Analyzing data on breastfeeding using dispersed statistical models

Naushad Mamode Khan, Cheika Jahangeer and Maleika Heenaye-Mamode Khan

Abstract—Exclusive breastfeeding is the feeding of a baby on no other milk apart from breast milk. Exclusive breastfeeding during the first 6 months of life is very important as it supports optimal growth and development during infancy and reduces the risk of obliterating diseases and problems. Moreover, it helps to reduce the incidence and/or severity of diarrhea, lower respiratory infection and urinary tract infection. In this paper, we make a survey of the factors that influence exclusive breastfeeding and use two dispersed statistical models to analyze data. The models are the Generalized Poisson regression model and the Com-Poisson regression models.

Keywords—Exclusive breastfeeding, Regression model, Generalized Poisson, Com-Poisson.

I. Introduction

The Holy Quran states in chapter 2 (Surah Baqarah, verse 233) " And the mothers should suckle their children for two complete years". In chapter 31 (Surah Luqmaan, verse 14), " And we have stressed on man concerning his parents. The mother bears the child, undergoing weakness upon weakness.And his weaning takes two years". These verses clearly indicate that breast milk is an irreplaceable food for an infant. Moreover, if adequately supplied, it should meet most of the nutritional requirements. In fact, WHO and AAP also recommend that exclusive breastfeeding for the first six months of life followed by nutritionally adequate and safe complementary foods with continued breastfeeding up to two years of age or beyond [3] [4]. Breastfeeding of infants provides advantages with regards to general health, growth, and development, while significantly decreasing the risk for a large number of acute and chronic diseases such as respiratory infection, bacterial meningitis and botulism. Other studies have also shown possible protective effect of human milk feeding against sudden infant death syndrome, insulin dependant diabetes mellitus, Crohn's disease, ulcerative colitis, lymphoma, allergic diseases and chronic digestive diseases [11]. Moreover, exclusive breastfeeding also improves the motor and language skills as compared to infants who have not been breastfed [8]. However, modernization and the fast changing evolution have led to a decrease in both the incidence and duration of exclusive breastfeeding.
In this paper, we use the Generalized Poisson regression model(GPR) [5], [6] and the Com-Poisson model (CMP) [19],

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[21], [20] to analyze the practices of exclusive breastfeeding based on a random subset of data collected from a survey on breastfeeding in Mauritius over the period 2005-2008. The organization of the paper is as follows: In section 2, we describe the factors influencing exclusive breastfeeding in Mauritius since 2005. We review the Generalized Poisson regression model (GPR) in section 3 and Com-Poisson model (CMP) in section 4. Estimations of parameters via the quasi-likelihood estimation technique are also presented in these sections. In section 5, we analyze the exclusive breastfeeding data and present the results and conclusion.

II. Factors influencing exclusive breastfeeding

Maternal age is a factor that can adversely affect breastfeeding rates among mothers . Employment, maternity leave and the length of maternity leave are influential on the incidence of exclusive breastfeeding and thus affect mother's choice of feeding practice. Despite the fact that the working mothers may be aware of the benefits of breastfeeding, many of them are rather reluctant to practice exclusive breastfeeding as compared to unemployed mothers. In Mauritius, according to the 2003 report from the Pay Research Bureau, only 12 weeks of maternity leaves are granted to public officers for 3 confinements only (PRB, 2003). However, working outside the home and being a full-time worker is related to shorter duration of breastfeeding. Other studies have also reported that one of the most important reasons for mothers to stop breastfeeding at 6 months or earlier was "returning to work" [9]. Other studies have shown that information on breastfeeding can influence a mother's choice of feeding practice. Several authors have stated that health education could improve the present status on infant feeding practices [10]. The lack of proper information on breastfeeding sometimes acts as a barrier to its practice though women are strongly determined to breastfeed. Caesarian section is becoming an increasingly common practice in the private hospitals among the upper and middle income groups, and this seems to be an obstacle to successful breastfeeding [15]. The place of antenatal treatment and the place of delivery can also have an impact on the feeding practices of mothers. There are two types of hospital set-up in Mauritius namely the public hospitals and the private hospitals. Both differ in the ways in which antenatal care, perinatal and postnatal care are being provided. Enthusiasm, support and pediatricians involvement are also very essential in the promotion and practice of breastfeeding towards the achievement of optimal infant and child health, growth and development [11].

