

# Analysis of the Elastic Scattering of $^{12}\text{C}$ on $^{11}\text{B}$ at Energy near Coulomb Barrier Using Different Optical Potential Codes

Sh. Hamada, N. Burtebayev, A. Amar and N. Amangeldy

**Abstract**—the aim of that work is to study the proton transfer phenomenon which takes place in the elastic scattering of  $^{12}\text{C}$  on  $^{11}\text{B}$  at energies near the coulomb barrier. This reaction was studied at four different energies 16, 18, 22, 24 MeV. The experimental data of the angular distribution at these energies were compared to the calculation prediction using the optical potential codes such as ECIS88 and SPIVAL. For the raising in the cross section at backward angles due to the transfer process we could use Distorted Wave Born Approximation (DWUCK5). Our analysis showed that SPIVAL code with  $l$ -dependent imaginary potential could be used effectively.

**Keywords**—Transfer reaction, DWBA, Elastic Scattering, Optical Potential Codes.

## I. INTRODUCTION

THE elastic scattering of heavy ions which differ only by one or a few nucleons (valence nucleons) is special case of heavy ion elastic scattering. The measured angular distribution show deviations from the purely decreasing shape which is normally expected in the elastic scattering process of non-identical heavy ions. The diffraction structure shown at the backward angles in the angular distribution is characteristic for a transfer of the difference of the two colliding nuclei, a reaction of type  $A(B,A)B$ . Simultaneously often a large rise of the cross section towards  $180^\circ$  is observed. Where, that behavior cannot be described by standard optical model calculation. In several cases, these phenomena were explained in a consistent way by the competing transfer process  $B(A,B)A$  (elastic transfer), where the elastic scattering is given by the process  $B(A,A)B$ . The two processes are related by their center of mass angles  $\theta_B = \pi - \theta_A$ . In that paper, the reaction  $^{11}\text{B}(^{12}\text{C}, ^{12}\text{C})^{11}\text{B}$  was studied at four different energies 16, 18, 22, 24 MeV. The experimental data were taken from the work of H. P. Duck et al[1]. At energies well above the coulomb barrier for that reaction, we remark that there is increasing in the differential cross section at the

backward angles. That behavior can be described in the framework of DWBA[2]. In that work we managed from obtaining good optical potential parameters which can describe that behavior at different energies. First, the optical model ECIS88 for interpreting the first part of the angular distribution curve as shown in Fig.1 and DWBA was used for interpreting the second part of the curve which arises as a result of proton transfer mechanism. Where, a good result could be obtained also with the usage of the optical model program SPIVAL with different two options 1) optical potential and 2) optical potential with  $l$ - dependent imaginary potential. In approach of Chatwin et al.[3] and Robson[4] the large cross sections at backward angles are obtained by introducing an  $l$ - dependent imaginary potential in the optical model.

## II. RESULTS AND DISCUSSION

For higher energies greater than the coulomb barrier, both of DWBA (Distorted Wave Born Approximation) with ECIS88, and SPIVAL code were used in the comparison between the experimental and the calculated results and a good agreement were obtained as shown in figure (1). For smaller energies which close to the coulomb barrier the effect of raising cross section at backward angles is strongly unobserved so, the results from DWBA for energies 16, 18, 22 MeV doesn't required to be used as shown in figures (2, 3, 4). Where, the optical model with  $l$ - dependent imaginary potential can be used effectively in that range of energies or individual potential without  $l$ - dependent imaginary potential. The best obtained optical potential parameters are listed down in table 1 and table 2.

## III. OPTICAL MODEL WITH $l$ -DEPENDENT IMAGINARY POTENTIAL

It was suggested [5] that especially in heavy ion elastic scattering-in a certain energy region-the use of a  $l$ -dependent imaginary potential is necessary. This should take into account the fact that high angular momentum waves are weakly absorbed relative to low angular momentum waves. In the optical model the imaginary potential  $W$  is replaced by

Sh. Hamada is with Faculty of Physics, Kazakh National University, Almaty, Kazakhstan. Phone: 8707-344-7658; fax: 8707-356-99-42; (e-mail: sh.m.hamada@gmail.com).

