzed and the correspondence between the ranges and the orders. Sections 4 and 5 show the analysis methods for multi-labeled data which are based on the role of a label against a set of labels. Section 6 concludes the paper.

## II. Introducing Orders for Sets of Labels

Individual data or an object is classified by certain types of characteristic, which are called attributes. For example, an object is classified into countries, states, cities, etc., where the attribute is region. While there is classification for multiple attributes, this paper discusses one specific attribute for simplicity, and assumes that a classification hierarchy of the attribute is given in advance and objects will be classified based on the hierarchy.

Let $o$ be an object and $L$ be a label which is used to classify objects. Let $\overline{L}$ be the set of the objects expressed by $L$, and $\widetilde{o}$ be the label of $o$ for the classification attribute. An object is usually classified into the lowest category (or categories in multi-label classification) related to the object in a given classification hierarchy [5] [6]. $\widetilde{o}$ is the label (or the set of labels) of the category (or the categories) into which $o$ is classified. Objects may be classified into intermediate categories, which are not leaves in the hierarchy [3] [10]. For example, the label of an object on Japan is on the country

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level, which is not a label of a leaf category if the hierarchy has still city level categories.

For labels $L_1$ and $L_2$, $L_2$ is higher than $L_1$ ($L_1$ is lower than $L_2$) if the category of $L_2$ is a higher concept of the category of $L_1$, denoted by $L_1 \prec L_2$. $L_1 \preceq L_2$ denotes that $L_2$ is higher than or equal to $L_1$. The membership of single-labeled objects to $\overline{L}$ is decided by the label of the objects as $\overline{L} = \{o \mid \widetilde{o} \preceq L\}$.

When an object is classified into more than one category, the label of this object is a set of labels, called set-label.

For an object $o$ with a set-label, $\overline{L}$ is defined by introducing an order between the set-label $\widetilde{o}$ and a label $L$. A set of labels $\boldsymbol{L}$ is usually interpreted as conjunction or disjunction of the objects described by the labels in $\boldsymbol{L}$. The orders for these interpretations are follows.

1) Conjunction: For a label $L$ and a set of labels $\boldsymbol{L}$, $\boldsymbol{L}$ is lower than or equal to $L$ if every label of $\boldsymbol{L}$ is lower than or equal to $L$, denoted by $\boldsymbol{L} \preceq_C L$.
2) Disjunction: For a label $L$ and a set of labels $\boldsymbol{L}$, $\boldsymbol{L}$ is lower than or equal to $L$ if some label of $\boldsymbol{L}$ is lower than or equal to $L$, denoted by $\boldsymbol{L} \preceq_D L$.

A label used to express objects is extended to a set of labels. Let $\overline{\boldsymbol{L}}$ be the set of the objects expressed by a set of labels $\boldsymbol{L}$, and conjunction and disjunction interpretations of a set of labels for a single label are extended to for a set of labels. Generally a set of labels $\boldsymbol{L}$ is interpreted as the intersection or the union of the sets of objects expressed by the labels of $\boldsymbol{L}$. Let $\overline{\boldsymbol{L}}^{CI} = \bigcap_{L \in \boldsymbol{L}} \{o \mid \widetilde{o} \preceq_C L\}$ and $\overline{\boldsymbol{L}}^{CU} = \bigcup_{L \in \boldsymbol{L}} \{o \mid \widetilde{o} \preceq_C L\}$ be the intersection and the union of the sets of objects expressed by the labels in $\boldsymbol{L}$ for conjunction, respectively, and $\overline{\boldsymbol{L}}^{DI} = \bigcap_{L \in \boldsymbol{L}} \{o \mid \widetilde{o} \preceq_D L\}$ and $\overline{\boldsymbol{L}}^{DU} = \bigcup_{L \in \boldsymbol{L}} \{o \mid \widetilde{o} \preceq_D L\}$ be the intersection and the union for disjunction, respectively.

Since the set of objects expressed by $\boldsymbol{L}$ is decided by the order of $\widetilde{o}$ and $\boldsymbol{L}$, orders for sets of labels have to be introduced. The orders corresponding to $\overline{\boldsymbol{L}}^{CI}$, $\overline{\boldsymbol{L}}^{CU}$, $\overline{\boldsymbol{L}}^{DI}$, and $\overline{\boldsymbol{L}}^{DU}$ are defined as follows.

**Definition 1** For sets of labels $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$,
$\boldsymbol{L}_1 \preceq_{CI} \boldsymbol{L}_2$ if $\forall L_2 \in \boldsymbol{L}_2$, $\forall L_1 \in \boldsymbol{L}_1$, $L_1 \preceq L_2$,
$\boldsymbol{L}_1 \preceq_{CU} \boldsymbol{L}_2$ if $\exists L_2 \in \boldsymbol{L}_2$, $\forall L_1 \in \boldsymbol{L}_1$, $L_1 \preceq L_2$,
$\boldsymbol{L}_1 \preceq_{DI} \boldsymbol{L}_2$ if $\forall L_2 \in \boldsymbol{L}_2$, $\exists L_1 \in \boldsymbol{L}_1$, $L_1 \preceq L_2$, and
$\boldsymbol{L}_1 \preceq_{DU} \boldsymbol{L}_2$ if $\exists L_2 \in \boldsymbol{L}_2$, $\exists L_1 \in \boldsymbol{L}_1$, $L_1 \preceq L_2$. □

**Theorem 1** [4] For a set of labels $\boldsymbol{L}$, $\overline{\boldsymbol{L}}^{CI} = \{o \mid \widetilde{o} \preceq_{CI} \boldsymbol{L}\}$, $\overline{\boldsymbol{L}}^{CU} = \{o \mid \widetilde{o} \preceq_{CU} \boldsymbol{L}\}$, $\overline{\boldsymbol{L}}^{DI} = \{o \mid \widetilde{o} \preceq_{DI} \boldsymbol{L}\}$, and $\overline{\boldsymbol{L}}^{DU} = \{o \mid \widetilde{o} \preceq_{DU} \boldsymbol{L}\}$. □

There can be, on the other hand, the extension of these orders for a set of labels and a single-labeled object. There are two interpretations of a set of labels for single-labeled objects, intersection and union, which are formally expressed as $\bigcap_{L \in \boldsymbol{L}} \overline{L}$ and $\bigcup_{L \in \boldsymbol{L}} \overline{L}$, respectively.

Suppose that interpretation of $\boldsymbol{L}$ is intersection. For a single-labeled object $o$ and $L' = \widetilde{o}$, $L'$ is lower than or equal to every label in $\boldsymbol{L}$, and $\overline{L'} \subseteq \bigcap_{L \in \boldsymbol{L}} \overline{L} = \bigcap_{L \in \boldsymbol{L}} \{o \mid \widetilde{o} \preceq L\}$. Thus the set of multi-labeled objects expressed by $\boldsymbol{L}$ with conjunction is $\bigcap_{L \in \boldsymbol{L}} \{o \mid \forall L' \in \widetilde{o}, L' \preceq L\}$, denoted by $\overline{\boldsymbol{L}}^{IC}$.

