

Analytical Solutions for Corotational Maxwell Model Fluid Arising in Wire Coating inside a Canonical Die

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Abstract—The present paper applies the optimal homotopy perturbation method (OHPM) and the optimal homotopy asymptotic method (OHAM) introduced recently to obtain analytic approximations of the non-linear equations modeling the flow of polymer in case of wire coating of a corotational Maxwell fluid. Expression for the velocity field is obtained in non-dimensional form. Comparison of the results obtained by the two methods at different values of non-dimensional parameter λ_{10} , reveal that the OHPM is more effective and easy to use. The OHPM solution can be improved even working in the same order of approximation depends on the choices of the auxiliary functions.

Keywords—Wire coating die, Corotational Maxwell model, optimal homotopy asymptotic method, optimal homotopy perturbation method.

I. INTRODUCTION

WIRE coating is often used for the purpose of high and low voltage and protection against corrosion. The wire coating is performed by pulling the wire in the molten polymer inside the coating unit. Due to the shear stress between the wire and the molten polymer the wire is coated. The thickness of the coated wire is same as the thickness of the canonical die at the exit.

Akhter et al. [1], [2] investigated wire coating analysis using inelastic power-law fluid and studied the effect of the change in viscosity on various parameters. Siddiqui et al. [3] studied the wire coating in cylindrical die by assuming that the coating materials obeys the constitutive equation of a third grade fluid. Fenner et al. [4] carried out an analysis of the flow in the tapering section of a pressure type die.

The exact solution of the Navier-Stokes equations is notoriously difficult to find because the non-linearity existing in these equations due to the convection term. To handle the non-linearity, different approximate analytical and numerical methods have been widely used in fluid mechanics and engineering. The perturbation methods are the most widely applied analytic tools for non-linear problems to obtain approximate solutions of these equations. Most of the perturbation methods require the presence of a small or large parameter in that equation, but not every equation has a small or large parameter. Therefore there is a strong need to develop new methods [5]. Recently, Marinca and Herisanu developed a new method known as OHAM. OHAM combines the He's homotopy perturbation method (HPM) and the

method of least squares to optimally identify the unknown constants of the series solutions [6]. They proved this new technique to be a reliable approach to strongly nonlinear problems. In a series of papers by Marinca et al. [7]-[9], Islam et al. [10] and Javed et al. [11] have not only applied this method successfully to obtain the solution to some important problems in engineering and fluid mechanics, but also they have shown that this method is a more powerful tool than other perturbation tools for non linear problems. More recently, Marinca and Herisanu introduced the OHPM [12], [13], which provides a convenient way to control the convergence of the solutions to strongly non-linear differential equations. In this method the non-linear term is expanded in a series with respect to the parameter p and a number of auxiliary functions are introduced that depend on variable r and some constants.

Polymer liquids consist of networks of molecular chains with a large range of length scales and therefore relaxation times. Also for short time scales (high frequencies, high deformation rates) the Maxwell model [14], [15] shows an elastic behavior, and therefore, the present analysis aims at finding the effects of relaxation times on the steady flow created by dragging the wire in the molten polymer (Maxwell liquid) inside the die. Such work seems to be important and useful because attention has hardly been given to the study of corotational Maxwell fluid. Also, some non-Newtonian (second/third grades) models take into account normal stress differences and shear thinning/thickening effects, but lack other features such as stress relaxation.

To the authors' knowledge, no previous investigation has been reported to develop the governing equations for steady incompressible flow of a corotational Maxwell model fluid in case of wire coating. In this work, it is intended to construct the equations for an incompressible flow corotational Maxwell model. The non-linear differential equations are made dimensionless and solved for velocity field by means of OHAM and OHPM.

II. BASIC GOVERNING EQUATIONS

The basic equations governing the flow of an incompressible corotational Maxwell model fluid neglecting the thermal effects are:

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

$$\rho \frac{D\mathbf{u}}{Dt} = \nabla \cdot \mathbf{T} + \rho \mathbf{f}, \quad (2)$$

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where \mathbf{u} is the velocity vector, ρ is the constant density, \mathbf{f} is the body force, \mathbf{T} is the Cauchy stress tensor, and D/Dt denote the material derivative.