N. Burtebayev is with the Institute of Nuclear Physics of National Nuclear Center, Almaty, Kazakhstan. (e-mail: nburtebayev@yandex.ru).

A. Amar is with Faculty of Physics, Kazakh National University, Almaty, Kazakhstan. Phone: 8707-356-99-41; (e-mail: amar.physics@yahoo.com).

N. Amangeldy is with the Institute of Nuclear Physics of National Nuclear Center, Almaty, Kazakhstan. (e-mail: nur19792@mail.ru).

$$W(l) \approx W \frac{l}{1 + \exp(l - L_c)\Delta L} \quad (1)$$

and good fits for various elastic scattering data were reported. There are several suggestions concerning the interpretation and definition of the value  $L_c$  [3, 4, 5]. One estimation of  $L_c$  is based on the maximum angular momentum, which can be carried away by any of the non-elastic, direct reaction channels [5] and the  $l$ -dependence should be most important in those cases in which this maximum angular momentum is much smaller than that brought in by the entrance channel. Nevertheless, the problem of how to determine  $L_c$  is still open. It is therefore obvious, that  $L_c$  cannot be extracted by just fitting the experimental data without having a physical model to determine  $L_c$  or  $W_0$ . It is not astonishing that an optical model with an  $l$ -dependent imaginary potential can account for the rise of the differential cross section structure at the backward angles in the angular distributions. The  $l$ -dependent imaginary potential gives an enhancement of a few partial waves at the nuclear surface. This is, however, just the same effect which is generated by the two processes proposed earlier, the elastic scattering, which may be described by strongly absorbing potentials and the competing elastic transfer. The transfer reaction takes place at the nuclear surface. A coherent summation of elastic scattering and transfer therefore results in an enhancement of a few partial waves at the nuclear surface in addition to the behavior of the partial waves obtained by a simple optical model.

In the optical potential codes ECIS88 and SPIVAL, the Woods-Saxon form factor was used for both the real and imaginary potential

$$\begin{aligned} U &= V + iW, \\ V &= V_0 [1 + \exp(r - R_r)/a_r]^{-1} \\ W &= W_0 [1 + \exp(r - R_w)/a_w]^{-1} \end{aligned} \quad (2)$$

$V_0$  and  $W_0$ ,  $a_r$  and  $a_i$ ,  $R_r$  and  $R_w$  being the depth, diffuseness and radii of the real and imaginary potentials, respectively. The radii are expressed in terms of the mass numbers  $A_1$  and  $A_2$  of the nuclei involved given by

$$R = r_0 (A_1^{1/3} + A_2^{1/3}) \quad (3)$$

By definition, we take the scattering amplitude  $f(\theta)$  in a partial wave expansion to be

$$f(\theta) = f_c(\theta) + \frac{i}{2k} \sum_{L=0}^{\infty} (2L+1) e^{2i\sigma_L} (1 - \eta_L) P_L(\cos(\theta)) \quad (4)$$

Where  $f_c(\theta)$  is the Coulomb scattering amplitude,  $\sigma_L$  the Coulomb phase shifts,  $k$  the wave number and  $\eta_L$  the scattering matrix elements.

They can be represented by  $\eta_L = A_L \exp(2i\delta_L)$  with  $A_L$  being the reflection coefficients and  $\delta_L$  the real nuclear Phase shifts.