In the same way as $\overline{\boldsymbol{L}}^{IC}$, the sets of objects expressed by $\boldsymbol{L}$ for intersection interpretation of $\boldsymbol{L}$ with disjunction of multi-labeled objects, and for union interpretation of $\boldsymbol{L}$ with conjunction and disjunction of multi-labeled objects are defined as $\overline{\boldsymbol{L}}^{ID} = \bigcap_{L \in \boldsymbol{L}} \{o \mid \exists L' \in \widetilde{o}, L' \preceq L\}$, $\overline{\boldsymbol{L}}^{UC} = \bigcup_{L \in \boldsymbol{L}} \{o \mid \forall L' \in \widetilde{o}, L' \preceq L\}$, and $\overline{\boldsymbol{L}}^{UD} = \bigcup_{L \in \boldsymbol{L}} \{o \mid \exists L' \in \widetilde{o}, L' \preceq L\}$.

Since the set of objects expressed by $\boldsymbol{L}$ consists of the objects whose labels are lower than or equal to $\boldsymbol{L}$, the orders corresponding to $\overline{\boldsymbol{L}}^{IC}$, $\overline{\boldsymbol{L}}^{ID}$, $\overline{\boldsymbol{L}}^{UC}$, and $\overline{\boldsymbol{L}}^{UD}$ are introduced.

**Definition 2** For sets of labels $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$,
$\boldsymbol{L}_1 \preceq_{IC} \boldsymbol{L}_2$ if $\forall L_1 \in \boldsymbol{L}_1$, $\forall L_2 \in \boldsymbol{L}_2$, $L_1 \preceq L_2$,
$\boldsymbol{L}_1 \preceq_{ID} \boldsymbol{L}_2$ if $\exists L_1 \in \boldsymbol{L}_1$, $\forall L_2 \in \boldsymbol{L}_2$, $L_1 \preceq L_2$,
$\boldsymbol{L}_1 \preceq_{UC} \boldsymbol{L}_2$ if $\forall L_1 \in \boldsymbol{L}_1$, $\exists L_2 \in \boldsymbol{L}_2$, $L_1 \preceq L_2$, and
$\boldsymbol{L}_1 \preceq_{UD} \boldsymbol{L}_2$ if $\exists L_1 \in \boldsymbol{L}_1$, $\exists L_2 \in \boldsymbol{L}_2$, $L_1 \preceq L_2$. □

**Theorem 2** [4] For a set of labels $\boldsymbol{L}$, $\overline{\boldsymbol{L}}^{IC} = \{o \mid \widetilde{o} \preceq_{IC} \boldsymbol{L}\}$, $\overline{\boldsymbol{L}}^{ID} = \{o \mid \widetilde{o} \preceq_{ID} \boldsymbol{L}\}$, $\overline{\boldsymbol{L}}^{UC} = \{o \mid \widetilde{o} \preceq_{UC} \boldsymbol{L}\}$, and $\overline{\boldsymbol{L}}^{UD} = \{o \mid \widetilde{o} \preceq_{UD} \boldsymbol{L}\}$. □

There are eight kinds of orders which can be integrated to three kinds according to their definitions. Since the definitions of orders $\preceq_{CI}$ and $\preceq_{UD}$ are equal to $\preceq_{IC}$ and $\preceq_{DU}$, respectively, $\preceq_{CI}$ and $\preceq_{UD}$ are excluded. It is obvious that $\overline{\boldsymbol{L}}^{ID}$ is a special case of $\overline{\boldsymbol{L}}^{DI}$, and both $\overline{\boldsymbol{L}}^{IC}$ and $\overline{\boldsymbol{L}}^{CU}$ are special cases of $\overline{\boldsymbol{L}}^{UC}$ by the definitions of those orders. Orders $\preceq_{ID}$, $\preceq_{IC}$, and $\preceq_{CU}$ are excluded from our considerations. Thus the orders are summarized to three kinds which are the orders $\preceq_{DI}$, $\preceq_{UC}$, and $\preceq_{DU}$.

While the set of objects described by a set of labels $\boldsymbol{L}$ is decided by the order of $\boldsymbol{L}$ and $\widetilde{o}$, there may exist some labels in $\boldsymbol{L}$ and $\widetilde{o}$ which do not affect this membership.

**Example 1** Suppose $\boldsymbol{L}_1$ and $\widetilde{o_1}$ are {*Japan, U.S.A*} and {*Tokyo, New York, Shanghai*}, respectively. $o_1$ is in $\overline{\boldsymbol{L}_1}^{DI}$ because there is a lower label in $\widetilde{o_1}$ for each label in $\boldsymbol{L}_1$. *Shanghai* in $\widetilde{o_1}$ does not affect this membership. Although there must be a label in $\widetilde{o_1}$ for each label of $\boldsymbol{L}_1$, $\widetilde{o_1}$ may include labels unrelated to $\boldsymbol{L}_1$. On the other hand, the labels of object $o_2$ labeled {*Tokyo, Kyoto*} in $\overline{\boldsymbol{L}_1}^{UC}$ are not lower than or equal to label *U.S.A* in $\boldsymbol{L}_1$. Object $o_3$ labeled {*Tokyo, Shanghai*} is in $\overline{\boldsymbol{L}_1}^{DU}$, where $U.S.A$ in $\boldsymbol{L}_1$ and $Shanghai$ in $\widetilde{o_3}$ do not affect the membership of $o_3$ to $\overline{\boldsymbol{L}_1}^{DU}$ at all. Fig. 1 illustrates these memberships. □

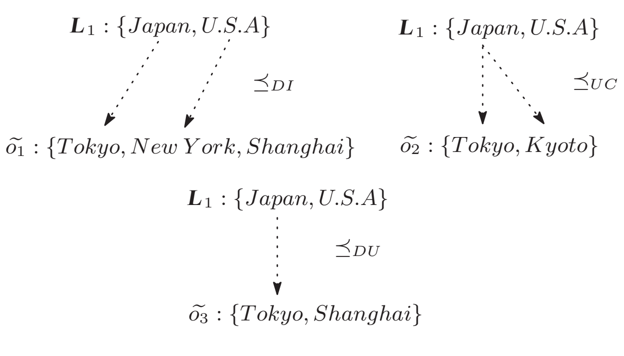


Fig. 1. Labels for Membership

For sets of labels $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$, $\boldsymbol{L}_1 \preceq_{DI} \boldsymbol{L}_2$ requires that each label of $\boldsymbol{L}_2$ is lower than or equal to some label in $\boldsymbol{L}_1$, which is a restriction on the higher set $\boldsymbol{L}_2$. In the same way, $\boldsymbol{L}_1 \preceq_{UC} \boldsymbol{L}_2$ has the restriction on the lower set $\boldsymbol{L}_1$. There is no restriction for $\boldsymbol{L}_1 \preceq_{DU} \boldsymbol{L}_2$. Thus $\preceq_{DI}$, $\preceq_{UC}$, and $\preceq_{DU}$ are renamed to $\preceq_{RU}$, $\preceq_{RL}$, and $\preceq_{RN}$, respectively.

