International Journal of Engineering, Mathematical and Physical Sciences
ISSN: 2517-9934
Vol:11, No:10, 2017

Analytical Formulae for the Approach Velocity Head Coefficient

Abdulrahman Abdulrahman

Abstract**—**Critical depth meters, such as abroad crested weir, Venture Flume and combined control flume are standard devices for measuring flow in open channels. The discharge relation for these devices cannot be solved directly, but it needs iteration process to account for the approach velocity head. In this paper, analytical solution was developed to calculate the discharge in a combined critical depth-meter namely, a hump combined with lateral contraction in rectangular channel with subcritical approach flow including energy losses. Also analytical formulae were derived for approach velocity head coefficient for different types of critical depth meters. The solution was derived by solving a standard cubic equation considering energy loss on the base of trigonometric identity. The advantage of this technique is to avoid iteration process adopted in measuring flow by these devices. Numerical examples are chosen for demonstration of the proposed solution.

Keywords**—**Broad crested weir, combined control meter, control structures, critical flow, discharge measurement, flow control, hydraulic engineering., hydraulic structures, open channel flow.

I. Introduction

BROAD crested weir is a critical depth meter that is, if the weir is high enough, critical depth occurs on the crest of the weir. The discharge over a broad crested weir in rectangular channel is given by: [2]

$$Q = (\frac{2}{3})^{3/2} C_d .B.\sqrt{g}.\left(h + \frac{v^2}{2g}\right)^{3/2} \quad (1)$$

in which C_d – Discharge coefficient, $B-$ Channel width, $h-$ Head above the crest (Fig. 1), $v-$ Approach velocity.

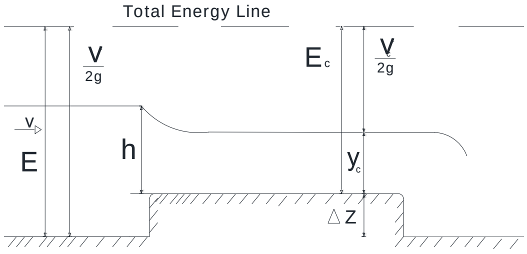


Fig. 1 Flow over a broad crested weir

Venture flume is a critical flow-meter wherein the critical depth is created by a contraction in width of the channel. The discharge in a critical flow-meter is given by [5]:

$$Q = (\frac{2}{3})^{3/2} C_d .b.\sqrt{g}.\left(y + \frac{v^2}{2g}\right)^{3/2} \quad (2)$$

in which $b-$ Contraction width (throat width), $y-$ Subcritical approach depth (Fig. 2).

Combined control flume is a critical flow meter formed by a combination of a hump with contraction, in which a control section is achieved, with a critical depth occurring over it as shown in Fig. 3. The discharge in a critical flow flume is given by [6]:

$$Q = (\frac{2}{3})^{3/2} C_d .b\sqrt{g}.\left(h + \frac{v^2}{2g}\right)^{3/2} \quad (3)$$

Equations (1)-(3) cannot be solved directly for the discharge, since the approach velocity head $\frac{v^2}{2g}$ is unknown.

Some authors neglected the approach velocity head to avoid iteration process; others replaced it by an approach velocity head coefficient [3], [6] thus

$$Q = (\frac{2}{3})^{3/2} C_d .C_v .B.\sqrt{g}.h^{3/2} \quad (4)$$

- For contracted flume:

$$Q = (\frac{2}{3})^{3/2} C_d .C_v .b\sqrt{g}.y^{3/2} \quad (5)$$

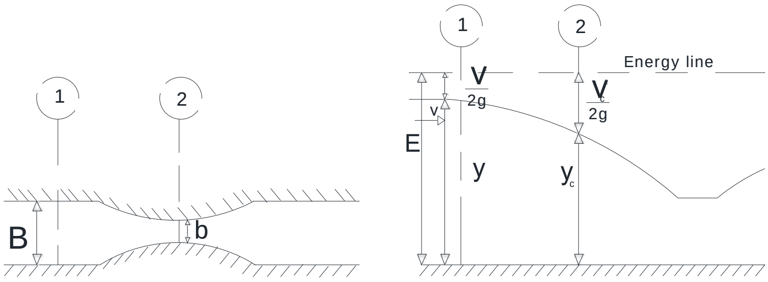
- For combined control flume:

$$Q = (\frac{2}{3})^{3/2} C_d .C_v .b.\sqrt{g}.h^{3/2} \quad (6)$$

The solution was made by assuming $C_V = 1$ then; the approach velocity head can be calculated to update the value of C_V for a second calculation.