The Cauchy stress tensor \mathbf{T} is defined as:

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \quad (3)$$

where p is the dynamic pressure, \mathbf{I} course is the identity tensor, and \mathbf{S} is the shear stress tensor.

The constitutive equation for corotational Maxwell fluid [14], [15] is:

$$\mathbf{S} + \lambda_1 \overset{\nabla}{\mathbf{S}} + \frac{1}{2} \lambda_1 (\mathbf{A}_1 \mathbf{S} + \mathbf{S} \mathbf{A}_1) = \eta_0 \mathbf{A}_1, \quad (4)$$

in which λ_1 is the relaxation time, η_0 is the coefficient of viscosity of the fluid, the line kinematic tensors \mathbf{A}_1 and the upper contra-variant convected derivative designed by $\overset{\nabla}{\mathbf{S}}$ over \mathbf{S} are defined by:

$$\mathbf{A}_1 = \mathbf{L}^T + \mathbf{L}, \quad \overset{\nabla}{\mathbf{S}} = \frac{D\mathbf{S}}{Dt} - [\mathbf{L}^T \mathbf{S} + \mathbf{S} \mathbf{L}] \quad (5)$$

where \mathbf{L} is the gradient of velocity vector \mathbf{u} , and the superscript T denotes the transpose of the matrix.

III. FORMULATION OF THE PROBLEM

Consider an incompressible corotational Maxwell model fluid flowing in a stationary canonical die of radius R_d . The wire of radius R_w is translating with velocity U_w in a die to the direction of the fluid flow. The wire and die are concentric and the coordinate system is chosen at the centre of the wire, in which r is taken perpendicular to the direction of fluid flow, and z is taken in the direction of fluid flow.

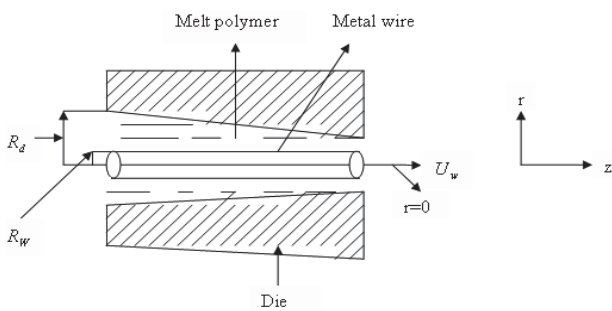


Fig. 1 Schematic profile of wire coating in a pressure type die

Boundary conditions are:

$$w(R_w) = U_w, \text{ and } w(R_d) = 0, \quad (6)$$

Since the flow is axisymmetric and unidirectional, so the velocity field is defined as:

$$\mathbf{u} = [0, 0, w(r)], \quad \mathbf{S} = \mathbf{S}(r). \quad (7)$$

Further, it is assumed that the flow is steady and laminar. Moreover, the gravitational force is considered to be negligible.

Under the above consideration of the velocity field, the continuity equation (1) is satisfied identically, and (4) gives the non-zero components of the extra stress tensor \mathbf{S} as:

$$S_{rr} + \lambda_1 S_{rz} \frac{dw}{dr} = 0, \quad (8)$$

$$S_{r\theta} + \frac{1}{2} \lambda_1 S_{z\theta} \frac{dw}{dr} = 0, \quad (9)$$

$$S_{\theta z} - \frac{1}{2} \lambda_1 S_{\theta r} \frac{dw}{dr} = 0, \quad (10)$$

$$S_{zz} - \lambda_1 S_{rz} \frac{dw}{dr} = 0, \quad (11)$$

$$S_{rz} = \frac{\eta_0 \frac{dw}{dr}}{1 + \lambda_1^2 \left(\frac{dw}{dr} \right)^2}. \quad (12)$$