A survey was carried out since 2005. We choose a random subset of this data consisting of 1800 mothers. We noted that the average practices of exclusive breastfeeding during the first six months of the baby's life is approximately 750 while the variance is 256. This indicates that the data is under-dispersed. To model such data under a regression set-up, we use the GPR and CMP models. In the next section, we provide an overview of the two models and develop two sets of quasi-likelihood estimating equations to estimate the regression and under-dispersion parameters.

III. GENERALIZED POISSON REGRESSION MODEL

(GPR) let y_i be a count response and X_i be a p-dimensional vector of covariates for subject $i(i = 1, \ldots, I)$. Let β be the vector of regression parameters such that β_j $(j = 1, 2, \ldots, p)$ is the regression effect of the jth covariate on the incidence of exclusive breastfeeding among mauritian mothers. The density function of y_i is given by

$$f(y_i, \theta_i, \alpha) = (\frac{\theta_i}{1 + \alpha\theta_i})^{y_i} \frac{(1 + \alpha y_i)^{y_i - 1}}{y_i!} \exp[\frac{-\theta_i(1 + \alpha y_i)}{1 + \alpha_{\theta_i}}] \quad (1)$$

, $y_i = 0, 1, 2, \ldots$; where $\theta_i = \exp(X_i^T\beta)$. The mean of y_i is given by θ_i and the variance of y_i is given by $\theta_i(1 + \alpha\theta_i)^2$, $\alpha < 0$ represents count data with under-dispersion. Since GPR does not belong to the family of exponential dispersion, it becomes difficult to apply the popular maximum likelihood estimation technique (MLE) to estimate the parameters. Moreover, the partial derivatives of β and α are quite complicated. Thus, we propose to use the quasi-likelihood estimation (QLE) technique [14] to estimate the regression and under-dispersion parameters.

A. Quasi-likelihood estimation technique

Wedderburn [14] developed a quasi-likelihood estimation technique (QLE) to estimate parameters under generalized linear model. In this section, we extend his approach and develop two marginal QLEs under GPR. The first QLE is to estimate the vector of regression parameters β based on observations y_i while the second QLE is to estimate the dispersion index α. The QLE to estimate β is given by

$$\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}^{-1}(y_i - \theta_i) = 0, \quad (2)$$

where $V_{i,\beta} = \theta_i(1 + \alpha\theta_i)^2$. $D_{i,\beta} = \frac{\partial\theta_i}{\partial\beta^T} = \theta_i X_i^T$ is a $p \times 1$ matrix. The QLE to estimate α is given by

$$\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}^{-1}(y_i^2 - \eta_i) = 0, \quad (3)$$

where $\eta_i = \theta_i(1 + \alpha\theta_i)^2 + \theta_i^2$ and $D_{i,\alpha} = 2\theta_i^2(1 + \alpha\theta_i)$. $V_{i,\alpha}$ is the variance of Y_i^2 and is calculated using

$$V_{i,\alpha} = E(Y_i^4) - E(Y_i^2)^2 \quad (4)$$

where

$$E(Y_i^4) = \frac{3\theta_i^2}{(1 - \alpha\theta_i)^6} + \theta_i(\frac{15}{(1 - \alpha\theta_i)^2} - \frac{20}{(1 - \alpha\theta_i)} + 6)\frac{1}{(1 - \alpha\theta_i)^5} \quad (5)$$

following [6] and Johnson and Kotz [12]. The Newton-Raphson technique is then applied to the two estimating equations. The iterative equations are given as follows: At the rth iteration,

