

# Analysis of Partially Shaded PV Modules Using Piecewise Linear Parallel Branches Model

Yaw-Juen Wang, and Po-Chun Hsu

**Abstract**—This paper presents an equivalent circuit model based on piecewise linear parallel branches (PLPB) to study solar cell modules which are partially shaded. The PLPB model can easily be used in circuit simulation software such as the ElectroMagnetic Transients Program (EMTP). This PLPB model allows the user to simulate several different configurations of solar cells, the influence of partial shadowing on a single or multiple cells, the influence of the number of solar cells protected by a bypass diode and the effect of the cell connection configuration on partial shadowing.

**Keywords**—Cell Connection Configurations, EMTP, Equivalent Circuit, Partial Shading, Photovoltaic Module.

## I. INTRODUCTION

SOLAR electricity is becoming an important source of electrical energy because of the ever increasing prices and scarcity of fossil petroleum. Environmental issues and concerns on global warming are other incentives that favor the development of photovoltaic (PV) power.

The nonlinear I-V characteristic of PV cells is the first problem to deal with in the analysis of a PV generation system. Since both solar irradiance and cell temperature have strong influence on the I-V relation of PV cells, modeling of PV cells must take these effects into account. Existing PV cell models can roughly be categorized into two types: the equation oriented models and the circuit oriented models.

The equation oriented models [1, 2] express the cell I-V characteristic at a given temperature and a given level of irradiance in an analytical form, normally as a nonlinear implicit function, but with some approximation, the I-V characteristic can be expressed in an explicit form. When several PV cells are connected to form a PV module, analysis of the module electrical performance can be carried out by the Newton-Raphson method.

In the circuit oriented models, a PV cell is represented by its equivalent circuit that allows the PV system to be simulated using circuit simulation software. Different PV cell equivalent circuits have been developed to suit different simulation programs. For example, Gow and Manning [3] used a double

exponential cell equivalent circuit to simulate PV arrays. Castaner and Silvestre [4] used PSpice for PV system modeling. Veerachary [5] developed a PV cell model for use in PSIM. Wang and Pierrat [6] proposed an equivalent circuit model for use in EMTP. When using a circuit oriented model to study the PV system, we save the time for programming and we can make full use of the available components of the simulation software.

Partial shading is a commonly encountered mismatch problem in a PV system. It is caused by snow, tree shadow or bird dung covering PV module surface. In a large PV system occupying a wide area of land, moving clouds can also lead to this phenomenon. In building integrated PV systems, PV modules may be installed with different orientations to fit the building outer wall, which makes the modules receive different levels of irradiance, a situation similar to partial shading.

This paper starts with the development of a novel PV cell equivalent circuit model based on piecewise linear parallel branches (PLPB). This is followed by the application of the proposed model to the analysis of partial shading of a PV module. The effects of partial shading on PV modules with different cell connection configurations: series-parallel (SP), total-cross-tied (TCT), bridge-linked (BL) and honeycomb (HC) configurations, will be studied.

## II. DEVELOPMENT OF THE PLPB MODEL

Fig. 1 shows the single-diode equivalent circuit model [7] of PV cell which is commonly used in many studies and provides sufficient accuracy for most applications. In Fig. 1,  $I_L$  is the light-generated current,  $I_d$  and  $V_d$  the diode current and voltage,  $I$  and  $V$  the cell current and voltage, and  $R_s$  and  $R_p$  the cell series and parallel resistances. It can be seen from Fig. 1 that the nonlinearity of a PV cell is indeed attributed to the nonlinearity of the diode.

