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Analysis of Stress Concentration and Deflection in Isotropic and Orthotropic Rectangular Plates with Central Circular Hole under Transverse Static Loading

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***Abstract*—**The distributions of stresses and deflection in rectangular isotropic and orthotropic plates with central circular hole under transverse static loading have been studied using finite element method. The aim of author is to analyze the effect of *D/A* ratio (where *D* is hole diameter and *A* is plate width) upon stress concentration factor (SCF) and deflection in isotropic and orthotropic plates under transverse static loading. The *D/A* ratio is varied from 0.01 to 0.9. The analysis is done for plates of isotropic and two different orthotropic materials. The results are obtained for three different boundary conditions. The variations of SCF and deflection with respect to *D/A* ratio are presented in graphical form and discussed. The finite element formulation is carried out in the analysis section of the ANSYS package.

Keywords—Finite Element Method, SCF, Deflection, Plate, Boundary conditions

I. INTRODUCTION

A rectangular isotropic or orthotropic plate with central circular hole under transverse static loading, have found widespread applications in various fields of engineering such as aerospace, marine, automobile and mechanical. For design of plates with hole, accurate knowledge of deflection, stresses and stress concentration are required. Stress concentration arises from any abrupt change in geometry of plate under loading. As a result, stress distribution is not uniform throughout the cross section. Failures such as fatigue cracking and plastic deformation frequently occur at points of stress concentration.

Paul and Rao [1,2] presented a theory for evaluation of stress concentration factor of thick and FRP laminated plate with the help of Lo-Christensen-Wu higher order bending theory under transverse loading. Shastry and Raj [3] have analyzed the effect of fibre orientation for a unidirectional composite laminate with finite element method by assuming a plane stress problem under in plane static loading. Xiwu et al. [4,5] evaluated stress concentration of finite composite laminates with elliptical hole and multiple elliptical holes based on classical laminated plate theory. Iwaki [6] worked on stress concentrations in a plate with two unequal circular holes. Ukadgaonker and Rao [7] proposed a general solution for stresses around holes in symmetric laminates by introducing a general form of mapping function and an arbitrary biaxial loading condition in to the boundary conditions. Ting et al. [8] presented a theory for stress analysis by using rhombic array of alternating method for multiple circular holes. Chaudhuri [9] worked on stress concentration around a part through hole weakening a laminated plate by finite element method. Peterson [16] has developed good theory and charts on the basis of mathematical analysis and presented excellent methodology in graphical form for evaluation of stress concentration factors in isotropic plates under in-plane loading with different types of abrupt change, but no results are presented for transverse loading.

In this article a study of rectangular isotropic and orthotropic plate with central circular hole upon the effect of *D/A* ratio on SCF under transverse static loading is made. The purpose of this research work is to investigate the effect of *D/A* ratio on SCF for normal stress in *X*, *Y* directions (σ_x, σ_y) and shear stress in *XY* plane (τ_{xy}), and on deflection in transverse direction (U_z). The U_z for different ratio of *D/A* is compared with deflection in transverse direction in plate without hole (U_z^*). Results are obtained for three different boundary conditions. The analytical treatment for such type of problem is very difficult and hence the finite element method adopt for whole analysis.

II. DESCRIPTION OF PROBLEM

To study the influence of *D/A* ratio upon deflection and SCF for different stresses, a rectangular plate of dimension 200 *mm* X 100 *mm* X 1 *mm* with a circular hole of diameter *D* at centre under uniformly distributed static loading of P = 0.02 *Newton* in transverse direction for all cases is analyzed by finite element method. Fig. 1 shows the basic model of the problem. The entire dimensions are also shown in Fig.1. The D/A ratio is varied from 0.1 to 0.9.

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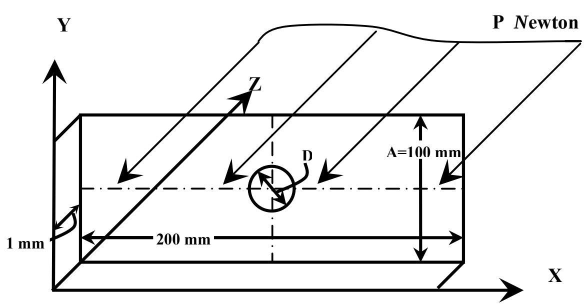

