Analytical Estimation of Rotor Loss Due to Stator Slotting of Synchronous PM Machines

Adel Bettayeb, Robert Kaczmarek and Jean-Claude Vannier

Abstract**—**In this paper, we analyze the rotor eddy currents losses provoqued by the stator slot harmonics developed in the permanent magnets or pole pieces of synchronous machines. An analytical approach is presented to evaluate the effect of slot ripples on rotor field and losses calculation. This analysis is then tested on a model by 2D/3D finite element (FE) calculation. The results show a good agreement on loss calculations when skin effect is negligible and the magnet is considered.

Keywords**—**Analytical modeling, Eddy-currents, Finite-element methods, Power losses, Slot harmonics effect.

I. Introduction

THE synchronous machines with permanent magnets rotor are considerably used in recent years due to their simple structure and high power density [1-2]. However, the rotor's pole pieces and the permanent magnets of synchronous machines (PMSM) are exposed to several harmonics which rotates asynchronously with the rotor. This represents a source of losses which can be significant. Moreover, the high conductivity of PM leads to significant eddy current losses and this may cause the PM thermal demagnetization |3-4].

To avoid such impediments, the losses in the magnets and pole pieces must be evaluated during the machine design.

In order to estimate precisely the rotor power losses, the contribution of the following three types of harmonics should be considered: a) times harmonics from the Supply Pulse width Modulation (PWM), b) the space harmonics which depend on stator winding slots, c) the harmonics created by the stator slotting when the rotor moves.

However, the effect of the stator slotting is not assessed in detail. Hereby, the permeability is decreased, due to the openings of stator slots, which cause a variation of induction, and therefore lead to induced currents provoqued into the surface layer of the rotor [5-6].

Different opinions for calculating the rotor losses due to stator slotting are presented in literature. Some authors estimate negligible these losses, others believe that they should be considered [8-12].

A. Bettayeb is with the Department of Electrical Machines & Power Systems of Supélec, 91192 Gif-sur-Yvette, France (adel.bettayeb@supelec.fr)

R. Kaczmarek is with the Department of Electrical Machines & Power Systems of Supélec, 91192 Gif-sur-Yvette, France (robert.kaczmarek@supelec.fr)

J-C. Vannier is with the Department of Electrical Machines & Power Systems of Supélec, 91192 Gif-sur-Yvette, France (jean-claude.vannier@supelec.fr)

Reference [11], proposed an analytical method to estimate losses due to space harmonics with slot ripple included a surface machine magnet. The loss calculations are not applied for the geometry of the machine's rotor, but in a simplified 2D multilayer rotor model. These analytical equations show that the eddy current losses are proportional to the slots opening, the number of stator slots and the number of pole pair of the machine.

In reference [5], an analytical formulation has been found for surface eddy current losses noted 'P_s'. The hypothesis was taken into account that the depth of penetration of eddy currents is equal to half of the skin thickness. The surface losses as a function of velocity 'v' of the rotor, the slot pitch 't_d', the amplitude of flux density created by the slot ripple 'ΔB' and the characteristic factor of material K_s , are given by:

$$K_S = \frac{1}{32\pi}\sqrt{\frac{10^7}{\mu_r \rho}} \tag{1}$$

$$P_s = K_S (\Delta B)^2 v^{\frac{3}{2}} t_d \tag{2}$$

Recently, reference [12] presented both 2-D analytical and finite elements approaches to estimate eddy current losses in the non-magnetic conductive region on a rotor of a synchronous machine with salient poles. The analytical equation shows that the losses are proportional to the square of velocity. The result is in agreement with the reference [13]. Nevertheless, the comparative study gives a difference of 50% between the analytical model and the FE 2D loss calculation. This justified the lower accuracy besides the more complexity of the analytical model for loss calculation.

In this study, an Interior Permanent Magnet Synchronous Machine (IPMSM) as described in Fig.1, with 72 stator slots fed by voltage supply is designed by quasi sinusoidal winding distribution. The harmonics analysis is concerned with the slot harmonics because the short pitch winding.

The objective of this paper is to estimate the rotor eddy current losses due to the slot harmonics of stator by an analytical model. The level of magnetic flux induction caused by these harmonics will be evaluated, using the FE method with the technique of rotation of the rotor. Then both analytical model and FE is proposed to estimate the corresponding losses. Finally, a comparison of both analytical model and FE is carried out to check the validity of the loss model for permanent magnet and pole piece.