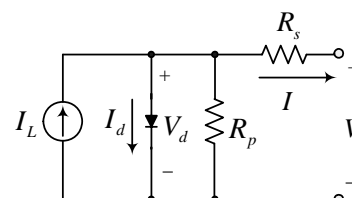


Fig. 1 Single-diode model of PV cell

### A. Basic Principle

The basic principle of the PLPB model is to approximate the

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nonlinear  $I$ - $V$  curve of the diode by  $2^h$  linear segments as shown in Fig. 2. The diode  $I$ - $V$  curve for  $V_d$  between zero and the cell open-circuit voltage  $V_{oc}$  is depicted in Fig. 2. The curve is first approximated by two line segments (dotted lines) with their corner voltage  $E_2$ . The curve to the left of  $E_2$  is again approximated by two segments with a corner voltage  $E_1$ ; and the portion to the right of  $E_2$  approximated by another two segments with a corner voltage  $E_3$ . Hence, the diode  $I$ - $V$  curve is now approximated by four line segments (dashed lines) with three corner voltages  $E_1, E_2$  and  $E_3$ . This procedure continues until a required accuracy is obtained. It is noted that  $2^h$  line segments are used in the  $h$ th approximation step, and the larger the number  $h$ , the better the approximation.

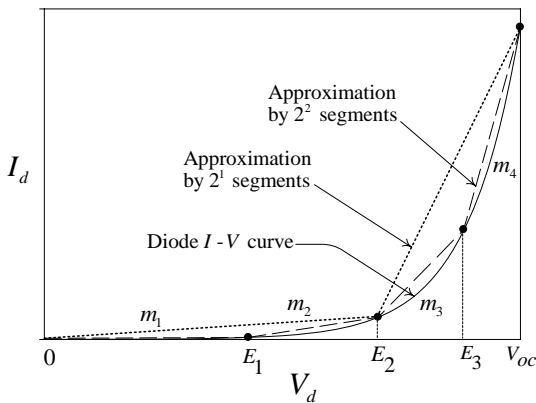


Fig. 2 Diode  $I$ - $V$  characteristic (solid curve) approximated by two (dotted lines) and four (dashed lines) line segments

**B. Realization of Equivalent Circuit**

In Fig. 2, when the diode  $I$ - $V$  curve is approximated by four line segments with corner voltages  $E_1, E_2$  and  $E_3$ , and their respective slopes  $m_1, m_2, m_3$  and  $m_4$ , the piecewise linearized  $I$ - $V$  relation can be represented by four linear branches in parallel as shown by Fig. 3. Each branch consists of an ideal diode, a resistance and a voltage source in series. In Fig. 3 the voltage source  $E_0$  is actually zero. We draw  $E_0$  to make all the branches have the same circuit structure. The resistances in Fig. 3 are related to the segment slopes by the following equations:

$$R_1 = \frac{1}{m_1}; R_2 = \frac{1}{m_2 - m_1}; R_3 = \frac{1}{m_3 - m_2}; R_4 = \frac{1}{m_4 - m_3} \quad (1)$$

When  $V_d < 0, I_d = 0$ . When  $0 \leq V_d \leq E_1$ , the diode of the first branch turns on. This state is represented by the segment between zero and  $E_1$ . When  $E_1 \leq V_d \leq E_2$ , the first and second branches conduct. This state is represented by the segment between  $E_1$  and  $E_2$ . When  $E_2 \leq V_d \leq E_3$ , the first, second, and third branches turn on. This state is represented by the segment between  $E_2$  and  $E_3$ . When  $V_d \geq E_3$ , all branches turn on. This state is represented by the segment between  $E_3$  and  $V_{oc}$  in Figure 2.

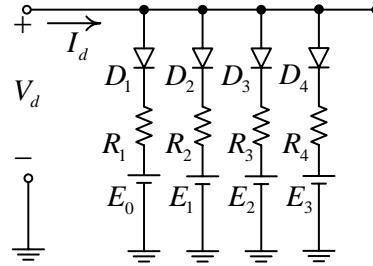


Fig. 3 Diode equivalent circuit using four parallel branches

To generalize, we replace the diode in Fig. 1 with a  $k$ -branch diode equivalent circuit ( $k=2^h$ ). The result is shown in Fig. 4 which is the basic circuit configuration of the PLPB model for PV cell. The resistances in Fig. 4 have a more general expression

$$R_1 = \frac{1}{m_1}; \quad (2)$$

$$R_k = \frac{1}{m_k - m_{k-1}} \text{ for } k \geq 2, k = 2, 3, 4, \dots, 2^h \quad (3)$$