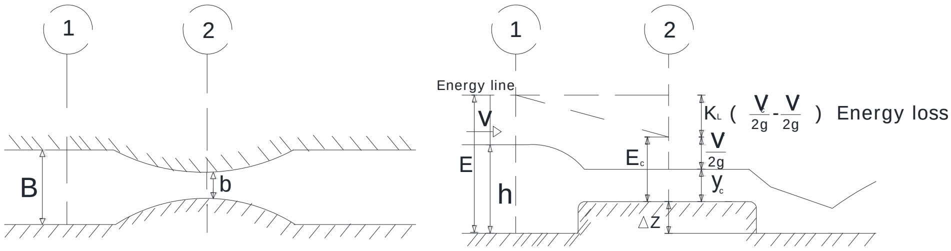
We derived analytical formulae for approach velocity head coefficient C_V which allows us to obtain a direct solution for measuring discharge by these devices.

A. Abdulrahman is with Aleppo University, Aleppo Faculty of Civil Eng. Water Eng. Dept., Syria. He is now the Dean of Civil Eng. Faculty of AL-hasakeh, AL-furat University, Syria (e-mail: abdulrahman21959@gmail.com).



(b) Plan (a) section

Fig. 2 Venturi Flume (Standing wave flume)



(b) plan (a) section

Fig. 3 Combined control Flume

II. Analytical Solution

A combined control flume is an excellent flow-measuring device, formed by a combination of a raised floor over a reach of channel with lateral contraction in which a control section is achieved, with a critical depth occurring over it [6] as shown in Fig. 3.

Applying energy equation from subcritical approach flow section i.e., section to control section including energy loss term, gives:

$$y+\frac{Q^2}{2gA^2}=E_C+\Delta z+k_L\left(\frac{Q^2}{2gA_C{}^2}-\frac{Q^2}{2gA^2}\right)$$

$$y+\frac{Q^2}{2g.B^2.y^2}=\frac{3}{2}y_C+\Delta z+k_L\frac{Q^2}{2g}\left(\frac{1}{b^2.y_C{}^2}-\frac{1}{B^2.y^2}\right) \quad (7)$$

where: y_C – Critical depth at the control section, k_L – Energy loss coefficient, and Δz – Hump height above the bed.

It is convenient to replace the term $\frac{Q^2}{g}$ in (7) by $b^2 y_C{}^3$ and to denote $M=\frac{b}{B}$ (Contraction ratio), so that

$$y+\left(\frac{1+k_L}{2y^2}\right)M^2 y_C{}^3=\left(k_L+\frac{3}{2}\right)y_C+\Delta z \quad (8)$$

Also denoting $\frac{1}{\xi^2}=1+k_L$ and rearranging (8), it becomes:

$$y_C{}^3-(2\xi^2+1)\left(\frac{y}{M}\right)^2 y_C+2(y-\Delta z)\left(\frac{\xi.y}{M}\right)^2=0 \quad (9a)$$

Equation (9a) has a standard form of a cubic equation [1]:

$$y_C{}^3+p.y_C+r=0 \quad (9b)$$

in which

$$p=-(2\xi^2+1)\left(\frac{y}{M}\right)^2 \quad (10a)$$

$$r=2(y-\Delta z)\left(\frac{\xi.y}{M}\right)^2 \quad (10b)$$

The discriminant of (9b) is:

$$\Delta=4p^3+27q^2$$

$$\Delta=108y^6\left(\frac{\xi}{M}\right)^4\left(\left(1-\frac{\Delta z}{y}\right)^2-\frac{\lambda}{M^2}\right) \quad (11)$$

where:

$$\lambda=\frac{(2\xi^2+1)^3}{27\xi^4}$$

The variable λ has a minimum value $\lambda_{\min}=1$ for $\xi=1$

The term $\left(1-\frac{\Delta z}{y}\right)$ in (11) is less than one since $\left(\frac{\Delta z}{y}<1\right)$

and the minimum value of the term $\frac{\lambda}{M^2}$ in the same equation equals to 1 for $\lambda_{\min} = 1$ and $M_{\max} = 1$. Therefore, the discriminant is negative, and (9b) has three real roots. To find these roots, we seek value of y_C, θ such that three terms on the left hand side of the cubic $y_C{}^3 + py_C + r = 0$ are respectively proportional to the terms on the left hand side of the trigonometrical identity [4]. It follows that y_C and θ must satisfy the ratios:

$$\frac{y_C{}^3}{4\cos^3\theta} = -\frac{py_C}{3\cos\theta} = -\frac{r}{\cos 3\theta} \tag{12a}$$