II. Rotor Harmonic Field Analysis

The rotor of IPMSM uses the "focus principle" in which magnets are located with a circumferential magnetization. It is composed of a shaft which is often made of magnetic iron, with a nonferromagnetic hub in order to prevent magnetic flux leakage through the shaft. The magnets are inserted between the pole pieces.

FE 2D model formulated by the potential vector A_z is used to analyze the time variation of the magnetic field in the machine when the rotor moves.

Fig. 1 shows the flux lines simulated by 2D-FE by ANSYS package for one pole pair of the IPMSM, when the stator slots are fed by sinusoidal supply and the PM's are actuated.

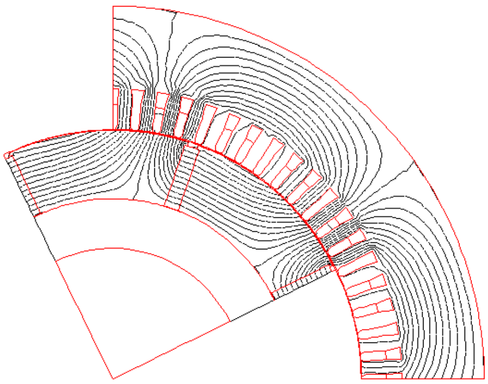

Fig. 1 Distribution of flux lines in one pole pair in the IPMSM

The problem concerns two sources of harmonic fields which perturb the air gap flux density, and provoqued rotor losses in the machine.

First, when the 3-phase machine is fed by balanced sinusoidal voltages and the magnets are demagnetized, time variation of magnetic flux density in the air gap can be represented by the superposition of two fields, one is related to the fundamental field of stator and the other is due to the sinusoid of the slot harmonic [14].

Secondly, the effect of slotting can be visualized when the rotor rotates at synchronous speed N_s and only the magnets are actuated. The surface slot ripple flux density in the rotor can be approximated by the slot's first harmonic.

However, to evaluate the losses due to slot ripples, the contribution of both harmonics magnetic field must be taken into account. Indeed, the magnetic materials of rotor see both harmonic fields and, it is difficult to consider the contribution of one harmonic field without other in losses calculation.

Fig. 2 shows the flux density components waveforms on the rotor surface of the magnet for 90° mech rotation. It can be observed that the slot ripple's magnetic field distribution depends on the number of stator slots n_s, the slot pitch and geometry of slot-tooth. If the opening of the slot corresponds to the width of the slot, the frequency of the stator tooth is then:

$$F_s = n_s N_s \quad (3)$$

The magnetic flux density due to stator opening can be found according to Richter [15]. References [16][12] presented the application of this analysis in the case of our study.

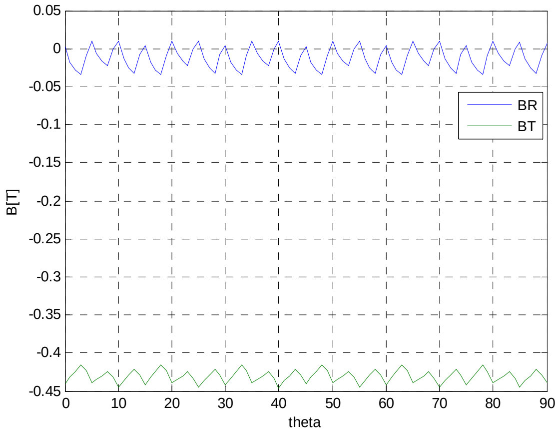

Fig. 2 Radial and tangential components (B_R, B_T) on magnet obtained with 2D-FE, versus rotor position mechanical angle.

The harmonics spectra content for flux density radial component in rotor surface are plotted in Fig. 3 and Fig. 4. It can be observed that the 18th harmonic is the most important. Consequently, the pulsation flux due to the slot effect can be represented by an analytical function of the wave that slides at velocity v equal to the linear speed on the rotor surface.