**Definition 3** For sets of labels $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$,
$\boldsymbol{L}_1 \preceq_{RU} \boldsymbol{L}_2$ if $\forall L_2 \in \boldsymbol{L}_2, \exists L_1 \in \boldsymbol{L}_1, L_1 \preceq L_2$,
$\boldsymbol{L}_1 \preceq_{RL} \boldsymbol{L}_2$ if $\forall L_1 \in \boldsymbol{L}_1, \exists L_2 \in \boldsymbol{L}_2, L_1 \preceq L_2$, and
$\boldsymbol{L}_1 \preceq_{RN} \boldsymbol{L}_2$ if $\exists L_1 \in \boldsymbol{L}_1, \exists L_2 \in \boldsymbol{L}_2, L_1 \preceq L_2$. □

Let $\overline{\boldsymbol{L}}^{RU}$, $\overline{\boldsymbol{L}}^{RL}$, and $\overline{\boldsymbol{L}}^{RN}$ be the sets of the objects expressed by a set of labels $\boldsymbol{L}$ with orders $\preceq_{RU}$, $\preceq_{RL}$, and $\preceq_{RN}$, respectively, then $\overline{\boldsymbol{L}}^{RU} = \overline{\boldsymbol{L}}^{DI}$, $\overline{\boldsymbol{L}}^{RL} = \overline{\boldsymbol{L}}^{UC}$, and $\overline{\boldsymbol{L}}^{RN} = \overline{\boldsymbol{L}}^{DU}$.

There may be other orders defined as that a set of labels $\boldsymbol{L}_1$ are lower than or equal to a set of labels $\boldsymbol{L}_2$ if $\boldsymbol{L}_1 \preceq_x \boldsymbol{L}_2$ and $\boldsymbol{L}_1 \preceq_y \boldsymbol{L}_2$ $(x, y \in \{RN, RU, RL\})$. The orders except the order defined with $x = RU$ and $y = RL$ are either $\preceq_x$ or $\preceq_y$. For example, the order defined with $x = RN$ and $y = RU$ is $\preceq_{RU}$.

The order where $x = RU$ and $y = RL$ has restrictions of $\preceq_{RU}$ and $\preceq_{RL}$. Such order is denoted by $\preceq_{RB}$, where $\preceq_{RB}$ restricts both of higher and lower sets of labels. Let $\overline{\boldsymbol{L}}^{RB}$ be the set of objects expressed by a set of labels $\boldsymbol{L}$ with order $\preceq_{RB}$. Since $\overline{\boldsymbol{L}}^{RB}$ is expressed as $\overline{\boldsymbol{L}}^{RB} = \{o \mid \tilde{o} \preceq_{RB} \boldsymbol{L}\} = \{o \mid \tilde{o} \preceq_{RU} \boldsymbol{L}, \tilde{o} \preceq_{RL} \boldsymbol{L}\}$, $\preceq_{RB}$ is defined as follows.

**Definition 4** For sets of labels $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$,
$\boldsymbol{L}_1 \preceq_{RB} \boldsymbol{L}_2$ if $\forall L_2 \in \boldsymbol{L}_2, \exists L_1 \in \boldsymbol{L}_1, L_1 \preceq L_2$ and
$\forall L_1 \in \boldsymbol{L}_1, \exists L_2 \in \boldsymbol{L}_2, L_1 \preceq L_2$. □

## III. Semantic Ranges of The Target Objects

If objects are annotated with a single-label, it is easy to specify the target objects to be analyzed. On the other hand, if objects are annotated with set-labels, it is usually supposed that there are several kinds of the relations between the set-labels of the objects and a given set of labels. This section shows the target objects to be changed with these interpretations and the correspondence between the objects and the orders proposed in Section 2.

The target objects are decided by the conditions for their set-labels against a given set of labels. There are two kinds of conditions. One is related to the labels which are included in $\boldsymbol{L}$, and the other is related to the labels which are not included in $\boldsymbol{L}$.

*The condition which is related to the labels included in* $\mathbf{L}$

There are two ways to decide the target objects, for the labels included in $\boldsymbol{L}$:

(1) The set-label of an object is only related to some of the labels in $\boldsymbol{L}$.

(2) The set-label of an object is related to all labels of $\boldsymbol{L}$.

The object described by (1) is the object with the set-label which has the label lower than or equal to some label of $\boldsymbol{L}$, namely the range of the target objects is specified by $\boldsymbol{L}$.

The object described by (2) is the object with the set-label, which has the label that is lower than or equal to $L$ for each label $L$ of $\boldsymbol{L}$, and each label of $\boldsymbol{L}$ is higher than or equal to some label of the set-label. It can be used to analyze the connection of $\boldsymbol{L}$.

*The condition between the set-label and the labels which are not included in* $\mathbf{L}$

There are two kinds of relations between the set-label and the labels which are not included in $\boldsymbol{L}$:

(a) The set-label of an object may be related to the labels not included in $\boldsymbol{L}$.

(b) The set-label of an object must not be related to the labels not included in $\boldsymbol{L}$.

The objects described by (a) are the objects with the set-labels which may have a label lower than or equal to a label not in $\boldsymbol{L}$, namely, the target objects are not limited within $\boldsymbol{L}$.

The objects described by (b) are the objects with the set-labels which do not have a label lower than or equal to any other labels not in $\boldsymbol{L}$, namely, the objects to be analyzed are limited within $\boldsymbol{L}$.

By combining these two conditions, there are four kinds of target objects which are annotated with set-labels.

(1-a) The set-label of an object is related to some label of $\boldsymbol{L}$ and may be related to labels not included in $\boldsymbol{L}$. For this kind of objects, the target of the objects is specified by $\boldsymbol{L}$ and the objects are not limited within $\boldsymbol{L}$.

(1-b) The set-label of an object is related to some label of $\boldsymbol{L}$ and must not be related to the labels which are not included in $\boldsymbol{L}$. For this kind of objects, the target of the objects is specified by $\boldsymbol{L}$ and the objects are limited within $\boldsymbol{L}$.