The first quality provides:

$$\frac{y_C}{\cos\theta} = \sqrt{-\frac{4}{3}p}\ , \tag{12b}$$

so

$$y_C = \cos\theta\sqrt{-\frac{4}{3}p} \tag{12c}$$

where it is understood that p is negative. Substituting value of $\frac{y_C}{\cos\theta}$ into the second equality of (12a), we obtain:

$$-\frac{1}{3}p\sqrt{-\frac{4}{3}p} = -\frac{r}{\cos 3\theta}$$

$$\cos 3\theta = \frac{3r}{p\sqrt{-\frac{4}{3}p}} \tag{12d}$$

Substituting values of p and r from (10a) and (10b) respectively into (12d) yields:

$$\cos 3\theta = \frac{M}{\lambda}\left(\frac{\Delta z}{y} - 1\right) \tag{12e}$$

It is seen from (12e) that

$$\cos 3\theta = -\frac{M}{\lambda} \quad as \quad \Delta z \to 0 \quad and \cos 3\theta = 0 \quad as \quad \frac{\Delta z}{y} \to 1$$

Therefore, the value of 3θ lies between π and $\frac{\pi}{2}$ since $M_{\max} = 1$ and $\lambda_{\min} = 1$.

We may take three distinct solutions of (9b), namely:

$$\theta_1 = \frac{1}{3}\cos^{-1}\left(\frac{M}{\lambda}\left(\frac{\Delta z}{y} - 1\right)\right) \tag{13a}$$

$$\theta_2 = \frac{1}{3}\cos^{-1}\left(\frac{M}{\lambda}\left(\frac{\Delta z}{y} - 1\right)\right) - \frac{2\pi}{3} \tag{13b}$$

$$\theta_3 = \frac{1}{3}\cos^{-1}\left(\frac{M}{\lambda}\left(\frac{\Delta z}{y} - 1\right)\right) + \frac{2\pi}{3} \tag{13c}$$

Substituting (10a) into (12b), yields:

$$\frac{y_C}{y} = \frac{2}{M}\sqrt{\frac{2\xi^2 + 1}{3}}\cos\theta \tag{14}$$

III. Discussion

By analyzing (13a), (13b), (13c) and (14), we note:

- As $\Delta z \to 0$:

$\frac{\pi}{3} \geq \theta_1 > \frac{\pi}{6}$ Since $1 \geq M > 0$ and $0 < \xi \leq 1$ $\Rightarrow \frac{y_C}{y} > 1$

$$-\frac{\pi}{3} \geq \theta_2 > -\frac{\pi}{6}, \Rightarrow 1 \geq \frac{y_C}{y} > 0$$

$$\pi \geq \theta_3 > \frac{5\pi}{6}, \Rightarrow \frac{y_C}{y} < 0$$

- As $\frac{\Delta z}{y} \to 1$

$$\theta_1 = \frac{\pi}{6}, \Rightarrow \left(\frac{y_C}{y}\right)_{\min} = \frac{1}{M} > 1$$

$$\theta_2 = -\frac{\pi}{2}, \Rightarrow \frac{y_C}{y} = 0$$

$$\theta_3 = \frac{5\pi}{6}, \Rightarrow \frac{y_C}{y} < 0$$

The above discussion provides us to the following results

1- The angle θ_1 generates critical depth in the control section greater than the subcritical approach depth, thus this root is not acceptable.
2- The angle θ_2 generates critical depth in the control section less than subcritical approach depth, thus this depth is acceptable.
3- The angle θ_3 generates negative critical depth which has no physical meaning.