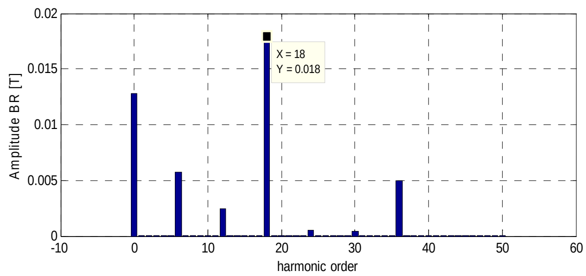

Fig. 3 Harmonic Spectrum for radial component of magnetic flux density in rotor magnet surface

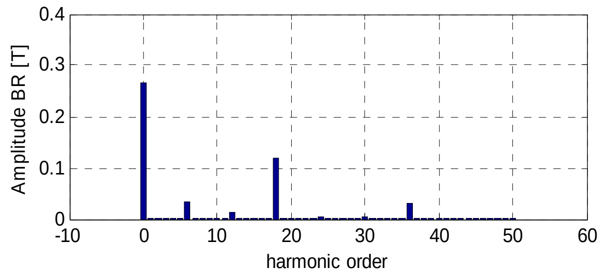

Fig. 4 Harmonic Spectrum for of magnetic flux density in rotor pole piece surface

In the following section, the analysis of a simplified model will be discussed.

III. Loss Model in the Magnet due to Stator Slot Harmonics

The effect of the stator slotting on the field and rotor eddy current losses can be modeled approximately by the study of a parallelepiped shape magnetic material submitted to the slot harmonics field.

A. 2-D Analytical model

The calculation of eddy current losses due to the stator slot ripples are based to the following assumptions:

- The induced currents are directed along the axis of the machine, so the study can be done with a 2D model [17].
- The pulsation flux which occurs on the stator tooth pitch due to the slot harmonic, has a single component B_y(x,t), and can be written in the form:

$$B_y(t,x) = B_0 \cos(\omega t - kx) \qquad (4)$$

Where B_0: the peak value of magnetic flux density (T)

ω: the tooth frequency (rad /s)

$v = \frac{\omega}{k} = \frac{\lambda}{T}$ is the linear velocity of the wave (m/s)

$k = \frac{2\pi}{\lambda}$ is a constant (rad/m)

$\lambda = \frac{2\pi R}{ns}$ is length of the wave or slot pitch (m)

R: radius of the rotor

For parallelepiped magnet, the study can be reduced to calculate loss for a conductor piece of wavelength λ, permeability $\mu_r = 1.1$, resistivity $\rho = 9$ nΩm and supposing the dimensions: $t*H*L$ described in Fig. 5.

For a layer of thickness h of the piece, and assuming homogeneous magnetic flux density along the x direction, the eddy currents $J_z(x,t)$ circulate in paths in rectangular form. Then, it can be represented by resistive coils covering the volume of the layer.

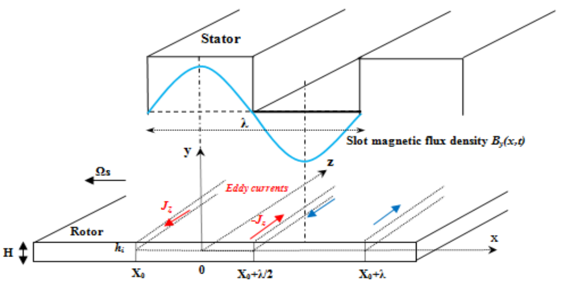

Fig. 5 Analytical model of 2D eddy current in one layer

Using Kirchoff's law, the current $i(t)$ in closed turn of a resistance r is given by:

$$\frac{d\Phi}{dt} + ri = 0 \qquad (5)$$

For a length of $\lambda/2$ raised from a coordinate $x=x_0$ and calculated for the length $L=1m$, the flux is defined as:

$$\Phi = 2\frac{LB_0}{k}\sin(wt - kx_0) \qquad (6)$$

Then, the time variant flux density B_y induces voltage e in one path of the turn which is expressed by:

$$e(t) = -\frac{d\Phi}{dt} = 2\frac{LB_0}{k}\omega\cos(\omega t - kx) \qquad (7)$$

where, the resistance r of this turn and the current density $J(t)$ are:

$$r = \frac{\rho L}{hdx} \qquad (8)$$

$$J(t) = \frac{B_0\omega}{\rho.k}\cos(\omega t - kx_0) \qquad (9)$$

Finally, the eddy current losses in the magnet's layer can be obtained by integration x from 0 to λ/2

$$P_e = n_s\int_0^{\frac{\lambda}{2}} rI_{avg}^2\,dx = n_s h\frac{L.\lambda}{2\rho}B_0^{\,2}v^2 \qquad (10)$$

Equation (10) gives the power losses in a single layer of permanent magnet. The losses are proportional to the square of velocity v, the amplitude of magnetic flux density B_0, the number of slots n_s and the volume of layer.