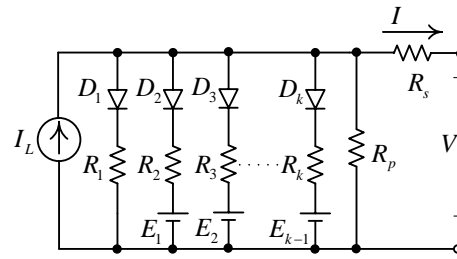


Fig. 4 Circuit configuration of the PLPB model

**C. Determination of Corner Voltages**

The resistances of the PLPB model are related to the slopes by (2) and (3). However, before all the slopes of line segments can be determined, we need to determine the corner voltages. An easiest way to determine the corner voltages  $E_1, E_2, \dots, E_{k-1}$  is to place them equally spaced over the interval of interest. In the case of Fig. 2, the interval was  $(0, V_{oc})$  but this can be changed according to our need. Equally spaced corner voltages do not allow the piecewise linear segments to best approximate the curve. In this paper we propose a more efficient method that, for a given number of linear segments, allows a better approximation.

Fig. 5 shows a known curve  $g(x)$  which is to be approximated by two linear segments  $L_1(x)$  and  $L_2(x)$  over the interval  $(x_L, x_R)$ . The corner point  $x_G$  needs to be determined. We propose an equal-area method that is to find  $x_G$  so that the areas formed by the curve  $g(x)$  and the two line segments  $L_1(x)$  and  $L_2(x)$  are equal, i.e.,  $A_1=A_2$ . The equal-area method has an advantage of fitting high curvature portion with shorter segment, which automatically improves the quality of approximation. A function  $f(x_G)$  is defined as the difference between areas  $A_1$  and  $A_2$ .

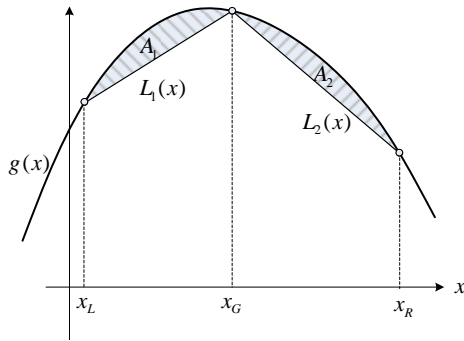


Fig. 5 Determination of the corner point using the equal-area method

$$f(x_G) = \int_{x_L}^{x_G} (g(x) - L_1(x)) \cdot dx - \int_{x_G}^{x_R} (g(x) - L_2(x)) \cdot dx \quad (4)$$

where  $L_1(x)$  and  $L_2(x)$  are straight-line equations

$$L_1(x) = g(x_L) + \frac{g(x_G) - g(x_L)}{x_G - x_L} \cdot (x - x_L) \quad (5)$$

$$L_2(x) = g(x_G) + \frac{g(x_R) - g(x_G)}{x_R - x_G} \cdot (x - x_G). \quad (6)$$

The corner point  $x_G$  can be found by equating (4) to zero and solving for  $x_G$  using the secant method or the false position method.

The equal-area method can readily be applied to the PLPB model. It suffices to replace the known function  $g(x)$  with the diode  $I$ - $V$  characteristic equation

$$I_d(V_d) = I_0 \cdot \left( \exp\left(\frac{qV_d}{nkT}\right) - 1 \right) \quad (7)$$

where  $I_0$  is the reverse saturation current,  $q$  the electron charge ( $q=1.602 \times 10^{-19}$  C),  $n$  the ideality factor,  $k$  the Boltzmann's constant ( $k = 1.38 \times 10^{-23}$  J/K).