Fig. 1 Basic model of the problem

III. FINITE ELEMENT ANALYSIS

An eight nodded Structural 3-D Shell Element (specified as Shell93 in ANSYS package) with element length of 2 *mm*, was selected based on convergence test and used through out the study. Each node has six degrees of freedom, making a total 48 degrees of freedom per element. In order to construct the graphical image of the geometries of models for different *D/A* ratios, rectangular isotropic and orthotropic plates examined using the ANSYS (Advanced Engineering Simulation). Mapped meshing are used for all models so that more elements are employed near the hole boundary. Due to the symmetric nature of different models investigated, it was necessary to discretize the quadrant plate for finite element analysis. Main task in finite element analysis is selection of suitable element type. Numbers of checks and convergence test are made for selection of suitable element type from different available elements and to decide the element length. Fig. 2 provides the example of the discretized models for *D/A* =0.2, used in study.

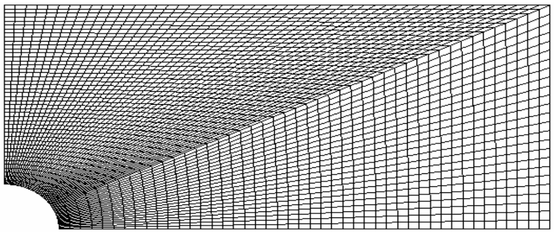

Fig. 2 Typical example of finite element model for *D/A*=0.2

IV. RESULTS AND DISCUSSION

Numerical results are presented for isotropic and orthotropic rectangular plates with a central circular hole. The material properties of different used materials are shown in following Table I. Where; E, G and μ represent modulus of elasticity, modulus of rigidity and poisson's ratio respectively.

TABLE I
MATERIAL PROPERTIES OF USED MATERIALS IN ANALYSIS

Properties	**Materials**		
	Isotropic	**Orthotropic-1 (E-glass/epoxy)**	**Orthotropic-2 (Boron Aluminum)**
E_x	39 GPa	39 GPa	235 GPa
E_y	------------	8.6 GPa	137 GPa
E_z	------------	8.6 GPa	137 GPa
G_{xy}	------------	3.8 GPa	47 GPa
G_{yz}	------------	3.8 GPa	47 GPa
G_{zx}	------------	3.8 GPa	47 GPa
μ_{xy}	0.3	0.28	0.30
μ_{yz}	------------	0.28	0.30
μ_{zx}	------------	0.28	0.30

Plates with three different boundary conditions, as plate (a), (b) and (c) are analyzed. In plate (a); all edges are simply supported, in plate (b); all edges are fixed, in plate (c); two edges are fixed and other two are simply supported. Fig. 3 provides the boundary conditions at all edges of plates (a), (b) and (c).

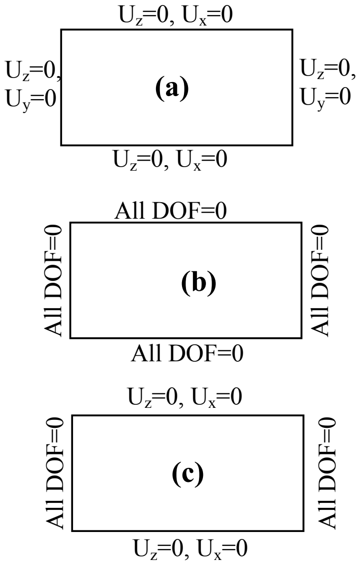

Fig. 3 Boundary conditions at all edges of plates (a), (b) and (c)

Stresses and deflections are obtained for uniformly distributed loads P = 0.02 *Newton* for all cases and *D/A* ratios. The stresses and deflection in transverse direction (U_z*) for full plate (a), (b) and (c), made of different materials under uniformly distributed load of 0.02 *Newton* are listed in Table II.

Fig. 4 shows the effect of *D/A* ratio on SCF (for σ_x, σ_y and τ_{xy}) and U_z/U_z^* in plates (a), (b) and (c) of isotropic material. Following observations can be made from these results.