Therefore, we have a unique solution for (9a), which may be achieved by substituting (13b) into (14)

$$\frac{y_C}{y} = \frac{2}{M}\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(M\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2}\left(\frac{\Delta z}{y} - 1\right)\right) - \frac{2\pi}{3}\right) \tag{15}$$

Equation (15) gives a direct solution for critical depth in a combined control flume. Therefore, a direct formula for discharge may be obtained as:

$$Q=\left(\frac{2}{M}\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(\left(M\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2}\left(-h^*\right)\right)-\frac{2\pi}{3}\right)\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2} \quad (16a)$$

or

$$Q=\left(\frac{2}{3}\right)^{3/2}\left(\frac{3}{M}\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(\left(M\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2}\left(-h^*\right)\right)-\frac{2\pi}{3}\right)\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2} \quad (16b)$$

Noting from Fig. 1, that:

$$\frac{\Delta z}{y}=\frac{y-h}{y},\quad \frac{h}{y}=h^*,\Rightarrow 1-\frac{\Delta z}{y}=1-h^*, and$$
$$\frac{\Delta z}{y}-1=-h^*$$

When the energy loss is neglected i.e.; $k_L=0\Rightarrow\xi=1$, then (16b) becomes

$$Q=\left(\frac{2}{3}\right)^{3/2}\left(\frac{3}{M}\cos\left(\frac{1}{3}\cos^{-1}\left(-M.h^*\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2} \quad (16c)$$

We can distinct the following special cases:

A. Broad Crested Weir with No Lateral Contraction

1. Including Energy Loss

Substituting $M=1$ into (16-a):

$$Q=\left(2.\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2}\left(\frac{\Delta z}{y}-1\right)\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2} \quad (17a)$$

or

$$Q=\left(\frac{2}{3}\right)^{3/2}\left(3.\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2}\left(h^*\right)\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2} \quad (17b)$$

2. Neglecting Energy Loss

Substituting $\xi=1$ into (17b)

$$Q=\left(2.\cos\left(\frac{1}{3}\cos^{-1}\left(\frac{\Delta z}{y}-1\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2}$$

or

$$Q=\left(\frac{2}{3}\right)^{3/2}\left(3.\cos\left(\frac{1}{3}\cos^{-1}\left(-h^*\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2} \quad (18)$$

B. Venture Flume Has No Raised Floor

1. Including Energy Loss:

We substitute $\Delta z=0$ into (16a)

$$Q=\left(\frac{2}{M}\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(-M\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2}\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2}$$

$$Q=\left(\frac{2}{3}\right)^{3/2}\left(\frac{3}{M}\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(-M\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2}\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2} \quad (19a)$$

2. Neglecting Energy Loss

Substituting $\xi=1$ into (19a)

$$Q=\left(\frac{2}{M}\cos\left(\frac{1}{3}\cos^{-1}\left(-M\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2}$$

or

$$Q=\left(\frac{2}{3}\right)^{3/2}\left(\frac{3}{M}\cos\left(\frac{1}{3}\cos^{-1}\left(-M\right)-\frac{2\pi}{3}\right)\right)^{3/2}.b.\sqrt{g}.y^{3/2} \quad (19b)$$

IV. Direct Formula for Approach Head Velocity Coefficient C_V

In order to get a direct formula for approach head velocity coefficient C_V, we should consider the cases of neglecting energy loss; i.e., $C_d = 1$.

A) *Case i)* – Broad crested weir with no lateral contraction: for this case, we compare (18) with (4) to obtain a direct formula for approach velocity head coefficient C_V.

$$C_V = \left(\frac{3.\cos\left(\frac{1}{3}\cos^{-1}\left(-h^*\right) - \frac{2\pi}{3} \right)}{h^*} \right)^{3/2} \tag{20}$$

B) *Case ii)* - Venture flume has no raised floor: for this case, we compare (19b) with (5) to obtain direct formula for approach velocity head coefficient C_V.

$$C_V = \left(\frac{3}{M}\cos\left(\frac{1}{3}\cos^{-1}\left(-M\right) - \frac{2\pi}{3} \right) \right)^{3/2} \tag{21}$$

C) *Case iii)* – Combined control flume: for this case, we compare (16c) with (6) to obtain direct formula for approach velocity head coefficient C_V.

$$C_V = \left(\frac{\frac{3}{M}\cos\left(\frac{1}{3}\cos^{-1}\left(-M.h^*\right) - \frac{2\pi}{3} \right)}{h^*} \right)^{3/2} \tag{22}$$