TABLE I  
THE OPTICAL POTENTIAL PARAMETERS USED BY CODE ECIS88  
AND DWUCK5 FOR E=24 MEV,  $R=r_0(A_p^{1/3} + A_T^{1/3})$

	E	$V_0$	$r_r$	$a_r$	$W_0$	$r_w$	$a_w$
1	24 MeV	88.89	1.403	0.362	24.18	1.512	0.068
2	24 MeV	100.0	1.362	0.367	27.0	1.475	0.023
3	24 MeV	192.9	1.190	0.435	83.1	1.29	0.223

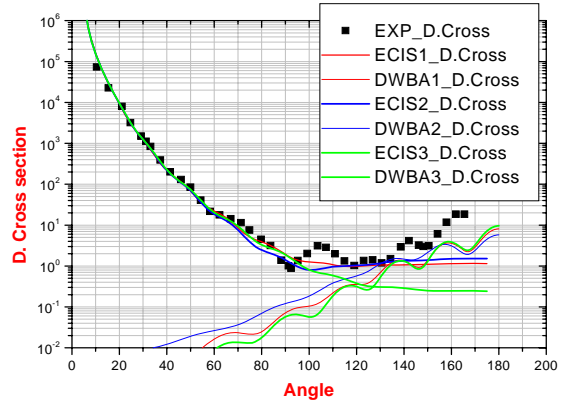


Fig. 1 Comparison between the experimental and the calculated angular distribution within the frame work of DWBA and ECIS88 at energy 24 MeV

TABLE II  
THE OPTICAL POTENTIAL PARAMETERS USED BY CODE SPIVAL  
FOR E=16, 18, 22 AND 24 MEV,  $R=r_0(A_p^{1/3} + A_T^{1/3})$ , THE COULOMB  
RADIUS WAS TAKEN 1.25 FM, AND THOSE WITH  $L$ -DEPENDENT.  
THE \* PARAMETERS WERE FIXED DURING THE SEARCH, WHERE  
THE POTENTIAL SETS A1, A2, A3, A4 WERE USED AS THE INPUT  
POTENTIAL PARAMETERS FOR DWUCK5 CODE

P. set	E	$V_0$	$r_r$	$a_r$	$W_0$	$r_w$	$a_w$	$L_c$	$\Delta L$
A1	16 MeV	103.7	1.181	0.489	31.70	1.356	0.263	---	---
B1	16 MeV	117.5	1.181	0.456	19.78	1.356	0.225	6	1
A2	18 MeV	95.85	1.178	0.476	31.18	1.267	0.33	---	---
B2	18 MeV	95.96	*1.190	0.492	12.66	*1.29	0.542	7	1
C2	18 MeV	95.50	*1.190	0.524	64.80	*1.29	0.423	6	1
D2	18 MeV	96.49	*1.190	0.499	122.35	*1.29	0.324	5	1
A3	22 MeV	90.0	1.176	0.482	31.90	1.274	0.330	---	---
B3	22 MeV	103.99	*1.190	0.430	24.59	*1.290	0.188	7	1
C3	22 MeV	100.88	*1.190	0.419	152.67	*1.290	0.167	5	1
A4	24 MeV	86.0	*1.258	0.409	32.0	*1.357	0.165	---	---
B4	24 MeV	102.17	*1.190	0.530	29.77	*1.290	0.296	8	1
C4	24 MeV	95.87	*1.190	0.456	81.02	*1.290	0.12	7	1
D4	24 MeV	100.31	*1.190	0.442	35.0	*1.290	0.157	6.8	1

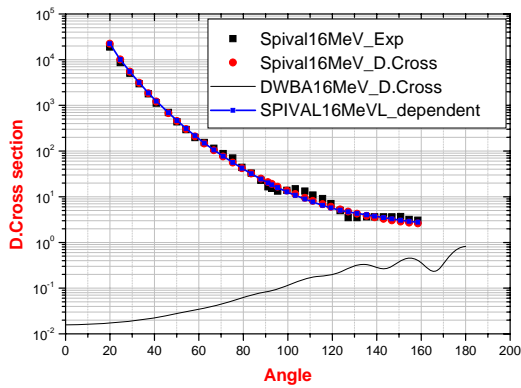


Fig. 2 Comparison between the experimental and the calculated angular distribution within the frame work of DWBA and SPIVAL at energy 16 MeV