TABLE II
STRESSES AND DEFLECTION FOR FULL PLATE

Material Type	Plate	σ_x (N/m^2)	σ_y (N/m^2)	τ_{xy} (N/m^2)	U_z^* (m)
Isotropic	(a)	2803.10	6122.60	3376.90	2.85E-07
	(b)	3415.00	4969.80	634.29	7.10E-08
	(c)	7151.00	5224.80	2061.30	2.37E-07
Orthotropic-1	(a)	6499.60	5301.40	2809.90	1.20E-06
	(b)	7345.10	4541.90	577.27	3.07E-07
	(c)	13725.00	3224.80	1534.00	7.23E-07
Orthotropic-2	(a)	3721.80	6146.00	2608.70	8.42E-08
	(b)	4473.70	4956.60	536.16	2.09E-08
	(c)	9290.90	4807.50	1699.10	6.42E-08

Variation of SCF for σ_x with respect to *D/A* ratio observed, maximum in case of plate (a) and significant in case of plates (b) and (c). In case of plate (a); SCF for σ_x increased from 1.65 to 2.11 with increase of *D/A* ratio from 0.01 to 0.06 and decreased from 2.11 to 0.89 with increase of *D/A* ratio from 0.08 to 0.9. In case of plate (b); SCF for σ_x almost unchanged with increase of *D/A* ratio from 0.01 to 0.5 and slight increased from 1.07 to 1.30 with increase of *D/A* ratio from 0.5 to 0.9. In case of plate (c); SCF for σ_x increased from 1.00 to 1.20 with increase of *D/A* ratio from 0.01 to 0.9. Variation of SCF for σ_y with respect to *D/A* ratio observed, maximum in case of plate (c), significant in case of plates (a) and almost negligible in case of plate (b). In case of plate (a); SCF for σ_y increased from 1.44 to 1.81 with increase of *D/A* ratio from 0.01 to 0.08 and decreased from 1.81 to 1.31 with increase of *D/A* ratio from 0.1 to 0.9. In case of plate (b); SCF for σ_y almost unchanged with increase of *D/A* ratio. In case of plate (c); SCF for σ_y increased from 1.44 to 1.82 with increase of *D/A* ratio from 0.01 to 0.1 and decreased from 1.82 to 0.88 with increase of *D/A* ratio from 0.1 to 0.9. Variation of SCF for τ_{xy} with respect to *D/A* ratio observed, maximum in case of plate (b) and significant in plates (a) and (c). In case of plate (a); SCF for τ_{xy} fluctuated between 0.9 to 1.25 with increase of *D/A* ratio from 0.01 to 0.9. In case of plate (b); SCF for τ_{xy} increased from 1.33 to 2.37 with increase of *D/A* ratio from 0.01 to 0.1, decreased from 2.37 to 1.31 with increase of *D/A* ratio from 0.1 to 0.6 and again increased from 1.31 to 1.61 with increase of *D/A* ratio from 0.6 to 0.9. In case of plate (c); SCF for τ_{xy} increased from 0.98 to 1.81 with increase of *D/A* ratio from 0.01 to 0.1 and decreased from 1.81 to 1.06 with increase of *D/A* ratio from 0.1 to 0.9. Variation of U_z/U_z^* with respect to *D/A* ratio observed, maximum in case of plate (a), significant in plate (c) and minimum in plate (b). In case of plate (a); the ratio of U_z/U_z^*increased from 1.00 to 1.33 with increase of *D/A* ratio from 0.01 to 0.6 and decreased from 1.33 to 1.26 with increase of *D/A* ratio from 0.6 to 0.9. In case of plate (b); the ratio of U_z/U_z^* increased from 1.00 to 1.10 with increase of *D/A* ratio from 0.01 to 0.2, decreased from 1.10 to 0.95 with increase of *D/A* ratio from 0.2 to 0.7 and again increased from 0.95 to 0.99 with increase of *D/A* ratio from 0.7 to 0.9. In case of plate (c); the ratio of U_z/U_z^* increased from 1.00 to 1.24 with increase of *D/A* ratio from 0.01 to 0.4 and decreased from 1.24 to 0.90 with increase of *D/A* ratio from 0.4 to 0.9. It is clear from table 1 that in case of isotropic material, for plates (a) and (b) σ_y attained more in compare to other stresses and in case of plate (c) σ_x attained more in compare to other stresses i.e. it is remembered that SCF for σ_y is more important in plates (a) and (b) and SCF for σ_x is more important in plate (c).