$$\left(\hat{\beta}_{r+1}\right) = \left(\hat{\beta}_r\right) + [\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}^{-1} D_{i,\beta}]_r^{-1}[\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}^{-1}(y_i - \theta_i)]_r \quad (6)$$

$$\left(\hat{\alpha}_{r+1}\right) = \left(\hat{\alpha}_r\right) + [\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}^{-1} D_{i,\alpha}]_r^{-1}[\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}^{-1}(y_i^2 - \eta_i)]_r \quad (7)$$

where $\hat{\beta}_r$ and $\hat{\alpha}_r$ are the values of $\hat{\beta}$ and $\hat{\alpha}_r$ at the rth iteration. $[.]_r$ is the value of the expression at the rth iteration. The estimators are consistent and under mild regularity conditions, for $I \to \infty$, it may be shown that $I^{\frac{1}{2}}((\hat{\beta}) - (\beta))^T$ has an asymptotic normal distribution with mean 0 and covariance matrix $I[\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}^{-1} D_{i,\beta}]^{-1}[\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}^{-1}(y_i - \theta_i)(y_i - \theta_i)^T V_{i,\beta}^{-1} D_{i,\beta}][\sum_{i=1}^{I} D_{i,\beta}^T V_{i,\beta}^{-1} D_{i,\beta}]^{-1}$ and $I^{\frac{1}{2}}((\hat{\alpha}) - (\alpha))^T$ has an asymptotic normal distribution with mean 0 and covariance matrix $I[\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}^{-1} D_{i,\alpha}]^{-1}[\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}^{-1}(y_i^2 - \eta_i)(y_i^2 - \eta_i)^T V_{i,\alpha}^{-1} D_{i,\alpha}][\sum_{i=1}^{I} D_{i,\alpha}^T V_{i,\alpha}^{-1} D_{i,\alpha}]^{-1}$ The algorithm to estimate the parameters works as follows: For an initial estimate of β and α, we iterate equation (2) until convergence, then use the updated β to update α in equation (3). We then replace the updated β and α in equation (2) and iterate until convergence. Having obtained the new β, we replace in equation (3) to obtain a new α and the cycle continues until both values converge.

IV. COM-POISSON REGRESSION MODEL (CMP)

Recently, Shmueli et al. [21] proposed the Com-Poisson distribution to model counts which may be equi-, over- and under- dispersed. Kadane et al. [20] and Shmueli et al. [21] studied the fitting of this distribution to over -and under - dispersed cross sectional count data. In regression set-up, Jowaheer and Mamode Khan [19] developed a Com Poisson generalized linear model (GLM) for the efficient estimation of the parameters of this model. Jowaheer and Mamode Khan [19] have developed a joint quasi-likelihood technique (JGQL). The Com Poisson regression model is given by:

$$f(y_i) = \frac{\lambda_i^{y_i}}{(y_i!)^\nu} \frac{1}{Z(\lambda_i, \nu)}, \quad (8)$$

where y_i is the number of incidence of exclusive breastfeeding practices corresponding to the ith individual and X_i is the vector of covariates corresponding to y_i. By letting β be the vector of regression parameters such that β_j is the regression effect of the jth covariate on the incidence of exclusive breastfeeding , we write

$$\lambda_i = \exp(x_i^T\beta) \quad (9)$$

In equation (1), the parameter ν corresponds the dispersion index. More specifically, the values $\nu = 1$, $\nu < 1$ and $\nu > 1$ correspond to under- dispersion. Since equation (1) does not

have closed form expressions, we use an asymptotic expression for $Z(\lambda_i, \nu)$ proposed by Shmueli et al. [16] given by

$$Z(\lambda_i, \nu) \simeq \frac{\exp(\nu \lambda_i^{\frac{1}{\nu}})}{\lambda_i^{\frac{\nu-1}{2\nu}} (2\pi)^{\frac{\nu-1}{2}} \sqrt{\nu}} \qquad (10)$$

and reformulate the equation (1) as

$$f(y_i) = \frac{[\exp(x_i^T \beta) y_i] \exp[(x_i^T \beta)(\frac{\nu-1}{2\nu})](2\pi)^{\frac{\nu-1}{2}} \sqrt{\nu}}{(y_i!)^\nu [\exp(\nu \exp(\frac{x_i^T \beta}{\nu}))]} \qquad (11)$$