(2-a) The set-label of an object is related to all labels of $\boldsymbol{L}$ and may be related to the labels which are not included in $\boldsymbol{L}$. For this kind of objects, the connection of $\boldsymbol{L}$ is analyzed and the objects are not limited within $\boldsymbol{L}$.

(2-b) The set-label of an object is related to all labels of $\boldsymbol{L}$ and must not be related to the labels which are not included in $\boldsymbol{L}$. For this kind of objects, the connection of $\boldsymbol{L}$ is analyzed and the objects are limited within $\boldsymbol{L}$.

The following shows that the four kinds of objects correspond to the orders proposed in Section 2 by discussing the objects expressed with each order.

For a given set of labels $\boldsymbol{L}$, $\overline{\boldsymbol{L}}^{RN}$ and $\overline{\boldsymbol{L}}^{RL}$ are the union of the objects expressed by the labels of $\boldsymbol{L}$, and $\overline{\boldsymbol{L}}^{RU}$ and $\overline{\boldsymbol{L}}^{RB}$ are the intersection of the objects expressed by the labels of $\boldsymbol{L}$. $\overline{\boldsymbol{L}}^{RN}$ and $\overline{\boldsymbol{L}}^{RU}$ include objects with labels which are not related to $\boldsymbol{L}$, while $\overline{\boldsymbol{L}}^{RL}$ and $\overline{\boldsymbol{L}}^{RB}$ do not. In the other words, the labels of the objects in $\overline{\boldsymbol{L}}^{RL}$ and $\overline{\boldsymbol{L}}^{RB}$ are within the range of $\boldsymbol{L}$.

**Example 2** For set of labels $\boldsymbol{L} = \{Japan, U.S.A\}$, $\overline{\boldsymbol{L}}^{RN}$ and $\overline{\boldsymbol{L}}^{RL}$ are the union of the objects expressed by the labels of $\boldsymbol{L}$, which include objects labeled {*Tokyo*}, {*Tokyo, New York*}, and {*Tokyo, New York, Shanghai*} for $\overline{\boldsymbol{L}}^{RN}$, and {*Tokyo*} and {*Tokyo, New York*} for $\overline{\boldsymbol{L}}^{RL}$. $\overline{\boldsymbol{L}}^{RU}$ and $\overline{\boldsymbol{L}}^{RB}$ are the intersection, which include the objects labeled {*Tokyo, New York*} and {*Tokyo, New York, Shanghai*} for $\overline{\boldsymbol{L}}^{RU}$, and {*Tokyo, New York*} for $\overline{\boldsymbol{L}}^{RB}$. While objects of $\overline{\boldsymbol{L}}^{RN}$ and $\overline{\boldsymbol{L}}^{RU}$ may include label *Shanghai* which is related to neither *Japan* nor

| | | The labels which are not contained in $\boldsymbol{L}$ (Range of a Set of Labels) | |
|---|---|---|---|
| | | Included (Not Exists) | Not Included (Exists) |
| The labels which are contained in $\boldsymbol{L}$ (Interpretation of a set of labels) | Some labels (Union) | 1-a (RN) | 1-b (RL) |
| | All labels (Intersection) | 2-a (RU) | 2-b (RB) |

Fig. 2. Objects to be Analyzed and Orders of Sets of Labels

*U.S.A*, the labels of objects of $\overline{\boldsymbol{L}}^{RL}$ and $\overline{\boldsymbol{L}}^{RB}$ are within the range of *Japan* and *U.S.A*. □

Since objects described by (1) are only related to some label of $\boldsymbol{L}$, the set of target objects are the union of the objects related to the labels in $\boldsymbol{L}$, and both $\preceq_{RN}$ and $\preceq_{RL}$ are orders for (1). Since objects described by (2) are related to all of labels in $\boldsymbol{L}$, the set of target objects are the intersection of the objects related to the labels in $\boldsymbol{L}$, and both $\preceq_{RU}$ and $\preceq_{RB}$ are orders for (2). Since objects described by (a) can be related to the labels not included in $\boldsymbol{L}$, $\preceq_{RN}$ and $\preceq_{RU}$ are orders for (a). Since objects described by (b) must not be related to the labels not included in $\boldsymbol{L}$, $\preceq_{RL}$ and $\preceq_{RB}$ are orders for (b). By analyzing the objects with a set of labels, the objects described by (1-a), (1-b), (2-a), and (2-b) are the objects expressed with orders $\preceq_{RN}$, $\preceq_{RL}$, $\preceq_{RU}$, and $\preceq_{RB}$, respectively. Thus the objects to be analyzed will be specified by the set of labels with these orders. The discussions of this section are summarized in Figure 2.

## IV. Analysis Methods by Adding a Label to a Set of Labels

This section shows the role of a label against a set of labels by discussing the change of objects expressed by a set of labels when a new label is added into the original set of labels. It gives the analysis method, which is based on the role of a label against a set of labels for multi-labeled objects.

In this section, it's assumed that $L_i \not\preceq L_j$ $(L_i, L_j \in \boldsymbol{L})$ for a set of labels, and such set $\boldsymbol{L}$ is called exclusive. In section 5, the discussion is extended to a set of labels where $L_i \preceq L_j$ may appear. Since deleting a label is the opposite operation to adding a label, only the case of the addition is discussed.

The objects expressed by adding a label are shown as Lemma 1, where Power $(\boldsymbol{L})$ is the power set of a set of labels $\boldsymbol{L}$.

**Lemma 1** For a set of labels $\boldsymbol{L}$ and a label $L$,
$\overline{\boldsymbol{L}\cup\{L\}}^{RN} = \overline{\boldsymbol{L}}^{RN} \cup \overline{\{L\}}^{RN}$,
$\overline{\boldsymbol{L}\cup\{L\}}^{RU} = \overline{\boldsymbol{L}}^{RU} \cap \overline{\{L\}}^{RU}$,
$\overline{\boldsymbol{L}\cup\{L\}}^{RL} = \overline{\boldsymbol{L}}^{RL} \cup \bigcup_{\boldsymbol{L}'\in Power(\boldsymbol{L})} \overline{\boldsymbol{L}'\cup\{L\}}^{RB}$, and
$\overline{\boldsymbol{L}\cup\{L\}}^{RB} \cap \overline{\boldsymbol{L}}^{RB} = \phi$. □

*Proof:* Since an object $o$ in $\overline{\boldsymbol{L}\cup\{L\}}^{RN}$ has the set-label whose label is lower than or equal to some label of $\boldsymbol{L}$ or $L$, $o$ is included in $\overline{\boldsymbol{L}}^{RN}$ or $\overline{\{L\}}^{RN}$, that is, $\overline{\boldsymbol{L}\cup\{L\}}^{RN} \subseteq \overline{\boldsymbol{L}}^{RN} \cup \overline{\{L\}}^{RN}$. Since an object $o$ in $\overline{\boldsymbol{L}}^{RN} \cup \overline{\{L\}}^{RN}$ has the set-label whose label is lower than or equal to some label of $\boldsymbol{L}$ or $L$, $o$ is included in $\overline{\boldsymbol{L}\cup\{L\}}^{RN}$, which indicates $\overline{\boldsymbol{L}\cup\{L\}}^{RN} \supseteq \overline{\boldsymbol{L}}^{RN} \cup \overline{\{L\}}^{RN}$, and then $\overline{\boldsymbol{L}\cup\{L\}}^{RN} = \overline{\boldsymbol{L}}^{RN} \cup \overline{\{L\}}^{RN}$.