Fig. 5 shows the effect of *D/A* ratio on SCF (for σ_x, σ_y and τ_{xy}) and U_z/U_z^* in plates (a), (b) and (c) of orthotropic material-1. Following observations can be made from these results. Variation of SCF for σ_x with respect to *D/A* ratio observed, maximum in case of plate (a) and significant in case of plates (b) and (c). In case of plate (a); SCF for σ_x decreased from 5.19 to 1.18 with increase of *D/A* ratio from 0.01 to 0.4 and increased from 1.18 to 1.35 with increase of *D/A* ratio from 0.4 to 0.9. In case of plate (b); SCF for σ_x increased from 0.70 to 1.00 with increase of *D/A* ratio from 0.01 to 0.9. In case of plate (c); SCF for σ_x decreased from 1.84 to 0.82 with increase of *D/A* ratio from 0.01 to 0.1, increased from 0.82 to 0.91 with increase of *D/A* ratio from 0.1 to 0.5 and again decreased from 0.91 to 0.83 with increase of *D/A* ratio from 0.5 to 0.9. Variation of SCF for σ_y with respect to *D/A* ratio observed, maximum in case of plates (a) and (c) and significant in case of plates (b). In case of plate (a); SCF for σ_y continuously decreased from 6.25 to 1.39 with increase of *D/A* ratio from 0.01 to 0.9. In case of plate (b); SCF for σ_y decreased from 1.50 to 1.26 with increase of *D/A* ratio from 0.01 to 0.03, increased from 1.26 to 1.29 with increase of *D/A* ratio from 0.03 to 0.1 and again decreased from 1.29 to 0.73 with increase of *D/A* ratio from 0.1 to 0.9. In case of plate (c); SCF for σ_y continuously decreased from 7.10 to 0.52 with increase of *D/A* ratio from 0.01 to 0.9. Variation of SCF for τ_{xy} with respect to *D/A* ratio observed, significant for all boundary conditions. In case of plate (a); SCF for τ_{xy} decreased from 4.06 to 2.84 with increase of *D/A* ratio from 0.01 to 0.07 and increased from 2.84 to 3.78 with increase of *D/A* ratio from 0.07 to 0.9. In case of plate (b); SCF for τ_{xy} decreased from 3.81 to 3.12 with increase of *D/A* ratio from 0.01 to 0.05, increased from 3.12 to 3.18 with increase of *D/A* ratio from 0.05 to 0.2 and again decreased from 3.18 to 1.64 with increase of *D/A* ratio from 0.2 to 0.9. In case of plate (c); SCF for τ_{xy} decreased from 5.25 to 1.91 with increase of *D/A* ratio from 0.01 to 0.2, increased from 1.91 to 1.98 with increase of *D/A* ratio from 0.2 to 0.4 and again decreased from 1.98 to 1.47 with increase of *D/A* ratio from 0.4 to 0.9. Variation of U_z/U_z^* with respect to *D/A* ratio observed, maximum in case of plate (a), significant in plate (c) and minimum in plate (b). In case of plate (a); the ratio of U_z/U_z^*increased from 0.89 to 1.26 with increase of *D/A* ratio from 0.01 to 0.4 and decreased from 1.26 to 1.14 with increase of *D/A* ratio from 0.4 to 0.9. In case of plate (b); the ratio of U_z/U_z^* increased from 0.51 to 0.72 with increase of *D/A* ratio from 0.01 to 0.8 and decreased from 0.72 to 0.70 with increase of *D/A* ratio from 0.8 to 0.9. In case of plate (c); the ratio of U_z/U_z^* increased from 0.95 to 1.17 with increase of *D/A* ratio from 0.01 to 0.2 and decreased from 1.17 to 0.57 with increase of *D/A* ratio from 0.2 to 0.9. It is clear from table 1 that in case of orthotropic material-1, for all plates (a), (b) and (c) σ_x attained more in compare to other stresses i.e. it is remembered that SCF for σ_x is more important than SCF for σ_y and τ_{xy}.