#### D. Implementation of the PLPB Model

The algorithm of the equal-area method described previously can be implemented in circuit simulation software such as PSIM or EMTP. Fig. 6 shows an example of four-branch model. In practical applications, more branches, *e.g.*, 8, 16 or 32 branches, can be used for better approximation of the PV cell  $I$ - $V$  characteristic. In Fig. 6, all the diodes are ideal. They actually serve as switches. The values of  $I_L$ ,  $R_1$ ,  $R_2$ ,  $R_3$ ,  $R_4$ ,  $E_1$ ,  $E_2$  and  $E_3$  are calculated and controlled at each simulation step by the control program according to the input values of solar irradiance and cell temperature. In EMTP for example, a control module called MODELS associated with the circuit serves to modeling the control behavior of the system to be simulated. Fig. 7 shows a screen snapshot of the four-branch circuit diagram that is plotted using the ATPDraw program. ATPDraw is a pre-processor of the EMTP that allows users to draw circuit diagrams.

### III. APPLICATIONS TO THE ANALYSIS OF PARTIAL SHADING

In this paper, a practical 50-W PV module is taken as example to study the effect of partial shading using the proposed PLPB model. The sample PV module SM50 is the product of

Shihlin Electric, Taiwan. It is composed of 36 PV cells in series. The electrical characteristics at  $25^\circ\text{C}$  and  $1.0 \text{ kW/m}^2$  solar irradiance are listed in Table I. The parameters of the cell used in SM50 PV module are listed in Table II. Normally the manufacturers do not provide cell parameters, so these data are obtained from experiments in our lab.

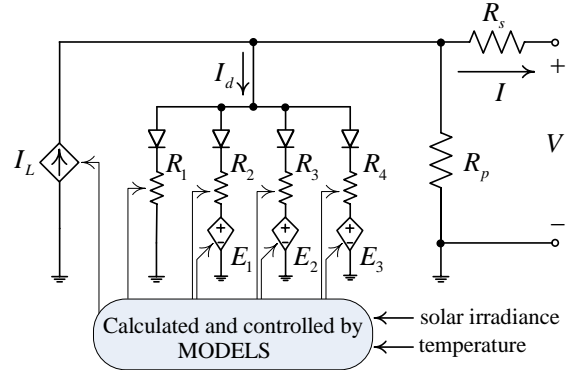


Fig. 6 Implementation of the PLPB model in EMTP

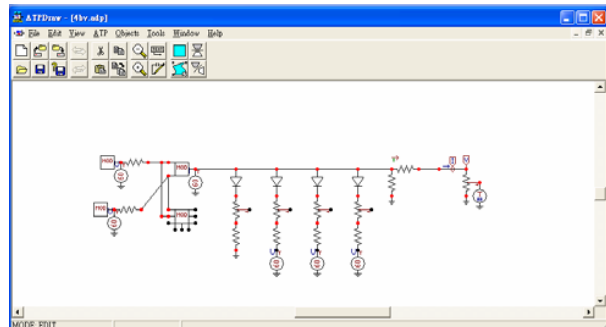


Fig. 7 Simulation circuit diagram in EMTP

 TABLE I  
SM50 PV MODULE ELECTRICAL SPECIFICATIONS

Specification	Quantity
Rated power	50 W
Current at maximum power point	2.7 A
Voltage at maximum power point	18 V
Average short-circuit current	3.3 A
Average open-circuit voltage	22 V

 TABLE II  
PARAMETERS OF THE CELL OF SM50 PV MODULE

Parameter	Quantity
Reverse saturation current at $T_{\text{ref}}$ ( $I_0$ )	$2 \times 10^{-7}$ A
Ideality factor ( $n$ )	1.4
Series resistance ( $R_s$ )	$0.031 \Omega$
Parallel resistance ( $R_p$ )	$15 \Omega$
Light-generated current at reference irradiance ( $I_{L0}$ )	3.3 A
Reference value of irradiance ( $G_0$ )	$1 \text{ kW}^2/\text{m}^2$
Reference temperature ( $T_{\text{ref}}$ )	$25^\circ\text{C}$

#### A. Influence of Percentage of Shading

For a PV array composed of modules in series, when one cell

is shaded, the effect of shading percentage of the shaded cell is studied. Fig. 8 shows the configurations of (a) three and (b) six modules in series. When one cell is shaded with shading percentage from 0% to 100%, the simulated  $I$ - $V$  characteristics using the PLPB model are shown in Fig. 9 in which the circles are the maximum power points. The irradiance and temperature for an unshaded cell are  $1 \text{ kW/m}^2$  and  $50^\circ\text{C}$ , and are zero to  $0.75 \text{ kW/m}^2$  and  $25^\circ\text{C}$  for the shaded cell.