The following section aims to compare formula (10) by the 2D model of one layer of conductor piece.

B. 2D-FE of slot effect

The 2D-FE model presented in Fig. 6 is an approximation of the harmonic slot effect on the machine losses. The distribution of magnetic flux density is modeled by a 3-phase currents sheet and periodicity on side edges.

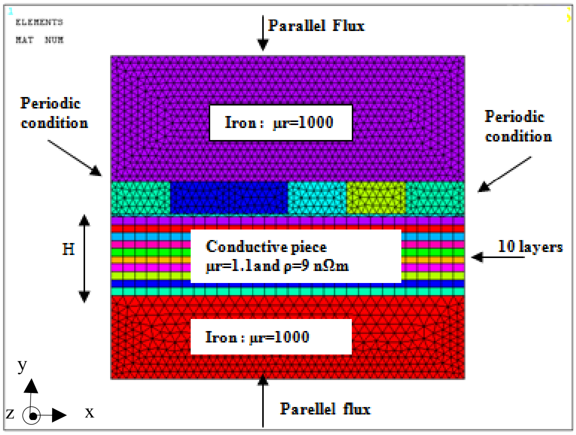

Fig. 6 2D-FE model of slot effect.

To take into account the effect of attenuation of the magnetic field, the magnet is divided by 10 conductive layers of high resistivity (i.e. *9 nΩm*) and permeability $\mu_r = 1.1$. The time variation of magnetic flux density in each layer is assumed homogeneous along x.

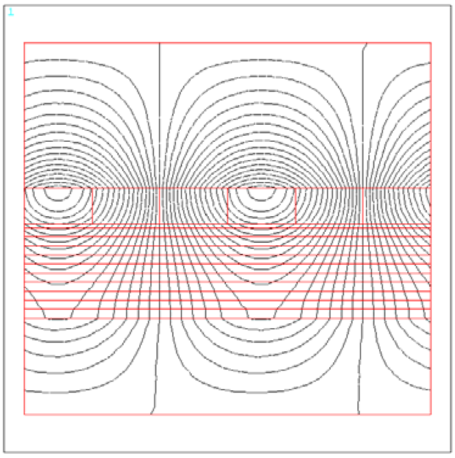

Fig. 7 Flux lines created by a 3-phase current sheet, at 8Hz

Fig. 7 clearly depicts two closed edges flux lines created by the 3-phase current sheet for one slot pitch in the FE model.

C. Comparison between analytical model and FE results

We will now proceed to verify the analytical formula (10) by the finite element calculation. A low frequency *8Hz* is chosen to verify if this approach is already applicable in the absence of skin effect.

C1. Study of one layer of the magnet

A single layer of finite element mesh structure on the magnet's surface such a choice permits to avoid the problem of non homogeneity of the field is considered. FE simulation is performed with the following data:

μ_r=1.1, ρ=9 nΩ.m, v=89.76e-3 m/s, h=0.25 mm, λ= 11.28 mm.

It can be observed a good agreement between analytical and 2D-FE-computations for induced currents in the magnets layer. The results are summarized in Table I.

A comparison at 8Hz between both computation methods is carried out to check the validity of the analytical method. The results presented in Table II show the good agreement between the obtained values in both methods.

Good agreement concerning the loss evaluation when current density quantities match well is resulted. Also, the calculation is verified for variable slot pitch wavelength λ.

Thus, we have confirmed the formula (10) in terms of homogeneity field. This is a perfect homogeneity imposed by the nature of the modeling layer meshes at the elementary level of FE modeling.

TABLE I
AMPLITUDE OF EDDY CURRENT FOR ONE LAYER MAGNET, AT 8HZ

B0 [mT]	Analytical [A/m²]	Finite Element [A/m²]	Deviation [%]
3.67	651	626	4.06
26.48	2640	2522	4.71
52.54	5020	5012	4.54

TABLE II
LOSSES IN A SINGLE LAYER'S SURFACE MAGNET, AT 8HZ

B0 [mT]	Formula (10) [w/m]	Finite Element [w/m]	Deviation [%]
6.57	3.36e-7	5.05e-7	6.15
26.48	8.8e-6	8.08e-6	8.92
52.54	3.46e-5	3.23e-5	7.2

C2. Study of multilayer of the magnet

Now we want to consider more layers on the magnet's surface. In this case, the attenuation of field should be taken into account.