If one takes into account the effect of energy loss then, a direct formula for the $C_V C_d$ may be derived as:

- For *case i)*,we compare (17b) with (4)

$$C_V.C_d = \left(\frac{3.\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(-\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2} h^* \right) - \frac{2\pi}{3} \right)}{h^*} \right)^{3/2} \tag{23}$$

- For *case ii)*, we compare (19a) with (5)

$$C_V.C_d = \left(\frac{3}{M}\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(-M\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2} \right) - \frac{2\pi}{3} \right) \right)^{3/2} \tag{24}$$

- For case iii, we compare (16b) with (6)

$$C_V.C_d = \left(\frac{\frac{3}{M}\sqrt{\frac{2\xi^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(-M.h^*.\xi^2\left(\frac{3}{2\xi^2+1}\right)^{3/2} \right) - \frac{2\pi}{3} \right)}{h^*} \right)^{3/2} \tag{25}$$

Generally it is recommended by (HERC-RAS) manual 1988 [7] the value of energy loss coefficient $k_L = 0.1$, thus $\xi = 0.953$.

V. Numerical Examples

It is proposed to demonstrate the design of combined control flume through numerical examples:

Example 1. Broad crested weir: in rectangular-depth meter 2 m wide with $\Delta z = 0.30m$, the subcritical approach depth y is measured to be $0.75m$. Find the discharge per unit depth in

- Case of neglecting energy losses $C_d = 1$,
- Case of considering energy loss with coefficient $k_L = 0.1$.

Solution: we have $\Delta z = 0.30m$, $y = 0.75m$,

$$h = y - \Delta z = 0.75 - 0.30 = 0.45m, h^* = \frac{h}{y} = \frac{0.45}{0.75} = 0.60$$

Substituting into (20), we obtain

$$C_V = \left(\frac{3.\cos\left(\frac{1}{3}\cos^{-1}\left(-0.60\right) - \frac{2\pi}{3} \right)}{0.60} \right)^{3/2} = 1.097979$$

Introducing value of C_V into (4), we get exact value of the discharge per unit depth

$$q = \frac{Q}{b} = \left(\frac{2}{3}\right)^{3/2} 1.097979\sqrt{9.81} * 0.45^{3/2} = 0.565m^2/\sec$$

The same answer would be achieved using (18)

$$q=\frac{Q}{b}=\left(\frac{2}{3}\right)^{3/2}\left(\begin{array}{l}3.\cos\\ \left(\frac{1}{3}\cos^{-1}\left(-0.60\ \right)-\frac{2\pi}{3}\right)\end{array}\right)^{3/2}$$

$$\sqrt{g}.0.75^{3/2}=0.565m^2/\sec$$

Considering energy loss: we substitute the above variables into (23)

$$C_V.C_d=\left(\frac{\begin{array}{l}3.\sqrt{\frac{2*0.953^2+1}{3}}\cos\\ \left(\begin{array}{l}\frac{1}{3}\cos^{-1}\\ \left(\begin{array}{l}-0.953^2\\ \left(\left(\frac{3}{2*0.953^2+1}\right)^{3/2}*0.60\right)\end{array}\right)-\frac{2\pi}{3}\end{array}\right)\end{array}}{0.60}\right)^{3/2}=1.0444$$

Introducing this value into (4), we get $q=\frac{Q}{b}=\left(\frac{2}{3}\right)^{3/2}1.0444\sqrt{9.81}*0.45^{3/2}=0.5375m^2/\sec\cdot$ The same answer would be achieved using (17b)

$$q=\frac{Q}{b}=\left(\frac{2}{3}\right)^{3/2}\left(\begin{array}{l}3.\sqrt{\frac{2*0.953^2+1}{3}}\cos\\ \left(\begin{array}{l}\frac{1}{3}\cos^{-1}\\ \left(\begin{array}{l}0..953^2\left(\frac{3}{20.953^2+1}\right)^{3/2}\\ (0.60)\end{array}\right)-\frac{2\pi}{3}\end{array}\right)\end{array}\right)^{3/2}$$

$$\sqrt{9.81}0.75^{3/2}=0.5375m^2/\sec$$

This problem was solved by trial and error, the value of $q=0.512m^2/\sec$ was obtained with as a second approximation [6].