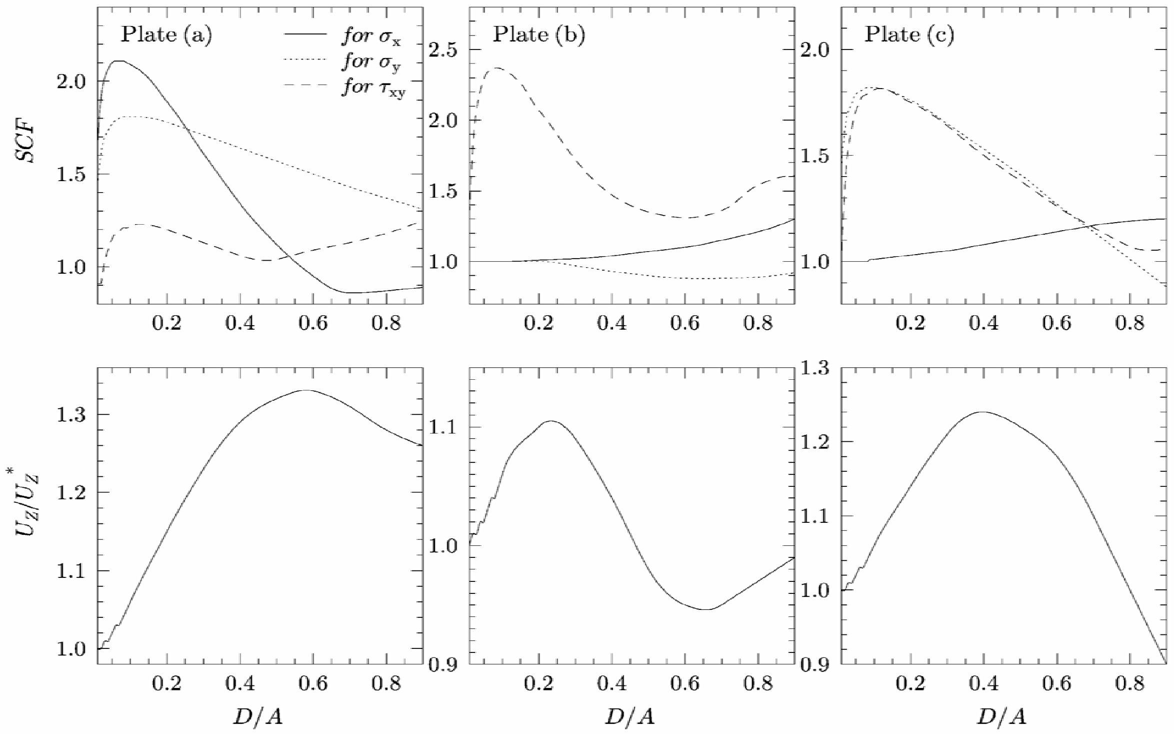

Fig. 4 Effect of D/A ratio on SCF (for σ_x, σ_y and τ_{xy}) and U_Z/U_Z^* in plates (a), (b) and (c) of isotropic material