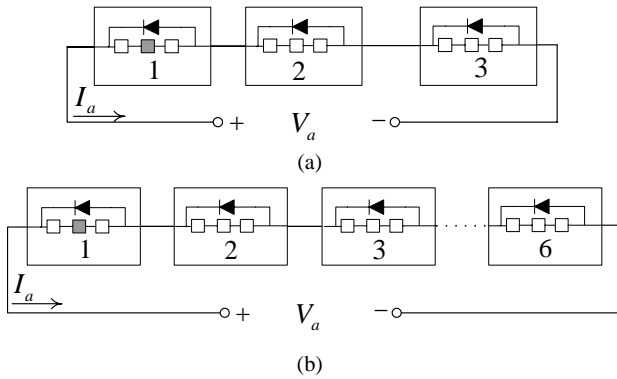


Fig. 8 Effect of shading percentage of one cell on the array composed of (a) three modules, and (b) six modules, in series.

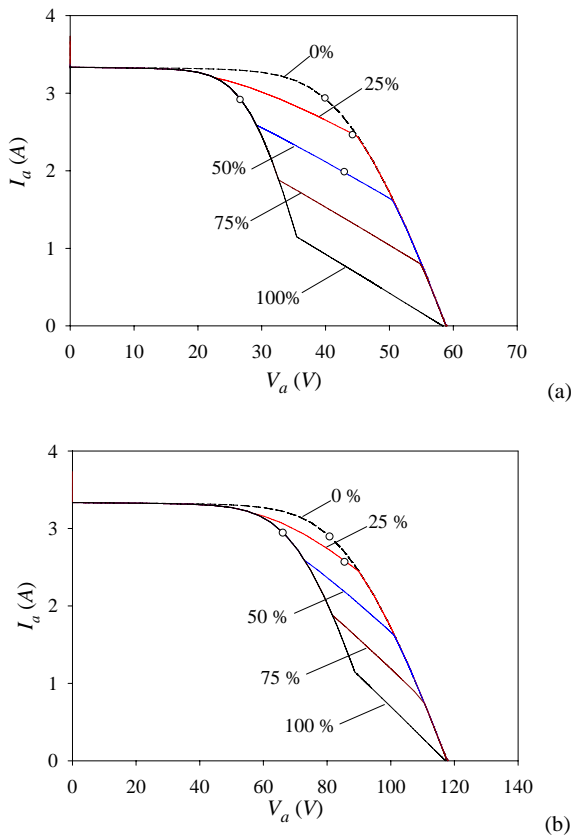


Fig. 9  $I$ - $V$  Characteristic of a PV array of (a) three and (b) six modules in series when one cell is shaded with various shading percentages.

**B. Effect of the Number of Shaded Cells**

In a PV module composed of 36 cells in series, the number of shaded cells influences the module performance. In this section, how the number of shaded cells affects the  $I$ - $V$  curve of an array is studied. The notations A and Bn used in [8] are also used in this paper. They are briefly explained as follows: A: All the 36 cells are unshaded with  $G=1.0 \text{ kW/m}^2$  and  $T_c=50^\circ\text{C}$ .

Bn: n cells are shaded with irradiance  $G=0.25 \text{ kW/m}^2$  and  $T_c=50^\circ\text{C}$  while  $(36-n)$  cells are unshaded with  $G=1.0 \text{ kW/m}^2$  and  $T_c=50^\circ\text{C}$ . Hence, B2 means the module has 2 shaded cells and 34 unshaded cells in series; B36 means all the cells are shaded, and so forth.