Formula (10) is applied to each layer of the piece conductor of magnet which has the height of *0.25 mm*. The peak value of magnetic flux density B_{0i} is calculated by FE for each layer.

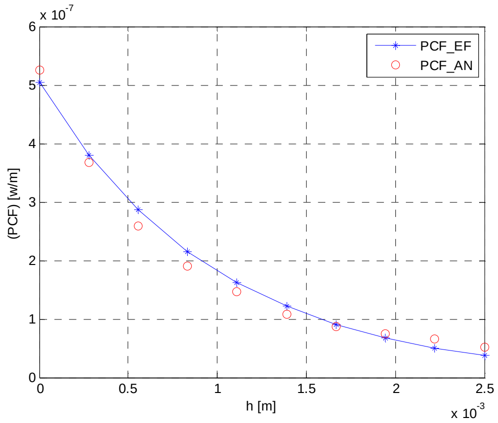

Fig. 8 Comparison between analytical and FE computations for eddy current losses along conductive piece of the depth *H*, at 8 Hz.

Whatever the value of B_0 and λ, the difference in the eddy current losses calculation between the analytical model and FE model is always less than 10% as shown in Fig.8.

The eddy current losses decrease with increasing the depth *H*. To propose an analytical expression for the peak value of magnetic flux density with depth from values calculated for each layer, it can be assumed that the amplitude of magnetic flux density with depth is the form:

$$B_{0i} = Ae^{-\frac{y}{\tau}} \tag{11}$$

A and τ representing the field attenuation in the depth of the magnet piece and B_{0i} being the peak value of the magnetic flux density in i^{th} layer.

The identification by curve fitting gives the constants:

A = 0.0133 and τ = 0.0012

The total eddy current losses in pole piece are the sum of losses for layer thickness *dy*, and can be expressed by:

$$P_{tot} = \frac{n_s \lambda}{2\rho} v^2 \int_0^H A^2 e^{-2.\frac{y}{\tau}} dy = n_s \lambda v^2 \frac{A^2.\tau}{4\rho}(1 - e^{\frac{-2H}{\tau}}) \tag{12}$$

The application of the analytical formula (12) in the case of the conductive magnet piece, gives good results close enough to FE model. A difference, at low frequency (8Hz), is about 20% which seems to be quite acceptable.

The good correspondence between the analytical and finite element loss calculation comes from the fact that in 2D, we only have one component of field and eddy current *Jz(x,t)*. This is not the case in 3D, where we will examine at the next section.

IV. Eddy Current Losses Model in Rotor Pole Piece Dues to the Slot Harmonics

A. 3D-FE model of slot effect

The losses in laminations steel of the pole piece due to the slot effect are very difficult to be estimated because the 2D study is not relevant in the laminated steel. The model will be developed in 3-D based on a 3D-FE model.

The magnetic field due to the slot effect is modelled by the three-phase current sheet, as show in the Fig. 9.

The flux lines in the model are calculated in single laminated steel modeled by the resistivity (i.e.$3.7\ n\Omega m$) and permeability $\mu_r = 1000$.

When the thickness of the laminated sheet is small, the induced currents tend to move in the *x* direction, as shown in Fig. 10, as λ >> e_z, the *z* direction can be ignored. This is the basis of our analytical development.

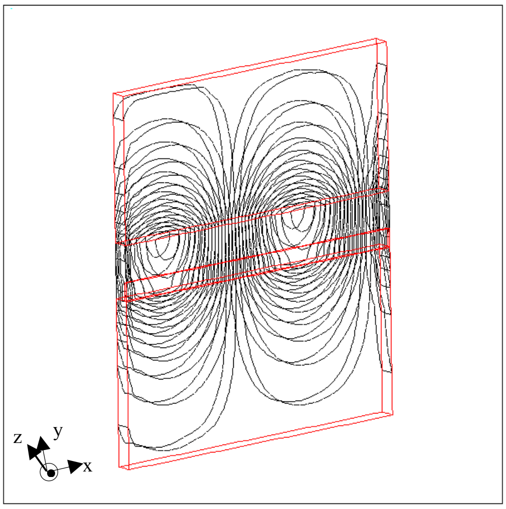

Fig. 9 Flux lines created by 3-phase current sheet in the 3D model, at 8Hz and for laminated sheet 0.35mm