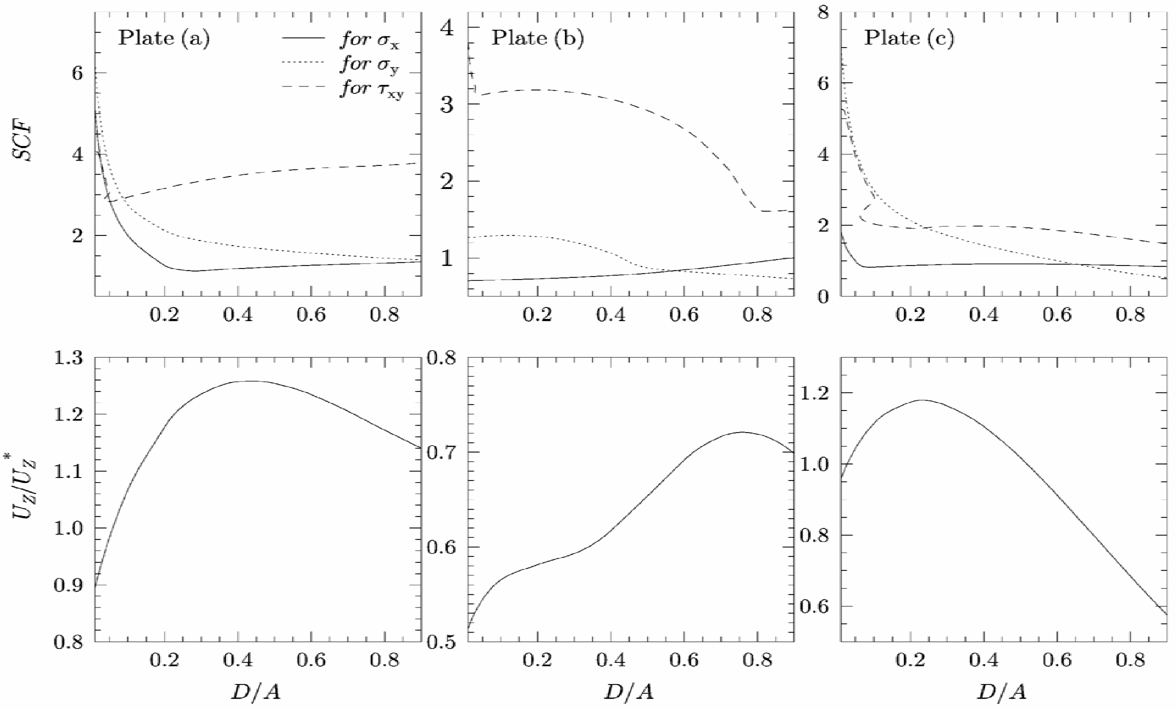

Fig. 5 Effect of D/A ratio on SCF (for σ_x, σ_y and τ_{xy}) and U_Z/U_Z^* in plates (a), (b) and (c) of orthotropic material-1

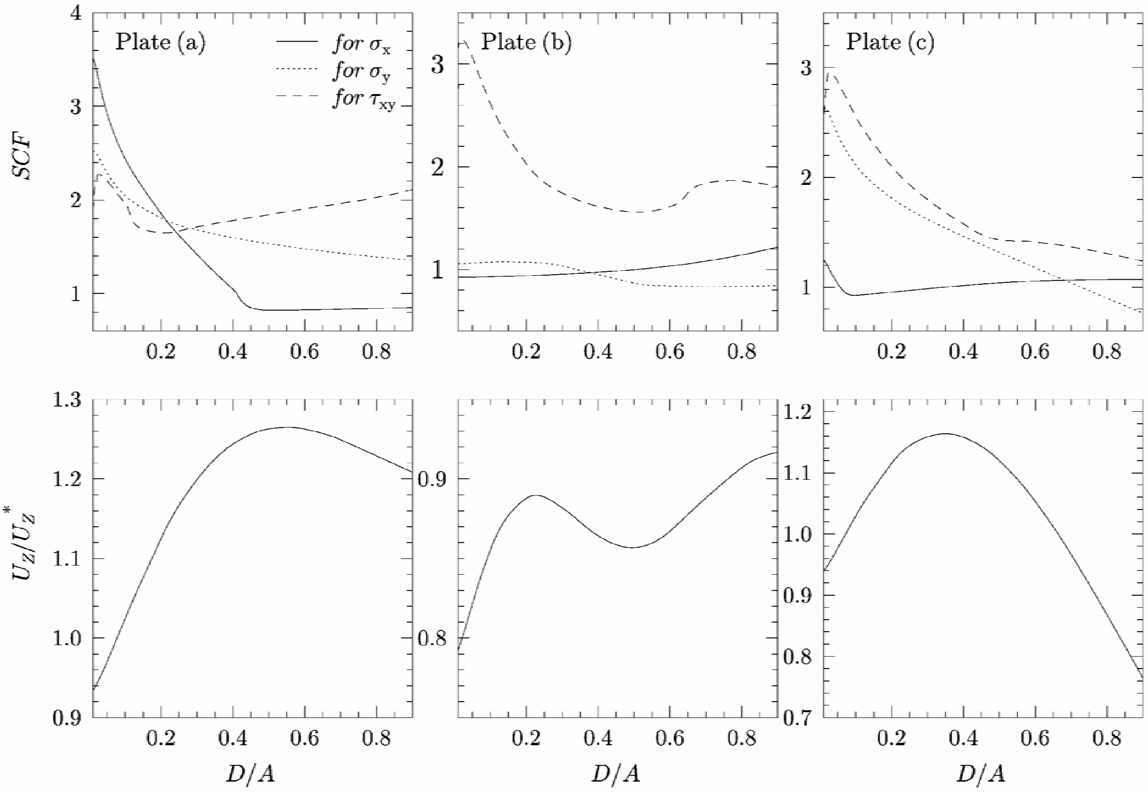

Fig. 6 Effect of D/A ratio on SCF (for σ_x, σ_y and τ_{xy}) and U_Z/U_Z^* in plates (a), (b) and (c) of orthotropic material-2