The simulation result in Fig. 10 shows the  $I$ - $V$  characteristic of a module with the number of shaded cells from zero to 36. For the effect of the number of shaded cells on array, two cases are considered. The first case is the shaded cells in the same module as shown in Fig. 11 where the right-most module has one, two and three shaded cells. The corresponding  $I$ - $V$  curves are shown in Fig. 12 in which it is observed that for this case the number of shaded cells does not affect the resulting maximum power point. In the second case, the shaded cells are located in different modules, and each module has at most one shaded cell, as shown in Fig. 13. The corresponding  $I$ - $V$  curves are depicted in Fig. 14 where it is seen that the number of shaded cells has significant influence on the maximum power point.

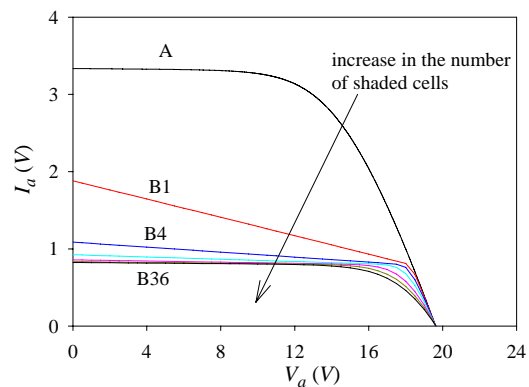


Fig. 10 Influence of the number of shaded cells upon the  $I$ - $V$  Characteristic of the PV module

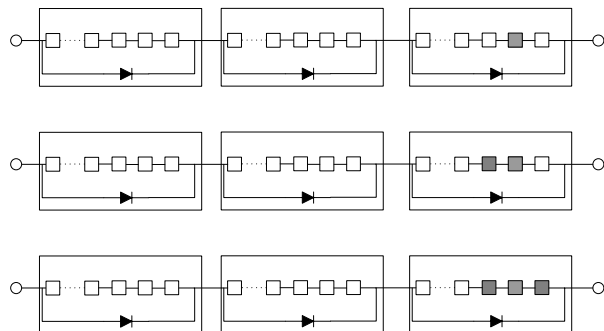


Fig. 11 A string of three modules with one, two and three shaded cells in the same module.

C. Effect of Bypass Diodes

Bypass diodes are often used to prevent PV cells from detrimental effects of partial shading. The number of cells protected by a bypass diode must be limited to ensure PV cells against reverse over-voltage. In this section, the case of complete shading and parameters  $G=1.0\text{ kW/m}^2$  and  $T_c=50^\circ\text{C}$  for unshaded cells and  $G=0\text{ kW/m}^2$  and  $T_c=25^\circ\text{C}$  for shaded cells are assumed. Fig. 15 shows two protection configurations [9]. In Fig. 15 (a), each bypass diode protects 18 cells without overlap. Whereas in Fig. 15 (b), diode D1 is connected across cells 1 to 20 and diode D2 across cells 13 to 36, with cells 13-20 overlapped. For the configuration of Fig. 15 (a), we simulate the case of shading cell 2; for the configuration of Fig. 15 (b), we shade cells 2, 18 and 35. The simulation results are shown in Fig. 16. It is noted that when overlapped cells are shaded, the short-circuit current can be much higher than the reference case (unshaded case). This is because when one or more overlapped cells are shaded, the circuit structure changes from a series string to two parallel strings.

Table III lists the maximum power and the corresponding voltage and current for each case shown in Fig. 16. The power loss  $\Delta P_{Mloss}$  is defined by (8)

$$\Delta P_{Mloss} = \frac{P_{M0} - P_{M1}}{P_{M0}} \times 100\% \quad (8)$$

where  $P_{M0}$  refers to the maximum power of a unshaded module,  $P_{M1}$  to the maximum power of the shaded module. It is noted that the case Cell 18(b) has a very high current at the maximum power point.