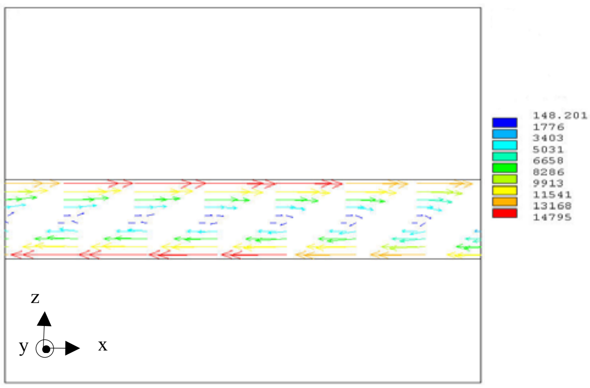

Fig. 10 Path of eddy currents in a laminated sheet of thickness e_z =0.35mm

International Journal of Electrical, Electronic and Communication Sciences
ISSN: 2517-9438
Vol:4, No:6, 2010

B. 3D Analytical model

In this model, we assume that the contribution of magnetic flux density B_x is negligible compared to the component B_y.

Only the path of currents induced in x direction is taken into account. Fig. 11 reveals the path model of eddy current.

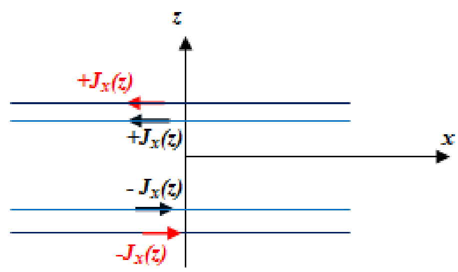

Fig. 11 Model of the path of eddy current in the laminated steel

With the same way as the 2D model, the flux and the EMF are calculated.

$$\Phi_z(x,t) = \int_x^{x+\lambda/2} B_y(x,t) 2z \cdot dx \quad (11)$$

The eddy current losses through the thickness of the sheet can be written by:

$$P_{ec} = 2\int_0^{\frac{e_z}{2}} rI_{avg}^{\;2} = \frac{2he_z^3 B_0^2 v^2}{3\rho\lambda} \quad (12)$$

C. Comparison between analytical model and FE results

The analytical formula (12) is tested in the case of the single layer of the steel. A comparison between the peak value of current density J_0 and the power loss for one sheet is described in Tab. III and Tab. IV.

The results show that the losses estimated by the analytical model are about 50% less than the losses calculated by the FE. The principle reason of this deviation can be explained by the contribution of the component of magnetic flux density B_x in the FE computation. Contrary, this component is neglected in the analytical approach.

TABLE III
AMPLITUDE OF EDDY CURRENT IN THE SHEET OF 0.35 MM

B0 [mT]	Analytical [A/m²]	Finite Element [A/m²]	Deviation [%]
40	387	432	11.1
56	541	619	12.44

TABLE IV
LOSSES IN ONE LAYER IN A SHEET 0.35 MM

B0 [mT]	Formula (12) [w]	Finite Element [w]	Deviation [%]
45.7	2.98-11	6.73e-11	55.7
56	4.37e-11	9.62e-11	54.57

V. CONCLUSION

The controversial problem of rotor losses generated by stator slotting may be resolved only by application of different methods, with cross-verification of results. We propose here analytical model 2D/3D in case of homogeneous field with aid of FE estimation of the field attenuation.

When applied in magnets, this method gives acceptable results in absence of skin effect. Its application in laminated steels didn't prove successful, but we think it can be improved.