Substituting the velocity field and (8)-(12) in the equation of balance of momentum (2) in the absence of body force takes the form:

$$\frac{\partial p}{\partial r} = \frac{1}{r} \frac{d}{dr} (r S_{rr}), \quad (13)$$

$$\frac{\partial p}{\partial \theta} = \frac{1}{r} \frac{d}{dr} (r^2 S_{r\theta}), \quad (14)$$

$$\frac{\partial p}{\partial z} = \frac{1}{r} \frac{d}{dr} \left(\frac{\eta_0 r \frac{dw}{dr}}{1 + \lambda_1^2 \left(\frac{dw}{dr} \right)^2} \right), \quad (15)$$

Assume that there is no pressure gradient along the axial direction and the flow is only due to drag of wire. Hence (15) with $\frac{\partial p}{\partial z} = 0$ yields.

$$r \frac{d^2 w}{dr^2} + \frac{dw}{dr} + \lambda_1^2 \left(\frac{dw}{dr} \right)^2 \left(\frac{dw}{dr} - r \frac{d^2 w}{dr^2} \right) = 0. \quad (16)$$

Let's introduce the following non-dimensional variables and parameters

$$r^* = \frac{r}{R_w}, \quad w^* = \frac{w}{U_w}, \quad \lambda_{10} = \frac{\lambda_1^2}{(R_w^2/U_w^2)}, \quad \delta = \frac{R_d}{R_w}. \quad (17)$$

Equations (17) and (7) after dropping the “*” take the following form:

$$r \frac{d^2 w}{dr^2} + \frac{dw}{dr} + \lambda_{10}^2 \left(\frac{dw}{dr} \right)^2 \left(\frac{dw}{dr} - r \frac{d^2 w}{dr^2} \right) = 0, \quad (18)$$

$$w(1) = 1, \text{ and } w(\delta) = 0, \quad (19)$$

The OHAM and the recently introduced method known as OHPM are used to solve (18) with respect to the boundary conditions (19) for the velocity field.

IV. OHAM SOLUTION [8]

We construct a homotopy $\phi(r, p): R \times [0, 1] \rightarrow R$ which satisfies

$$(1-p)[L(\phi(r, p) + g(r)) = H(p)[L(\phi(r, p) + g(r) + N(\phi(r, p))], \quad (20)$$

with boundary conditions

$$\phi(1, p) = 1, \phi(\delta, p) = 0. \quad (21)$$

Here $H(p)$ is a nonzero auxiliary function for $p \neq 0$, $H(0) = 0$ and $\phi(r, p)$ is an unknown function. The auxiliary function $H(p)$ depends either upon some constants [6]-[11] or upon some functions depending on a physical parameter [16], [17]. It was shown in the paper [16], [17] that a more complex function $H(p)$ leads to more accurate results.

We choose the auxiliary function $H(p)$ in the form:

$$H(p) = pC_1 + p^2C_2 + \dots \quad (22)$$

where C_1, C_2, \dots are constants to tackled the solution easily.

In (18) we have

$$L[\phi(r, p)] = r \frac{\partial^2 \phi(r, p)}{\partial r^2} + \frac{\partial \phi(r, p)}{\partial r}, \quad g(r) = 0 \quad (23)$$

and

$$N[\phi(r, p)] = \lambda_{10}^2 \left(\frac{\partial \phi(r, p)}{\partial r} \right)^2 \left(\frac{\partial \phi(r, p)}{\partial r} - r \frac{\partial^2 \phi(r, p)}{\partial r^2} \right). \quad (24)$$

Substituting (22)-(24) in the homotopy (20) and comparing the coefficient of p , we obtain the following equations:

Zeroth-order problem

$$p^0: r \frac{d^2 w_0}{dr^2} + \frac{dw_0}{dr} = 0, \quad (25)$$

subject to the boundary conditions

$$w_0(1) = 1, \quad w_0(\delta) = 0. \quad (26)$$

First-order problem

$$p^1: r \frac{d^2 w_1}{dr^2} + \frac{dw_1}{dr} - \frac{dw_0}{dr} - C_1 \frac{dw_0}{dr} - C_1 \lambda_{10} \left(\frac{dw_0}{dr} \right)^3 - r \frac{d^2 w_0}{dr^2} - r C_1 \frac{d^2 w_0}{dr^2} + r C_1 \lambda_{10} \left(\frac{dw_0}{dr} \right)^2 \frac{d^2 w_0}{dr^2} = 0 \quad (27)$$

subject to boundary conditions

$$w_1(1) = 0, \quad w_1(\delta) = 0. \quad (28)$$

Second-order problem

$$p^2: r \frac{d^2 w_2}{dr^2} + \frac{dw_2}{dr} - C_2 \frac{dw_0}{dr} - 2\lambda_{10} C_2 \left(\frac{dw_0}{dr} \right)^3 - \frac{dw_1}{dr} - C_1 \frac{dw_1}{dr} - r C_2 \frac{d^2 w_0}{dr^2} - 3C_1 \lambda_{10} \left(\frac{dw_0}{dr} \right)^2 \frac{dw_1}{dr} + r C_2 \lambda_{10} \left(\frac{dw_0}{dr} \right)^2 \frac{d^2 w_0}{dr^2} + 2r C_1 \lambda_{10} \frac{dw_0}{dr} \frac{dw_1}{dr} \frac{d^2 w_0}{dr^2} - r \frac{d^2 w_1}{dr^2} - r C_1 \frac{d^2 w_1}{dr^2} + r C_1 \lambda_{10} \left(\frac{dw_0}{dr} \right)^2 \frac{d^2 w_1}{dr^2} = 0 \quad (29)$$

subject to boundary conditions

$$w_2(1) = 0, \quad w_2(\delta) = 0. \quad (30)$$

The corresponding solutions of (25), (27) and (29) together with the boundary conditions (26), (28) and (30) are given as follows:

$$w_0(r) = 1 - \frac{\ln r}{\ln \delta}, \quad (31)$$

$$w_1(r) = \frac{C_1 \lambda_{10}}{(\ln \delta)^3} \left(\left(1 - \frac{1}{r^2} \right) - \frac{\ln r}{\ln \delta} \left(1 - \frac{1}{\delta^2} \right) \right). \quad (32)$$

$$w_2(r) = \frac{1}{(\ln \delta)^7} \left(\left(C_1 + C_1^2 + C_2 \left(1 - \frac{\ln r}{\ln \delta} \right) \right) 2(\ln \delta)^4 \lambda_{10} \left(1 - \frac{1}{r^2} \right) - \left(2(\ln \delta)^3 \ln r (1 + C_1) + \left(\ln \delta - \ln r - \left(\frac{\ln \delta}{\delta^2} - \frac{\ln r}{r^2} \right) \right) 3C_1 \lambda_{10} \right) C_1 \lambda_{10} \left(1 - \frac{1}{\delta^2} \right) - 2(\ln \delta)^2 C_1^2 \lambda_{10}^2 \left(\left(1 - \frac{1}{r^4} \right) - \frac{\ln r}{\ln \delta} \left(1 - \frac{1}{\delta^4} \right) \right) \right) \quad (33)$$

The second order approximation is:

$$w(r) = w_0(r) + w_1(r) + w_2(r) \quad (34)$$

Therefore, the second order approximate solution to velocity distribution is given by:

$$w(r) = 1 - \frac{\ln r}{\ln \delta} + \frac{C_1 \lambda_{10}}{(\ln \delta)^3} \left(\left(1 - \frac{1}{r^2} \right) - \frac{\ln r}{\ln \delta} \left(1 - \frac{1}{\delta^2} \right) \right) + \frac{1}{(\ln \delta)^7} \left(C_1 + C_2 + C_2 \left(1 - \frac{\ln r}{\ln \delta} \right) \right) 2(\ln \delta)^4 \lambda_{10} \left(1 - \frac{1}{r^2} \right) - \left(2(\ln \delta)^3 \ln r (1 + C_1) + \left(\ln \delta - \ln r - \left(\frac{\ln \delta}{\delta^2} - \frac{\ln r}{r^2} \right) \right) 3C_1 \lambda_{10} \right) C_1 \lambda_{10} \left(1 - \frac{1}{\delta^2} \right) - 2(\ln \delta)^2 C_1^2 \lambda_{10}^2 \left(\left(1 - \frac{1}{r^4} \right) - \frac{\ln r}{\ln \delta} \left(1 - \frac{1}{\delta^4} \right) \right) \quad (35)$$

where C_1 and C_2 are constants to be determined by using the method of least square [7]-[12]:

V. OHPM SOLUTION [13]

The OHPM is a combination of the classical homotopy concept and the perturbation technique [18], [19]. This method is fast convergence for both large and small parameters. The detail analyses are given in references [12], [13]. According to this technique the zeroth, first and second order problems are given as [12], [13]:

Zeroth-order problem

$$p^0: L(w_0) - L(u_0) + g(r) = 0, r \in \Omega, B\left(w_0, \frac{\partial w_0}{\partial r}\right) = 0, r \in \Gamma, \quad (36)$$

subject to the boundary conditions

$$w_0(1) = 1, w_0(\delta) = 0. \quad (37)$$

First-order problem

$$p^1: L(w_1) + L(u_0) + \Lambda_0(r, D_1)G(r, w_0, w'_0, w''_0) = 0, r \in \Omega, B\left(w_1, \frac{\partial w_1}{\partial r}\right) = 0, r \in \Gamma, \quad (38)$$

subject to boundary conditions

$$w_1(1) = 0, w_1(\delta) = 0. \quad (39)$$

Second-order problem

$$p^2: L(w_2) + \Lambda_{10}(r, D_j)w_1G_w(r, w_0, w'_0, w''_0) + \Lambda_{11}(r, D_k)w'_1G_w(r, w_0, w'_0, w''_0) + \Lambda_{12}(r, D_l)w''_1G_w(r, w_0, w'_0, w''_0) = 0, r \in \Omega, B\left(w_2, \frac{\partial w_2}{\partial r}\right) = 0, r \in \Gamma, \quad (40)$$

subject to boundary conditions

$$w_2(1) = 0, w_2(\delta) = 0, \quad (41)$$

where L is the linear operator and $g(r)$ is the known analytic function given in (23), $N(w) = G(r, w, w', w'')$ is the non-linear operator given in (24) for this problem, B is a boundary

operator, $w(r)$ is the unknown function, r denotes the spatial independent variable, Γ is the boundary of the domain Ω , $\Lambda_{ij}(r, D_k), i, j = 0, 1, 2, \dots$, are the auxiliary functions that depend on the variable r and some constants $D_1, D_2, D_3, D_4, \dots$ and $G_w = \frac{\partial G}{\partial w}, G_{w'} = \frac{\partial G}{\partial w'}, G_{w''} = \frac{\partial G}{\partial w''}$.

The auxiliary function $\Lambda_{ij}(r, D_k)$, can be constant, linear, quadratic, cubic or higher order polynomial [13], [14]. For higher degree polynomial the accuracy of this technique is more reliable than the lower degree polynomial for the same order problem. For the problem under discussion we take $\Lambda_0(r, D_1) = 1$, $\Lambda_{11}(r, D_k) = D_1 + D_2r$, and $\Lambda_{12}(r, D_l) = D_3 + D_4r$.