TABLE III  
MAXIMUM POWER POINTS AND POWER LOSS

case	$P_M$ (W)	$V_M$ (V)	$I_M$ (A)	$\Delta P_{Mloss}$
Ref	38.57	13.3	2.9	0%
Cell 2 (a)	19.43	6.7	2.9	49.6%
Cell 2 (b)	17.11	5.9	2.9	55.6%
Cell 18(b)	27.03	5.1	5.3	29.9%
Cell 35 (b)	13.92	4.8	2.9	63.9%

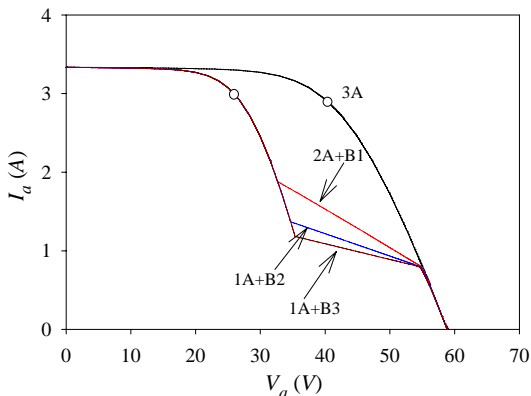


Fig. 12 I-V characteristics of a string of three modules with one, two and three shaded cells in the same module.

D. Effect of the Configuration of Cell Connection

A PV module is normally composed of 36 to 38 cells in series. In this paper four configurations of cell connection in a module

of 36 cells are considered. A module of  $9 \times 4$  cells are connected as shown in Fig. 17 where four configurations are studied: series-parallel (SP), total-cross-tied (TCT), bridge-linked (BL), and honeycomb (HC). The performances of the four configurations are compared when one, two, three and four cells are completely shaded in different locations of the PV module. For each configuration, we simulated six cases where the number and locations of shaded cells are listed in Table IV. Fig. 18 compares the power losses defined by (8) of the four configurations for each case.

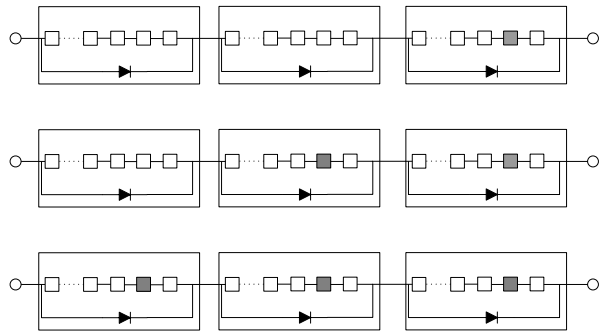


Fig. 13 A string of three modules with one, two and three shaded cells in different modules.

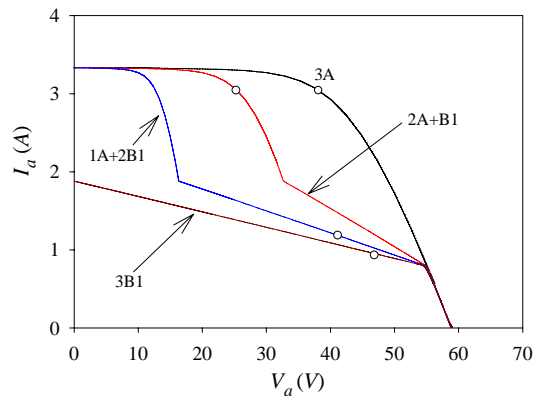


Fig. 14 I-V curves for a string with one, two and three shaded cells in different modules

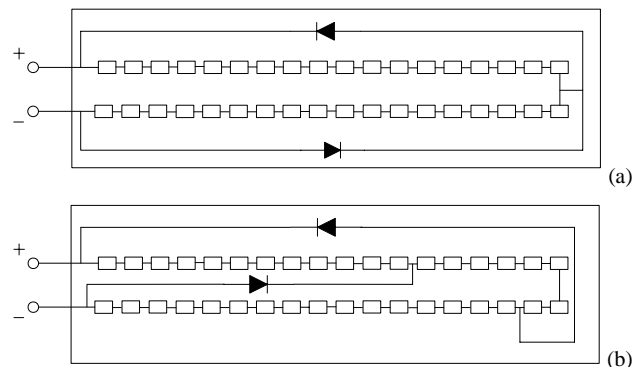


Fig. 15 Protection of PV module with bypass diodes, (a) without overlap cell, (b) with overlap cells.