Fig. 6 shows the effect of *D/A* ratio on SCF (for σ_x, σ_y and τ_{xy}) and U_z/U_z^* in plates (a), (b) and (c) of orthotropic material-2. Following observations can be made from these results. Variation of SCF for σ_x with respect to *D/A* ratio observed, maximum in case of plate (a) and significant in case of plates (b) and (c). In case of plate (a); SCF for σ_x decreased from 3.51 to 0.82 with increase of *D/A* ratio from 0.01 to 0.5 and increased from 0.82 to 0.85 with increase of *D/A* ratio from 0.5 to 0.9. In case of plate (b); SCF for σ_x continuously increased from 0.92 to 1.22 with increase of *D/A* ratio from 0.01 to 0.9. In case of plate (c); SCF for σ_x decreased from 1.25 to 0.92 with increase of *D/A* ratio from 0.01 to 0.1 and increased from 0.92 to 1.07 with increase of *D/A* ratio from 0.1 to 0.9. Variation of SCF for σ_y with respect to *D/A* ratio observed, maximum in case of plates (a) and (c) and significant in case of plates (b). In case of plate (a); SCF for σ_y continuously decreased from 2.52 to 1.36 with increase of *D/A* ratio from 0.01 to 0.9. In case of plate (b); SCF for σ_y increased from 1.05 to 1.07 with increase of *D/A* ratio from 0.01 to 0.1, decreased from 1.07 to 0.83 with increase of *D/A* ratio from 0.1 to 0.7 and again increased from 0.83 to 0.85 with increase of *D/A* ratio from 0.7 to 0.9. In case of plate (c); SCF for σ_y continuously decreased from 2.64 to 0.76 with increase of *D/A* ratio from 0.01 to 0.9. Variation of SCF for τ_{xy} with respect to *D/A* ratio observed, significant for all boundary conditions. In case of plate (a); SCF for τ_{xy} increased from 1.87 to 2.27 with increase of *D/A* ratio from 0.01 to 0.03, decreased from 2.27 to 1.65 with increase of *D/A* ratio from 0.03 to 0.2 and again increased from 1.65 to 2.11 with increase of *D/A* ratio from 0.2 to 0.9. In case of plate (b); SCF for τ_{xy} increased from 2.71 to 3.21 with increase of *D/A* ratio from 0.01 to 0.03, decreased from 3.21 to 1.56 with increase of *D/A* ratio from 0.03 to 0.5, again increased from 1.56 to 1.86 with increase of *D/A* ratio from 0.5 to 0.8 and again decreased from 1.86 to 1.80 with increase of *D/A* ratio from 0.8 to 0.9. In case of plate (c); SCF for τ_{xy} increased from 2.43 to 2.95 with increase of *D/A* ratio from 0.01 to 0.03 and decreased from 2.95 to 1.24 with increase of *D/A* ratio from 0.03 to 0.9. Variation of U_z/U_z^* with respect to *D/A* ratio observed, maximum in case of plate (a), significant in plate (c) and minimum in plate (b). In case of plate (a); the ratio of U_z/U_z^*increased from 0.93 to 1.26 with increase of *D/A* ratio from 0.01 to 0.5 and decreased from 1.26 to 1.21 with increase of *D/A* ratio from 0.5 to 0.9. In case of plate (b); the ratio of U_z/U_z^* increased from 0.79 to 0.89 with increase of *D/A* ratio from 0.01 to 0.2, decreased from 0.89 to 0.86 with increase of *D/A* ratio from 0.2 to 0.5 and again increased from 0.86 to 0.92 with increase of *D/A* ratio from 0.5 to 0.9. In case of plate (c); the ratio of U_z/U_z^* increased from 0.94 to 1.16 with increase of *D/A* ratio from 0.01 to 0.3 and decreased from 1.16 to 0.76 with increase of *D/A* ratio from 0.3 to 0.9. It is clear from table 1 that in case of orthotropic material-2, for plates (a) and (b) σ_y attained more in compare to other stresses and in case of plate (c) σ_x attained more in compare to other stresses i.e. it is remembered that SCF for σ_y is more important in plates (a) and (b) and SCF for σ_x is more important in plate (c).