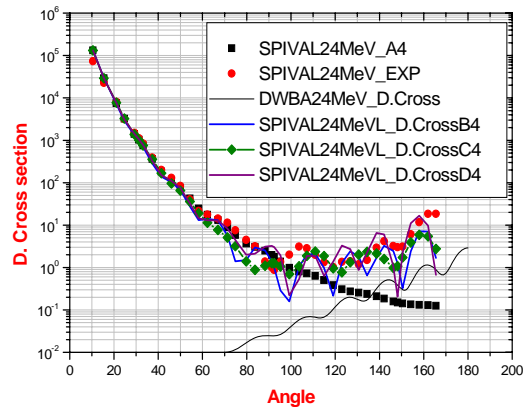


Fig. 5 Comparison between the experimental and the calculated angular distribution within the frame work of DWBA and SPIVAL at energy 24 MeV

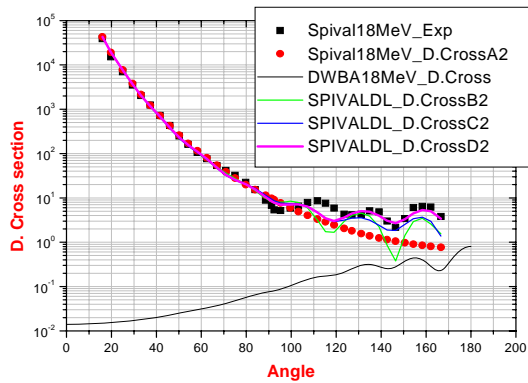


Fig. 3 Comparison between the experimental and the calculated angular distribution within the frame work of DWBA and SPIVAL at energy 18 MeV

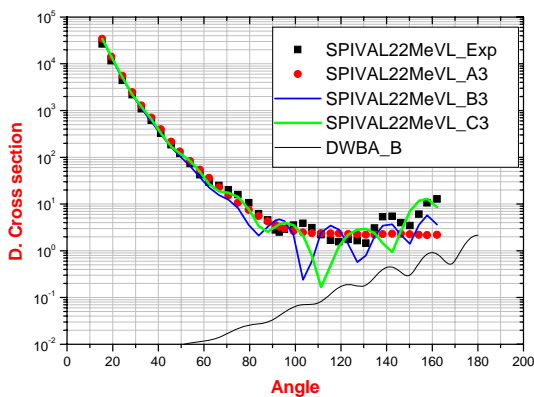


Fig. 4 Comparison between the experimental and the calculated angular distribution within the frame work of DWBA and SPIVAL at energy 22 MeV

#### IV. CONCLUSION AND RECOMMENDATIONS

In this work both of DWBA with ECIS88, and SPIVAL with  $l$ -dependent imaginary potential were used for comparing the results of the angular distribution for  $^{11}\text{B}$  ( $^{12}\text{C}, ^{12}\text{C}$ )  $^{11}\text{B}$  at four different energies 16, 18, 22, 24 MeV with the available experimental data. We remark that the effect of raising the differential cross section at backward angles was strongly observed at energy 24 MeV, which is greater than the coulomb barrier potential of the reaction of  $^{12}\text{C}$  on  $^{11}\text{B}$  and that behaviour is effectively interpreted by DWBA (DWUCK5 CODE) and also with SPIVAL with  $l$ -dependent imaginary potential. While, at small energies, close to the coulomb barrier the effect of raising the differential cross section at backward angles is not strongly observed and the good results can be obtained by using only SPIVAL with  $l$ -dependent imaginary potential code.

The optical model code SPIVAL with  $l$ -dependent imaginary potential can be used successfully for interpreting the raising of the differential cross section at backward angles for energies near the coulomb barrier which take place due to the proton transfer mechanism.

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