REFERENCES

[1] M. Jahns, G. B. Kliman, and T. W. Neumann, "Interior permanent magnet synchronous motors for adjustable-speed drives," *IEEE Trans. Ind. Appilcat.*, vol. IA-22,pp.738-745,1986
[2] A. Arkkio, N. Bianchi, S. Bolognani, T. Jokinen, F. Luise, and Rosu, "Design of synchronous PM motor for submerses marine propulsion system, " in *ICEM* 2002 Proc., Brugge, Belgium, Aug.2002,pp. 25-28
[3] J. Nerg, M. Niemela, J. Pyrhonen, and J. Partananen, "FEM calculation of rotor losses in medium speed direct torque controlled PM synchronous motor at different load condition," *IEEE trans. Magn.*, vol. 38, pp.5070-5072, Sep. 1997
[4] S. M. Abu Sharkh, M.R Harris, N. Taghizadeh. Irenji, "Calculation of rotor Eddy current loss in High-speed PM alternators," *electrical Machines and Drives,* pp.170-174, Sep.1997.
[5] G. Grellet, " Pertes dans les machines tournantes," technique de l'ingénieur, traiter d'électricité, vol. D3450, Dec. 1989
[6] Y. Katsumi, A. Atsushi, "Loss Investigation of Interior Permanent-Magnet Motors Considering Carrier Harmonics and Magnet Eddy Currents," *IEEE Transactions on Industry Application*, vol. 45, n° 4, April 2009, p.659-665.
[7] J. D. Ede, K. Attalah, G. W. Jewell, J. B. Wang, D. Howe, ''Effect of Axial Segmentation of Permanent Magnets on Rotor Loss in Modular Permanent –Magnet Brushless Machines, *IEEE Transactions on Industry Applications,* vol. 43, n° 5, September 2007, p.1207-1213.
[8] K. Altallah et al, "Rotor loss in Permanent-magnet brushless AC machines", *IEEE Trans. on Industry Applications*, vo. 36, no. 6, Nov. 2000, pp. 1612-1617
[9] Hiroaki Toda et al, "Rotor eddy-current loss in permanent magnet brushless machines", *IEEE Trans. on Magnetics*, vol. 40, no. 4, July 2004, pp.2104-2106
[10] H. Hofmann, S. R. Sanders, ''High-Speed Synchronous Reluctance Machine with Minimized Rotor Losses'', *IEEE Trans. on Industry Applications*, vo. 36, no. 2, April. 2000, pp. 531-539.
[11] N. Bianchi, S. Bolognani, and F. Luise, "Potentials and Limits of High-Speed PM Motors," *IEEE Trans. Industry Applications*, vol. 40, no. 6, pp. 1570–1578, Nov./Dec. 2004
[12] J. F. Gieras, A. C. Koening and L. D. Vanek, "Calculation of Eddy Current Losses in Conductive Sleeves of synchronous Machines", *Proceedings of the 2008 ICEM*, Paper ID1305.
[13] C. Mercrow, A. G. Jack, J. M. Masterman, "determination of rotor eddy current losses in permanent magnet machines", Sixth International

Conference on Electrical Machines and Drives. Publ. No. 376, pp. 299-304, Sep 1993
[14] T A. Lipo, " Introduction to AC machine design," Wisconsin power electronics research center, book chapter 5, pp. 287,297,
[15] R. Richter, "Elektrische Maschinen," Band IV. Die induction-maschinen. Birhauser, Basel, 1963.
[16] L. Alberti, E. Fornasiero, N. Bianchi, and S. Bolognani, "Impact of Rotor Losses in a 12–slot 10–pole
[17]] Chari, M. V.K and Silvester, P.P, Finite element in electrical and magnetic field problems, John Wiley et Sons, New York,1980.

Adel Bettayeb received the Ingénieur d'Etat Dergree in electrical power engineering from the U.S.T.H.B University, Algiers, Algeria, in 2004, and the Master Degree from the University of Paul Sabatier, Toulouse, F.R.., in 2007. In December 2007 He is currently a PhD student in Ecole Supérieure d'Electricité SUPELEC (France). His research interests are on the rotor iron losses in synchronous machines.

Robert Kaczmarek is professor in Ecole Supérieure d'Electricité (Supelec) in France. His research interest is with power networks (fault detection, network protection, and parameters identification), magnetism (domain observation, loss prediction and measurements in soft and hard magnetic materials) and design of electric machines.

Jean-Claude Vannier is professor and Head of Department of Electrical Machines and Power Systems in the Ecole Supérieure d'Electricité (Supelec) in France. His research interest is with energy conversion systems (motors, actuators, generators) and concerns the modeling, the design and the optimization of these equipments for specific applications.