The proof of $\overline{\boldsymbol{L}\cup\{L\}}^{RU} = \overline{\boldsymbol{L}}^{RU} \cap \overline{\{L\}}^{RU}$ is as the same as the above. For an object $o$ in $\overline{\boldsymbol{L}\cup\{L\}}^{RU}$, $o$ is also included in $\overline{\boldsymbol{L}}^{RU} \cap \overline{\{L\}}^{RU}$ because there exists such label $L'$ in $\widetilde{o}$ that $L' \preceq L''$ for each label $L''$ in $\boldsymbol{L}\cup\{L\}$. It indicates that $\overline{\boldsymbol{L}\cup\{L\}}^{RU} \subseteq \overline{\boldsymbol{L}}^{RU} \cap \overline{\{L\}}^{RU}$. Since it is obvious that $\overline{\boldsymbol{L}\cup\{L\}}^{RU} \supseteq \overline{\boldsymbol{L}}^{RU} \cap \overline{\{L\}}^{RU}$, $\overline{\boldsymbol{L}\cup\{L\}}^{RU} = \overline{\boldsymbol{L}}^{RU} \cap \overline{\{L\}}^{RU}$.

An object $o$ in $\overline{\boldsymbol{L}\cup\{L\}}^{RL} - \overline{\boldsymbol{L}}^{RL}$ is such an object that there must exist a label in $\widetilde{o}$ which is lower than or equal to $L$ and there may exist a label in $\widetilde{o}$ which is lower than or equal to some label of $\boldsymbol{L}$. Such objects are the union of the objects expressed by each element of the power set of $\boldsymbol{L}\cup\{L\}$ with order $\preceq_{RB}$.

The definition of order $\preceq_{RB}$ shows that $\overline{\boldsymbol{L}\cup\{L\}}^{RB} \cap \overline{\boldsymbol{L}}^{RB} = \phi$ if the sets of labels are different from each other. *Q.E.D.*

For a set of labels $\boldsymbol{L}$ and a label $L$, the objects expressed by $\boldsymbol{L}\cup\{L\}$ with $\preceq_{RN}$ ($\preceq_{RU}$) are the result of the union (intersection) of the objects expressed by $\boldsymbol{L}$ and the objects expressed by $L$.

The newly added objects by adding $L$ to $\boldsymbol{L}$ with $\preceq_{RL}$ are the result of the union of the objects expressed by $L$ and each element of the power set of $\boldsymbol{L}$ with $\preceq_{RB}$.

**Example 3** If label $\{China\}$ is added to set of labels $\{Japan, U.S.A\}$, the result of the union of $\overline{\{China\}}^{RB}$, $\overline{\{Japan, China\}}^{RB}$, $\overline{\{U.S.A, China\}}^{RB}$, and $\overline{\{Japan, U.S.A, China\}}^{RB}$ are newly added to $\overline{\{Japan, U.S.A\}}^{RL}$. □

The objects expressed by $\boldsymbol{L}\cup\{L\}$ with $\preceq_{RB}$ are totally different from the objects expressed by $\boldsymbol{L}$. Since it is impossible to compare mutually, $\preceq_{RB}$ is not discussed.

By Lemma 1, the change of target objects by adding a label $L$ and $\boldsymbol{L}$ are $\overline{\boldsymbol{L}}^{RN} \subseteq \overline{\boldsymbol{L}\cup\{L\}}^{RN}$, $\overline{\boldsymbol{L}}^{RU} \supseteq \overline{\boldsymbol{L}\cup\{L\}}^{RU}$, and $\overline{\boldsymbol{L}}^{RL} \subseteq \overline{\boldsymbol{L}\cup\{L\}}^{RL}$. Since $\preceq_{RN}$ and $\preceq_{RL}$ are used to specify the range of the analysis, the target objects are increased by adding a label to the set of labels. On the opposite, the objects expressed by a set of labels with $\preceq_{RU}$ are decreased by adding a label. Since $\preceq_{RU}$ is used to analyze the connection of a set of labels, the objects to be analyzed are decreased because some objects which are not related to the newly added label are deleted from the original set of objects.

Based on adding a new label, the rest of this section discusses the analysis method by investigating newly added or deleted objects. Let $\Omega$ be the set of labels which consists of all of the labels in the classification hierarchy. For a set of labels $\boldsymbol{L}$, the set of labels whose labels are not lower than or equal to any labels of $\boldsymbol{L}$ are formally expressed as $\boldsymbol{L}^C = \{L | L \in \Omega, \forall L' \in \boldsymbol{L}(L' \neq L), L \not\preceq L'\}$.

**Theorem 3** For a set of labels $\boldsymbol{L}$ and a label $L$,

$$\overline{\boldsymbol{L} \cup \{L\}}^{RN} - \overline{\boldsymbol{L}}^{RN} = \bigcup_{\boldsymbol{L}' \in Power(\boldsymbol{L}^C)} \overline{\boldsymbol{L}' \cup \{L\}}^{RB},$$
$$\overline{\boldsymbol{L}}^{RU} - \overline{\boldsymbol{L} \cup \{L\}}^{RU} = \bigcup_{\boldsymbol{L}' \in Power(\{L\}^C)} \overline{\boldsymbol{L}' \cup \boldsymbol{L}}^{RB}, \text{ and}$$
$$\overline{\boldsymbol{L} \cup \{L\}}^{RL} - \overline{\boldsymbol{L}}^{RL} = \bigcup_{\boldsymbol{L}' \in Power(\boldsymbol{L})} \overline{\boldsymbol{L}' \cup \{L\}}^{RB}. \quad \square$$

*Proof:* By Lemma 1, $\overline{\boldsymbol{L} \cup \{L\}}^{RN} - \overline{\boldsymbol{L}}^{RN} = \overline{\{L\}}^{RN} - \overline{\boldsymbol{L}}^{RN}$. An object $o$ in $\overline{\{L\}}^{RN} - \overline{\boldsymbol{L}}^{RN}$ is an object related to $L$ which may be related to some other labels than $\boldsymbol{L}$. Such objects are the union of the objects expressed by each element of the power set of $\boldsymbol{L}^C$ and $L$ with order $\preceq_{RB}$.