The initial approximation is:

$$u_0(r) = 1 - \frac{\ln r}{\ln \delta} \quad (42)$$

In view of all these, the corresponding solutions of (36), (38) and (40) together with the boundary conditions (39)-(41) are given as:

$$w_0(r) = 1 - \frac{\ln r}{\ln \delta}, \quad (43)$$

$$w_1(r) = -\frac{\lambda_{10}^2}{2(\ln \delta)^3} \left(\left(1 - \frac{1}{r^2} \right) - \frac{\ln r}{\ln \delta} \left(1 - \frac{1}{\delta^2} \right) \right), \quad (44)$$

$$w_2(r) = \frac{1}{144\delta^4 r^4 (\ln \delta)^7} \left(90\delta^4 r^2 D_1 \lambda_{10}^2 \ln \delta (\delta^2 \ln \delta - r^2 \ln r) \left(1 - \frac{1}{\delta^2} \right)^2 + 45D_1 \lambda_{10}^2 \ln \delta (\delta^4 \ln \delta - r^4 \ln r - \delta^4 r^4 (\ln \delta - \ln r)) + 360\delta^3 r^3 D_2 \lambda_{10}^2 (r \ln r - \delta \ln \delta - \delta r (\ln \delta - \ln r)) \left(1 - \frac{1}{\delta^2} \right) + 80\delta r D_2 \lambda_{10}^2 \ln r (\delta^3 \ln \delta - r^3 \ln r - \delta^3 r^3 (\ln \delta - \ln r)) + 18\delta^2 r^2 D_3 \lambda_{10}^2 (r^2 \ln r - \delta^2 \ln \delta - \delta^2 r^2 (\ln \delta - \ln r)) \left(1 - \frac{1}{\delta^2} \right) - 108\delta^4 r^2 D_3 \lambda_{10}^2 \ln \delta (\delta^2 \ln \delta - r^2 \ln r) \left(1 - \frac{1}{\delta^2} \right) + 432\delta^3 r^3 D_4 \lambda_{10}^2 \ln \delta (\delta \ln \delta - r \ln r - \delta r (\ln \delta - \ln r)) + 72\delta^3 r^3 D_4 \lambda_{10}^2 (r \ln r - \delta \ln \delta - r \delta (\ln \delta - \ln r)) \left(1 - \frac{1}{\delta^2} \right) \right) \quad (45)$$

The second order approximation is obtained in the following way:

$$w(r) = w_0(r) + w_1(r) + w_2(r) \quad (46)$$

where D_1, D_2, D_3 and D_4 are constants to be determined by using the method of least square [13], [14].

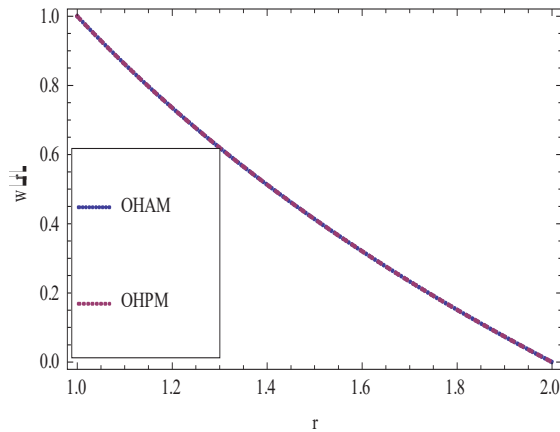


Fig. 2 Dimensionless velocity profile for $\lambda_{10} = 0.01$, and $C_1 = -1.01917329$, $C_2 = 0.00003253$, $D_1 = 2.413436440$, $D_2 = -1.4984336291$, $D_3 = 41.91954583$, $D_4 = -2.054242202$

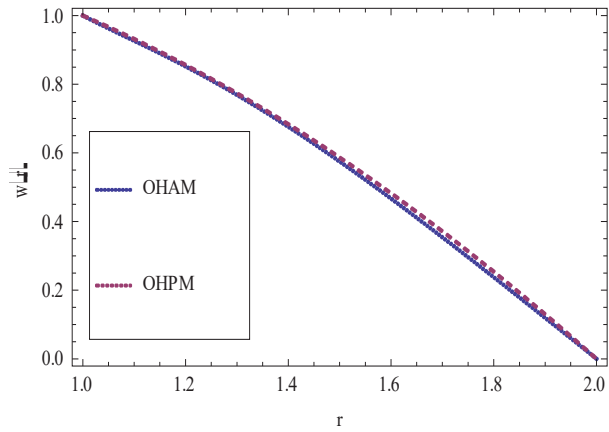


Fig. 5 Dimensionless velocity profile for $\lambda_{10} = 10$, and $C_1 = 0.05453425$, $C_2 = 0.0132347$, $D_1 = 0.1367199210$, $D_2 = -0.398798583$, $D_3 = 0.577538188$, $D_4 = -0.7307045360$