V. CONCLUSIONS

In general; for plates (a) and (c), the maximum stress concentration is always occurred on hole boundary and in case of plates (b), the maximum stress concentration is occurred on supports. The SCF for σ_x and σ_y varied maximum in plates (a) and (c) and minimum in plate (b), where the SCF for τ_{xy} varied significant in plates (a), (b) and (c) with respect to *D/A* ratio for all materials. The variation of SCF for all stresses with respect to *D/A* ratio observed more in orthotropic plates as compare to isotropic plates for all boundary conditions. The SCF for all stresses is achieved more in orthotropic plate as compare to isotropic plate for respective boundary conditions. It is also observed that variation of SCF for all stresses with *D/A* ratio; highly depends on elastic constants and differ with material to material. For all materials, stress concentration for σ_x and σ_y occurred maximum in plates (a) and (c) and minimum in plate (b), where Stress concentration for τ_{xy} occurred significant in plates (a), (b) and (c), hence the SCF for σ_x and σ_y plays an important role in design of plates (a) and (c) and a minor role in design of plate (b), where the SCF for τ_{xy} plays an important role in design of plates (a), (b) and (c). Maximum deflection in transverse direction occurred for plate (a) and minimum deflection in transverse direction occurred for plate (b) for all cases. The variation of U_z/U_z^* with *D/A* ratio has maximum in plate (a), significant in plate (c) and minimum in plate (b) for all materials. It is also observed that the trend of variation of U_z/U_z^* with *D/A* ratio is almost same for all materials for respective boundary conditions. For all materials; in case of plate (a) and (c), the U_z/U_z^* first increased with increase of *D/A* ratio and after some increase, decreased with increase of *D/A* ratio, but in case of plate (b) the U_z/U_z^* fluctuated between a small range with respect to *D/A* ratio.

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REFERENCES

[1] T. K. Paul, and K. M. Rao, "Stress analysis in circular holes in FRP laminates under transverse load," Computers & Structures, vol. 33(4), pp. 929-935, 1989.
[2] T. K. Paul, and K. M. Rao, "Finite element evaluation of stress concentration factor of thick laminated plates under transverse loading," Computers & Structures, vol. 48(2), pp. 311-317, 1993.
[3] B. P. Shastry, and G. V. Raj "Effect of fibre orientation on stress concentration in a unidirectional tensile laminate of finite width with a central circular hole," Fibre Science and Technology, vol. 10, pp. 151-154, 1997.
[4] X. Xiwu, S. Liangxin, and F. Xuqi, "Stress concentration of finite composite laminates with elliptical hole," Computers & Structures, vol. 57(1), pp. 29-34, 1995.
[5] X. Xiwu, S. Liangxin, and F. Xuqi, "Stress concentration of finite composite laminates weakened by multiple elliptical holes," International Journal of Solids Structures, vol. 32(20), pp. 3001-3014, 1995.
[6] T. Iwaki, "Stress concentrations in a plate with two unequal circular holes," International Journal of Engineering Sciences, vol. 18(8), pp. 1077-1090, 1980.

[7] V. G. Ukadgaonker, and D. K. N. Rao, “A general solution for stress around holes in symmetric laminates under in-plane loading,” Composite Structure, vol. 49, pp. 339-354, 2000.

[8] K. Ting, K. T. Chen, and W. S. Yang, “Stress analysis of the multiple circular holes with the rhombic array using alternating method,” International Journal of Pressure Vessels and Piping , vol. 76, pp. 503-514, 1999.

[9] R. A. Chaudhuri, “Stress concentration around a part through hole weakening laminated plate,” Computers & Structures, vol. 27(5), pp. 601-609, 1987.

[10] H. Mahiou, and A. Bekaou, “Local stress concentration and the prediction of tensile failure in unidirectional composites,” Composites Science and Technology, vol. 57, pp. 1661-1672, 1997.

[11] L. Toubal, M. Karama, and B. Lorrain, “Stress concentration in a circular hole in composite plate,” Composite Structures, vol. 68, pp. 31-36, 2005.

[12] N. T. Younis, “Assembly stress for the reduction of stress concentration,” Mechanics Research Communications, vol. 33, pp. 837-845

[13] G. B. Sinclair, “On the effect on stress concentration of rounding the edge of a hole through plate,” International Journal of Mechanical Sciences, vol. 22(12), pp. 731-734, 1980.

[14] N. Troyani, C. Gomes, and G. Sterlacci, “Theoretical stress concentration factors for short rectangular plates with centred circular holes,” Journal of Mechanical Design, ASME, vol. 124, pp. 126-128, 2002.

[15] M. Fillipini, “Stress Gradient calculations at notches,” International Journal of Fatigue, vol. 22(5), pp. 397-409, 2000.

[16] R. E. Peterson, “Stress concentration design factors,” New York: John Wiley and Sons, 1966.