By Lemma 1, $\overline{\boldsymbol{L}}^{RU} - \overline{\boldsymbol{L} \cup \{L\}}^{RU} = \overline{\boldsymbol{L}}^{RU} - \overline{\{L\}}^{RU}$. An object $o$ in $\overline{\boldsymbol{L}}^{RU} - \overline{\{L\}}^{RU}$ is the object related to $\boldsymbol{L}$ and may related to the other labels than $L$. Such objects are the union of the objects expressed by each element of the power set of $\{L^C\}$ and $L$ with order $\preceq_{RB}$.

It is obvious $\overline{\boldsymbol{L} \cup \{L\}}^{RL} - \overline{\boldsymbol{L}}^{RL} = \bigcup_{\boldsymbol{L}' \in Power(\boldsymbol{L})} \overline{\boldsymbol{L}' \cup \{L\}}^{RB}$ by Lemma 1. *Q.E.D.*

When a label $L$ is added to a set of labels $\boldsymbol{L}$, the objects related to $L$, and the objects related to $L$ and some other labels than $\boldsymbol{L}$ are newly added to the objects expressed by $\boldsymbol{L}$ with $\preceq_{RN}$. If the objects related to $L$, $\overline{\{L\}}^{RL}$ $(= \overline{\{L\}}^{RB})$ are deleted from $\overline{\boldsymbol{L} \cup \{L\}}^{RN} - \overline{\boldsymbol{L}}^{RN}$, the resulted objects show the connection of $L$ with some labels except $\boldsymbol{L}$.

In the case of $\preceq_{RU}$, the objects without the label which is lower than or equal to $L$ are excluded from the objects expressed by a set of labels $\boldsymbol{L}$. $\overline{\boldsymbol{L}}^{RU} - \overline{\boldsymbol{L} \cup \{L\}}^{RU}$ shows the connection of $L$ with $\boldsymbol{L}$.

In the case of $\preceq_{RL}$, the objects related to $L$, and the objects related to $L$ and some labels of $\boldsymbol{L}$ are newly added to the objects expressed by $\boldsymbol{L}$. If the objects related to $L$, namely, $\overline{\{L\}}^{RL}$ $(= \overline{\{L\}}^{RB})$ are deleted from $\overline{\boldsymbol{L} \cup \{L\}}^{RL} - \overline{\boldsymbol{L}}^{RL}$, the resulted objects show the connection of $L$ with some labels of $\boldsymbol{L}$ and such connection is within $L$ and $\boldsymbol{L}$.

**Example 4** For set of labels $\boldsymbol{L} = \{Japan, U.S.A\}$ and label $L = China$, the objects expressed by $\boldsymbol{L} \cup \{L\}$ are compared with the objects expressed by $\boldsymbol{L}$ with $\preceq_{RN}$, $\preceq_{RU}$, and $\preceq_{RL}$. If the aggregation values of $\overline{\{Japan, U.S.A, China\}}^{RN} - \overline{\{Japan, U.S.A\}}^{RN} - \overline{\{China\}}^{RL(=RB)}$ such as the number of the target objects is bigger, the conection of China with the other regions than Japan and U.S.A is stronger. On the other hand, $\preceq_{RU}$ is used to analyze the connection of China with Japan and U.S.A. If the aggregation values of $\overline{\{Japan, U.S.A\}}^{RU} - \overline{\{Japan, U.S.A, China\}}^{RU}$ is smaller, the connection of China with Japan and U.S.A is stronger. In the closed region of these countries, the connection of China with Japan or U.S.A is analyzed by $\preceq_{RL}$. If the aggregation values of $\overline{\{Japan, U.S.A, China\}}^{RL} - \overline{\{Japan, U.S.A\}}^{RL} - \overline{\{China\}}^{RL(=RB)}$ is bigger, the connection of China with Japan or U.S.A restricted in this region is stronger. $\square$

## V. Analysis for sets of labels which are not exclusive

In Section 4, a set of labels is assumed to be exclusive, that is there does not exist such labels in the set that the labels are higher or lower than other labels of the set. This section discusses the set of labels which are not exclusive.

Let $L$ be a label in $\boldsymbol{L}_1 - \boldsymbol{L}_2$ for sets of labels $\boldsymbol{L}_1$ and $\boldsymbol{L}_2$. The objects expressed by $\boldsymbol{L}_1$ are generally different from the objects expressed by $\boldsymbol{L}_2$ because there is $L$. Theorem 3 shows that the objects expressed by a set of labels $\boldsymbol{L}$ are changed by adding $L$ if $\{L\} \cup \boldsymbol{L}$ is exclusive. However, the objects expressed by the set of labels $\boldsymbol{L}$ added such a label $L$ that is higher or lower than some label of $\boldsymbol{L}$ are not always different from the objects expressed by $\boldsymbol{L}$.

A set of labels $\boldsymbol{L}$ can be reduced to the subset of $\boldsymbol{L}$ consisting of the labels which are not lower than any other labels of $\boldsymbol{L}$ for $\preceq_{RN}$ and $\preceq_{RL}$. Such subset is defined as the upper bound of $\boldsymbol{L}$, formally described as $u(\boldsymbol{L}) = \{L \mid L \in \boldsymbol{L}, \forall L' \in \boldsymbol{L} \ (L' \neq L), L \not\preceq L'\}$. In the same way as the upper bound of a set of labels, the lower bound of a set of labels $\boldsymbol{L}$ is introduced for $\preceq_{RU}$. The lower bound of $\boldsymbol{L}$ is the subset of $\boldsymbol{L}$ consisting of the labels which are not higher than any labels of $\boldsymbol{L}$, formally described as $l(\boldsymbol{L}) = \{L \mid L \in \boldsymbol{L}, \forall L' \in \boldsymbol{L} \ (L' \neq L), L' \not\preceq L\}$. Let $ul(\boldsymbol{L})$ be $u(\boldsymbol{L}) \cup l(\boldsymbol{L})$ for $\preceq_{RB}$.

The objects expressed by a set of labels $\boldsymbol{L}$ with $\preceq_{RN}$, $\preceq_{RU}$, $\preceq_{RL}$, and $\preceq_{RB}$, are the objects by $u(\boldsymbol{L})$, $l(\boldsymbol{L})$, $u(\boldsymbol{L})$, and $ul(\boldsymbol{L})$, respectively, which are shown as Lemma 2.