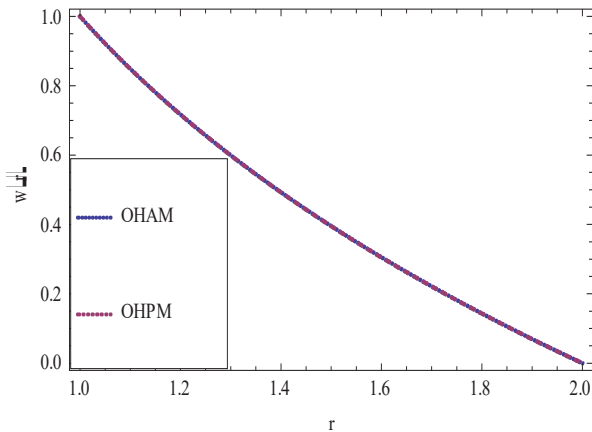


Fig. 3 Dimensionless velocity profile for $\lambda_{10} = 0.1$, and $C_1 = -1.2217968$, $C_2 = 0.00773987$, $D_1 = 16.80201139$, $D_2 = -28.34023866$, $D_3 = 56.38069670$, $D_4 = -38.58265248$

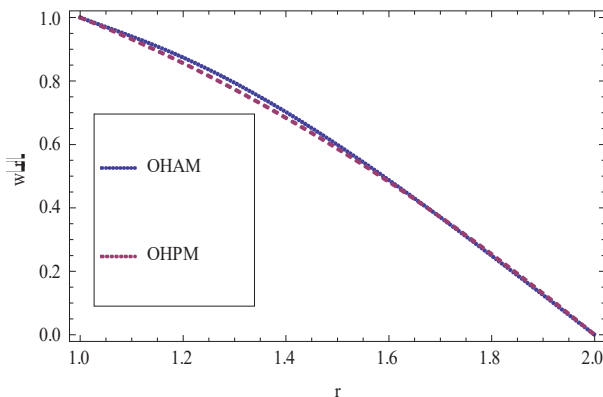


Fig. 4 Dimensionless velocity profile for $\lambda_{10} = 5$, and $C_1 = 0.10942059$, $C_2 = 0.04760301$, $D_1 = 0.13671992109$, $D_2 = -0.3987985838$, $D_3 = 0.57753818$, $D_4 = -0.730704536$

VI. RESULTS AND DISCUSSIONS

In the present paper, we have used two methods namely, the OHAM proposed by Herisanu et al. [7], [17] and OHPM proposed by Marinca et al. [12], [13] to find the solution of the flow problem governed by (18) subject to (19). The two solutions are in complete agreement for small values of the relaxation parameter (see Figs. 2 and 3), but there is little difference for large values of this parameter (see Figs. 4 and 5). Here, it is observed that with increase of relaxation parameter, the non-Newtonian effect increases in the melt polymer. The origin of OHPM is the He's HPM, but the construction of the homotopy is different, especially as regards to the auxiliary functions $\Lambda_{ij} = (r, D_k)$, involving the presence of some parameters $D_1, D_2, D_3, D_4, \dots$, that ensure a rapid convergence of the solution when they are optimally determined. For the same order of approximation, OHPM gives better results than OHAM depends on the choices of the auxiliary functions. Moreover, the key attribute of the OHPM is that it provides a simple and rigorous approach to control and adjust the convergence of a solution through the parameters D_i , which are optimally determined.

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