Example 2. Combined control flume: a Venture flume is formed in a horizontal channel of rectangular cross-section $1.40m$ wide by constricting the wide to $0.90m$ and raising the floor in the constricted section by $0.25m$ above that of the channel, the approach depth upstream is $0.60m$. Find the volume rate of the flow in case of neglecting energy loss, then considering energy loss with $k_L=0.1$.

Solution

- Case of Neglecting Energy Loss:

In this case, we have

Contraction ratio $M=\frac{b}{B}=\frac{0.90}{1.40}=0.6429$,

Energy loss coefficient $k_L=0\Rightarrow\xi=1, C_d=1$

By substituting the above variables into (22), we define C_V

$$C_V=\left(\frac{\frac{3}{0.6429}\cos\left(\frac{1}{3}\cos^{-1}(-0.6429*0.5833)-\frac{2\pi}{3}\right)}{0.6429}\right)^{3/2}=1.03365$$

Substituting into (6), we obtain

$$Q=\left(\frac{2}{3}\right)^{3/2}*1.03365*0.90*\sqrt{9.81}*0.35^{3/2}=0.3284m^2/\sec$$

The same answer would be achieved using (16c)

$$Q=\left(\frac{2}{3}\right)^{3/2}\left(\frac{3}{0.6429}\cos\left(\begin{array}{l}\frac{1}{3}\cos^{-1}\\ (-0.6429*0.5833)-\frac{2\pi}{3}\end{array}\right)\right)^{3/2}.$$

$$0.90.\sqrt{g}*0.60^{3/2}=0.3284m^3/\sec$$

This problem was solved by trial and error method; the same answer was obtained after the third trial [3].

- Case of Considering Energy Loss: $k_L=0.1\Rightarrow\xi=0.953$.

Introducing this value into (25), we get:

$$C_V.C_d=\left(\frac{\frac{3}{0.6429}\sqrt{\frac{2*0.953^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(\frac{-0.6429*0.5833*0.953^2}{\left(\left(\frac{3}{2*0.953^2+1}\right)^{3/2}\right)}\right)-\frac{2\pi}{3}\right)}{0.5833}\right)^{3/2}=0.9833$$

Substituting into (6),

$$Q=\left(\frac{2}{3}\right)^{3/2}*0.9833*0.90*\sqrt{9.81}*0.35^{3/2}=0.3124m^2/\sec$$

The same answer may be obtained from (16)

$$Q=\left(\frac{2}{3}\right)^{3/2}\left(\frac{3}{0.6429}\sqrt{\frac{2*0.953^2+1}{3}}\cos\left(\frac{1}{3}\cos^{-1}\left(-0.6429*0.5833*0.953^2\left(\frac{3}{20.953^2+1}\right)^{3/2}\right)-\frac{2\pi}{3}\right)\right)^{3/2}.$$

$$0.90*\sqrt{g}*0.6^{3/2}=0.3124m^3/\sec$$

VI. Conclusion

Analytical solution for discharge measurement by critical depth-meters in rectangular channel and head velocity coefficient has been presented. The derived solution can be applied to a broad crested weir, contracted flume and combined control flume. Numerical examples demonstrated that the equations are less time consuming and more accurate than existing relationships. It is hoped that the derived equations are helpful to irrigation engineer.

References

[1] Abdulrahman, A. Direct solution to problems of open-channel transitions: rectangular channels, Journal of irrigation and drainage engineering, Vol. 134, No. 4, August 1, 2008, ASCE.
[2] Chow, V.T. (1958). Open channel hydraulics. McGraw-Hill Book Co. Inc., New York, N.Y.
[3] Douglas, E. John F, Gasiorek, Joseph M. and Swaffield, John A. (2001). Fluid Mechanics, P.545, 4TH ED., Pearson, Edinburgh Gate, England.
[4] Heading, J. (1970). Mathematical Methods in Science and Engineering, P. 40, 2ND Ed., Edward Arnold, London.
[5] Henderson, F.M. (1966). Open channel hydraulics. Macmillan Company, New York, N.Y.
[6] Streeter, Victor L. Wylie, E. Beniamin (1981), Fluid Mechanics, P. 475, 4TH Ed., Pearson, Edinburgh Gate, England.
[7] Sturm, Terry W. (2001). Open Channel Hydraulics, P. 33, 1ST Ed., McGraw-Hill C0mpany, Inc., 1221 Avenue of the Americas, New York, NY 10020.