**Lemma 2** [4] For a set of labels $\boldsymbol{L}$,

$$\overline{\boldsymbol{L}}^{RN} = \overline{u(\boldsymbol{L})}^{RN},$$
$$\overline{\boldsymbol{L}}^{RU} = \overline{l(\boldsymbol{L})}^{RU},$$
$$\overline{\boldsymbol{L}}^{RL} = \overline{u(\boldsymbol{L})}^{RL}, \text{ and}$$
$$\overline{\boldsymbol{L}}^{RB} = \overline{ul(\boldsymbol{L})}^{RB}. \quad \square$$

A set of labels which is not exclusive can be reduced to the upper or the lower bound of the sets of labels. The reason why the objects expressed by a set of labels $\boldsymbol{L}$ with $\preceq_{RN}$ and $\preceq_{RL}$ are the objects expressed by $u(\boldsymbol{L})$ is that $\preceq_{RN}$ and $\preceq_{RL}$ are used to specify the range of the target objects and the higher concept labels cover the lower concept labels. Since $\preceq_{RU}$ is used to analyze the connection of $\boldsymbol{L}$ and the lower concept labels have harder connection than the higher concept labels, the objects expressed by $\boldsymbol{L}$ with $\preceq_{RU}$ agree with the objects expressed by $l(\boldsymbol{L})$. For $\preceq_{RB}$, the objects expressed by $\boldsymbol{L}$ are the objects expressed by the set of labels which consists of the upper and lower bounds of $\boldsymbol{L}$, because $\preceq_{RB}$ is defined with both conditions of $\preceq_{RU}$ and $\preceq_{RL}$.

When such a label $L$ is added to a set of labels $\boldsymbol{L}$ that $L$ is a higher concept than any other labels of $\boldsymbol{L}$, the inclusion relations between the objects expressed by $\boldsymbol{L}$ and the objects expressed by $\boldsymbol{L} \cup \{L\}$ are shown as Theorem 4.

**Theorem 4** For a set of labels $\boldsymbol{L}$ and a label $L$ such that $\exists L' \in \boldsymbol{L}, L' \preceq L$,

$$\overline{\boldsymbol{L}}^{RN} \subseteq \overline{\boldsymbol{L} \cup \{L\}}^{RN},$$
$$\overline{\boldsymbol{L}}^{RU} = \overline{\boldsymbol{L} \cup \{L\}}^{RU},$$

$\overline{\boldsymbol{L}}^{RL} \subseteq \overline{\boldsymbol{L} \cup \{L\}}^{RL}$, and
$\overline{\boldsymbol{L}}^{RB} \subseteq \overline{\boldsymbol{L} \cup \{L\}}^{RB}$. □

*Proof:* For a set of labels $\boldsymbol{L}$ and a label $L$ such that $\exists L' \in \boldsymbol{L}, L' \preceq L$, $\overline{\boldsymbol{L}}^{RU} = \overline{\boldsymbol{L} \cup \{L\}}^{RU}$ because of Lemma 2.

An object $o$ in $\overline{\boldsymbol{L}}^{RL}$ is included in $\overline{\boldsymbol{L} \cup \{L\}}^{RL}$ because there exists such label of $o$ that the label is lower than or equal to a label of $\boldsymbol{L}$, and $\overline{\boldsymbol{L}}^{RL} \subseteq \overline{\boldsymbol{L} \cup \{L\}}^{RL}$. In the same way, it is proved that $\overline{\boldsymbol{L}}^{RN} \subseteq \overline{\boldsymbol{L} \cup \{L\}}^{RN}$.

An object $o$ in $\overline{\boldsymbol{L}}^{RB}$ is included in $\overline{\boldsymbol{L} \cup \{L\}}^{RB}$ because for each label $L'$ of $\boldsymbol{L} \cup \{L\}$ there is a label of $\widetilde{o}$ that is lower than or equal to $L'$ and every label of $\widetilde{o}$ is lower than or equal to a label of $\boldsymbol{L} \cup \{L\}$. Thus $\overline{\boldsymbol{L}}^{RB} \subseteq \overline{\boldsymbol{L} \cup \{L\}}^{RB}$. *Q.E.D.*

By Theorem 4, the objects expressed by $\boldsymbol{L}$ with $\preceq_{RU}$ are not changed even $L$ is added, which means that adding such labels does not effect the analysis. For such the lowest label $L'$ in $\boldsymbol{L}$ that $L' \preceq L$, it is easy to prove that $\overline{\boldsymbol{L} \cup \{L\}}^{RN} - \overline{\boldsymbol{L}}^{RN} = \overline{\{L\}}^{RN} - \overline{\{L'\}}^{RN}$. Since the same objects as $\overline{\boldsymbol{L} \cup \{L\}}^{RN} - \overline{\boldsymbol{L}}^{RN}$ can be obtained with $L'$, it is not necessary to compare the objects expressed by $\boldsymbol{L}$ and the objects expressed by $\boldsymbol{L} \cup \{L\}$ even if the objects expressed with $\preceq_{RN}$ are changed by adding $L$. The objects expressed with $\preceq_{RL}$ are in the same situation. On the other hand, $\preceq_{RB}$ is used analyze the connection of $\boldsymbol{L}$ with labels within the range of $L$ because $\overline{\boldsymbol{L} \cup \{L\}}^{RB} - \overline{\boldsymbol{L}}^{RB}$ are the objects related to $\boldsymbol{L}$ and labels lower than or equal to $L$.

**Example 5** For set of labels $\boldsymbol{L} = \{Japan, China\}$ and label $L = Asia$, suppose the objects expressed by $\boldsymbol{L} \cup \{L\}$ are compared with the objects expressed by $\boldsymbol{L}$ with $\preceq_{RB}$. If the aggregation values of $\overline{\{Japan, China, Asia\}}^{RB} - \overline{\{Japan, China\}}^{RB}$ such as the number of the target objects is bigger, the connection of Japan and China with the other regions of Asia such as South Asia or India is stronger. □

When such a label $L$ is added to a set of labels $\boldsymbol{L}$ that $L$ is a lower concept than any other labels of $\boldsymbol{L}$, the inclusion relations between the objects expressed by $\boldsymbol{L}$ and the objects expressed by $\boldsymbol{L} \cup \{L\}$ are shown as Theorem 5.

**Theorem 5** For a set of labels $\boldsymbol{L}$ and a label $L$ where $\exists L' \in \boldsymbol{L}, L \preceq L'$,
$\overline{\boldsymbol{L}}^{RN} = \overline{\boldsymbol{L} \cup \{L\}}^{RN}$,
$\overline{\boldsymbol{L}}^{RU} \supseteq \overline{\boldsymbol{L} \cup \{L\}}^{RU}$,
$\overline{\boldsymbol{L}}^{RL} = \overline{\boldsymbol{L} \cup \{L\}}^{RL}$, and
$\overline{\boldsymbol{L}}^{RB} \supseteq \overline{\boldsymbol{L} \cup \{L\}}^{RB}$. □

*Proof:* For a set of labels $\boldsymbol{L}$ and a label $L$ where $\exists L' \in \boldsymbol{L}, L \preceq L'$, $\overline{\boldsymbol{L}}^{RN} = \overline{\boldsymbol{L} \cup \{L\}}^{RN}$ and $\overline{\boldsymbol{L}}^{RL} = \overline{\boldsymbol{L} \cup \{L\}}^{RL}$ because of Lemma 2.