From equation (4),

$$E(Y_i) = \theta_i = \lambda_i^{1/\nu} - \frac{\nu-1}{2\nu} \qquad (12)$$

and

$$Var(Y_i) = \frac{\lambda_i^{1/\nu}}{\nu} \qquad (13)$$

To estimate the parameters β and ν, we solve the joint quasi-likelihood equation given by

$$\sum_{i=1}^{I} D_i^T V_i^{-1} (f_i - \mu_i) = 0, \qquad (14)$$

where $f_i = (y_i, y_i^2)^T$, $\mu_i = E(f_i)$, $V_i = cov(f_i)$, $D_i = \frac{\partial E(f_i)}{\partial(\beta^T, \nu)}$. The components of equation (7) are derived by Jowaheer and Mamode Khan [19]. They are as follows:

$$D_i = \begin{pmatrix} \partial\theta_i/\partial\beta^T & \partial\theta_i/\partial\nu \\ \partial m_i/\partial\beta^T & \partial m_i/\partial\nu \end{pmatrix}$$

where

$$\partial\theta_i/\partial\beta^T = \frac{\lambda_i^{\frac{1}{\nu}}}{\nu} x_i^T \qquad (15)$$

$$\partial\theta_i/\partial\nu = \frac{1}{2}\frac{\nu-1}{2\nu^2} - \frac{1}{2\nu} - \frac{\lambda_i^{\frac{1}{\nu}} x_i^T \beta}{\nu^2} \qquad (16)$$

$$\partial m_i/\partial\beta^T = x_i^T \left(\frac{2\lambda_i^{\frac{1}{\nu}} + 2\nu\lambda_i^{\frac{2}{\nu}} - \nu\lambda_i^{\frac{1}{\nu}}}{\nu^2} \right) \qquad (17)$$

$$\partial m_i/\partial\nu = \frac{1}{2\nu^3}[2\lambda_i^{\frac{1}{\nu}}\nu\ln(\lambda_i) + \nu - 1 - 4\lambda_i^{\frac{2}{\nu}}\ln(\lambda_i)\nu - 4\lambda_i^{\frac{1}{\nu}}\nu$$
$$-4\lambda_i^{\frac{1}{\nu}}\ln(\lambda_i)]. \qquad (18)$$

The covariance matrix of f_i is expressed as

$$V_i = \begin{pmatrix} var(Y_i) & cov(Y_i, Y_i^2) \\ & var(Y_i^2) \end{pmatrix}$$

The elements in V_i are derived iteratively from the moment generating function of y_{it} which is given by

$$E[Y_i^{r+1}] = \lambda\frac{d}{d\lambda}E[Y^r] + E[Y]E[Y^r] \qquad (19)$$

By deriving the moments for y_i^2, y_i^3 and y_i^4, we obtain

$$cov(Y_i, Y_i^2) = E(Y_i^3) - E(Y_i)E(Y_i^2) \qquad (20)$$
$$= \frac{2\lambda_i^{\frac{1}{\nu}} + 2\nu\lambda_i^{\frac{2}{\nu}} - \nu\lambda_i^{\frac{1}{\nu}}}{\nu^2}$$

$$Var(Y_i^2) = E(Y_i^4) - E(Y_i^2)^2$$
$$= \frac{\lambda_i^{\frac{1}{\nu}}\nu^2 + 4\lambda_i^{\frac{3}{\nu}}\nu^2 + 10\lambda_i^{\frac{2}{\nu}}\nu - 4\lambda_i^{\frac{1}{\nu}}\nu + 4\lambda_i^{\frac{1}{\nu}} - 4\lambda_i^{\frac{2}{\nu}}\nu^2}{\nu^3}$$
$$\qquad (21)$$