For an object $o$ of $\overline{\boldsymbol{L}}^{RU}$, there is such label $L'$ in $\widetilde{o}$ that $L' \preceq L''$ for each label $L''$ in $\boldsymbol{L} \cup \{L\}$, and $o$ is included in $\overline{\boldsymbol{L} \cup \{L\}}^{RU}$. Thus $\overline{\boldsymbol{L}}^{RU} \supseteq \overline{\boldsymbol{L} \cup \{L\}}^{RU}$.

For an object $o$ of $\overline{\boldsymbol{L} \cup \{L\}}^{RB}$, $o$ is included in $\overline{\boldsymbol{L}}^{RB}$ because there is such label $L'$ in $\widetilde{o}$ that $L' \preceq L''$ for each label $L''$ in $\boldsymbol{L}$ and every label of $\widetilde{o}$ is lower than or equal to some label in $\boldsymbol{L}$, and then $\overline{\boldsymbol{L}}^{RB} \supseteq \overline{\boldsymbol{L} \cup \{L\}}^{RB}$. *Q.E.D.*

When such a label $L$ is added to a set of labels $\boldsymbol{L}$ that $L$ is a lower concept than any other labels of $\boldsymbol{L}$, the objects expressed by $\boldsymbol{L} \cup \{L\}$ with $\preceq_{RN}$ and $\preceq_{RL}$ are the same objects expressed by $\boldsymbol{L}$. Since the objects with $\preceq_{RN}$ and $\preceq_{RL}$ are used to specify the range of the target objects, $L$ does not extend the range because the range expressed by $L$ is covered by the range of the label of $\boldsymbol{L}$ which is higher than $L$. On the other hand, since $L$ is a lower concept than some label of $\boldsymbol{L}$, the connection of $\boldsymbol{L}$ is analyzed more precisely with $\preceq_{RU}$ and $\preceq_{RB}$.

**Example 6** For set of labels $\boldsymbol{L} = \{Japan, U.S.A\}$ and label $L = Tokyo$, suppose the objects expressed by $\boldsymbol{L} \cup \{L\}$ are compared with the objects expressed by $\boldsymbol{L}$ with $\preceq_{RU}$ and $\preceq_{RB}$. In the connection of Japan and U.S.A, the smaller the aggregation values of $\overline{\{Japan, U.S.A\}}^{RU} - \overline{\{Tokyo, U.S.A\}}^{RU}$ such as the number of the target objects is, the more important for the connection Tokyo is. As the same as the $\preceq_{RU}$, the smaller the aggregation values of $\overline{\{Japan, U.S.A\}}^{RB} - \overline{\{Japan, U.S.A, Tokyo\}}^{RB}$ is, the more important Tokyo is. □

## VI. Conclusions

This paper discussed the semantic ranges of objects to be analyzed precisely, and showed the correspondence of the objects to be analyzed and the objects expressed with orders. The objects to be analyzed are specified with $\preceq_{RN}$, $\preceq_{RU}$, $\preceq_{RL}$, and $\preceq_{RB}$. Based on the role of a label $L$ against a set of labels $\boldsymbol{L}$, this paper proposed some analysis methods for multi-labeled objects. The following three orders are used to analyze the connection about $L$ and $\boldsymbol{L}$. $\preceq_{RN}$, $\preceq_{RU}$, and $\preceq_{RL}$ are used to analyze the connection of $L$ with labels except $\boldsymbol{L}$, all labels of $\boldsymbol{L}$, and some labels of $\boldsymbol{L}$, respectively. For a set of labels which is not exclusive, $\preceq_{RB}$ can be used to analyze the connection of $\boldsymbol{L}$ in detail. The discussions of this paper are effective for the advanced analysis for multi-labeled objects. Economy globalization makes the connection among different countries more complicated such as China against Japan and U.S.A. With the results of this paper, the connection of China with Japan and U.S.A can be analyzed more precisely, for example.

## References


[1] Bertino, E., Fan, J., Ferrari, E., Hachi, M., and Elamagarmid, A.: A Hierarchical Access Control Model for Video Database Systems, *ACM Transactions on Information Systems*, Vol. 21, No. 2, pp. 151–191 (2003).

[2] Chakrabarti, K., Ganti, V., Han, J., and Xin, D.: Rankig Objects by Exploiting, Relationships: Computing Top-K over Aggregation, *Proc. ACM SIGMOD Int'l Conf. on Management of Data*, pp. 371–382 (2006).

[3] Furukawa, T. and Kuzunishi, M.: Hierarchical Classification of Heterogeneous Data, *Proc. IASTED Int'l Conf. on Databases and Applications* (DBA2005), pp. 252–257 (2005).

[4] Furukawa, T. and Kuzunishi, M.: "Multi-labeled Data Expressed by a Set of Labels", *Proc. World Academy of Science, Engineering and Technology*, Vol. 65, pp. 857–863 (2010).

[5] Ghamrawi, N. and McMallum, A.: Collecticve Multi-Label Classification, *Proc. Int'l Conf. on Information and Knowledge Management* (CIKM'05), pp. 195–200 (2005).

[6] Kuzunishi, M. and Furukawa, T.: Representation for Multiple Classified Data, *Proc. IASTED Int'l Conf. on Databases and Applications* (DBA2006), pp. 135–142 (2006).
[7] Silva, A. and Barbosa, D: Labeling Data Extracted from the Web, *Proc. On The Move to Meaningful Internet Systems 2007: CoopIS, DOA, and ODBASE*, pp. 1099–1116 (2007).
[8] Sun, A. and Lim, E.: Hierarchical Text Classification and Evaluation, *Proc. IEEE Int'l Conf. on Data Mining* (ICDM2001), pp. 521–528 (2001).
[9] Toutanova, K., Chen, F., Popat K., and Hofmann, T.: Text Classification in a Hierarchical Mixture Model for Small Training Sets, *Proc. Int'l Conf. on Information and Knowledge Management* (CIKM'01), pp. 105–112 (2001).
[10] Wang, K., Zhou, S., and He, Y.: Hierarchical Classification of Real Life Documents, *Proc. SIAM Int'l Conf. on Data Mining*, pp. 1–16 (2001).
[11] Wang, K., Zhou, S., and Liew, S. C.: Building Hierarchical Classifiers Using Class Proximity, *Proc. Int'l Conf. on Very Large Data Bases* (VLDB'99), pp. 363–374 (1999).