The QL estimates of β and ν are obtained by solving equation (7) iteratively until convergence using Newton-Raphson technique. At rth iteration,

$$\begin{pmatrix} \hat{\beta}_{r+1} \\ \hat{\nu}_{r+1} \end{pmatrix} = \begin{pmatrix} \hat{\beta}_r \\ \hat{\nu}_r \end{pmatrix} + [\sum_{i=1}^{I} D_i^T V_i^{-1} D_i]_r^{-1} [\sum_{i=1}^{I} D_i^T V_i^{-1} (f_i - \mu_i)]_r$$
$$\qquad (22)$$

where $\hat{\beta}_r$ is the value of $\hat{\beta}$ at the rth iteration. $[.]_r$ is the value of the expression at the rth iteration. The estimators are consistent and under mild regularity conditions, for $I \to \infty$, it may be shown that $I^{\frac{1}{2}}((\hat{\beta}, \hat{\nu}) - (\beta, \nu))^T$ has an asymptotic normal distribution with mean 0 and covariance matrix $I[\sum_{i=1}^{I} D_i^T V_i^{-1} D_i]^{-1} [\sum_{i=1}^{I} D_i^T V_i^{-1} (f_i - \mu_i)(f_i - \mu_i)^T V_i^{-1} D_i][\sum_{i=1}^{I} D_i^T V_i^{-1} D_i]^{-1}$

V. RESULTS

The application of GPR and CMP to the exclusive breast-feeding data using the equations derived in the above section provides the following estimates:

TABLE I
ESTIMATES OF THE REGRESSION PARAMETERS BASED ON GPR

Intercept	0.9732	(0.2425)
Age	-4.2284	(0.0242)
Length	1.8541	(0.0233)
treatment	-5.9201	(0.2101)
Info	9.8912	(0.0932)
Delivery	-1.4516	(0.1121)
Place	-1.4555	(0.1328)
dispersion parameter	-1.3122	(0.2111)

TABLE II
ESTIMATES OF THE REGRESSION PARAMETERS BASED ON CMP

Intercept	0.9932	(0.2218)
Age	-4.3421	(0.0226)
Length	1.9041	(0.0235)
treatment	-6.1211	(0.1888)
Info	10.8912	(0.0952)
Delivery	-1.3996	(0.1072)
Place	-1.3561	(0.1279)
dispersion parameter	3.5661	(0.1891)

These results are obtained by taking small initial values of the regression parameters. The entry in brackets represent the standard errors of each estimate. The negative value of the age factor indicates that age has an adverse effect on the practice of exclusive breastfeeding. This has been particularly observed among young mothers of less than 18 years old. The positive estimate of the length of maternity leave shows that as the number of days of maternity leave increases, it is more likely that the mothers will adopt a better infant feeding practice and the incidence of exclusive breastfeeding will increase. The estimated value of the place of antenatal treatment reflects the current situation of the private and public health institutions in Mauritius. In fact, in the public health sector, more information on proper infant feeding practices is being dispersed as compared to the private health sector, thus justifying the negative sign. In the same way, the estimate of the place of delivery is negative because there is a disparity at the level of the private and public health institutions where only

the latter have adopted the Baby Friendly Hospital Initiative (BFHI), thereby encouraging proper breastfeeding initiation and successful exclusive breastfeeding for the six months. The regression estimate corresponding to the type of delivery indicates that mothers undergoing caesarian section are less likely to practise exclusive breastfeeding. The information parameter estimate justifies that mothers who have been well informed about proper feeding practices are more likely to practise exclusive breastfeeding for the recommended time. The estimate of the under-dispersion parameter confirms the data is under-dispersed but is not considerable in the case of the GRP compared with the CMP model as compared to the mean-variance ratio of the exclusive breastfeeding data. This means CMP model provides more suitable fits of the data.

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