It is observed that SP configuration has a highest power loss in all the six cases while TCT configuration has a lowest loss.

Both BL and HC configurations are in between SP and TCT. In addition to  $\Delta P_{Mloss}$  we also compare the maximum power ( $P_M$ ) and the fill factor (FF) of the four configurations in each case. The results are listed in Table V. The TCT configuration is superior to others in both  $P_M$  and FF performance. SP has the poorest performance and BL and HC have comparable performance.

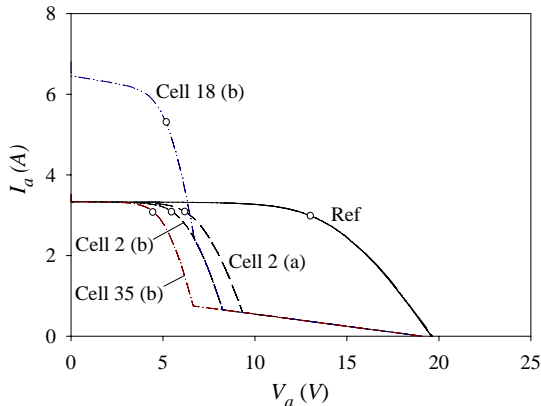


Fig. 16 I-V characteristics of a module with bypass diodes when one cell is completely shaded. Notations (a) and (b) correspond to the configurations of Fig. 15 (a) and (b).

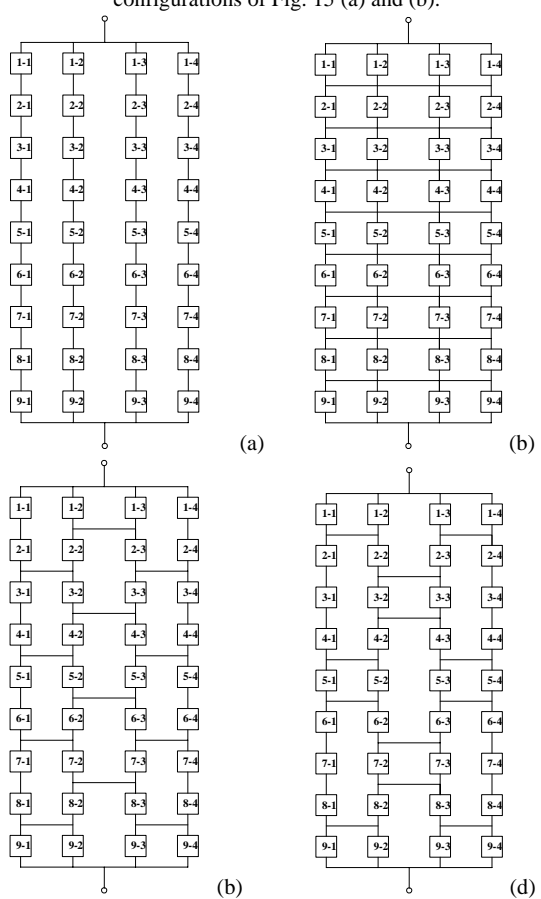


Fig. 17 Cell connection configurations: (a) series-parallel (SP), (b) total-cross-tied (TCT), (c) bridge-linked (BL), (d) honey comb (HC)

TABLE IV  
CASES OF SIMULATION

cas	Locations of shaded cells
1	1-1
2	2-1
3	1-1,2-2
4	2-1,3-2
5	1-1,2-2,3-3,4-4
6	2-1,3-2,4-3,5-4

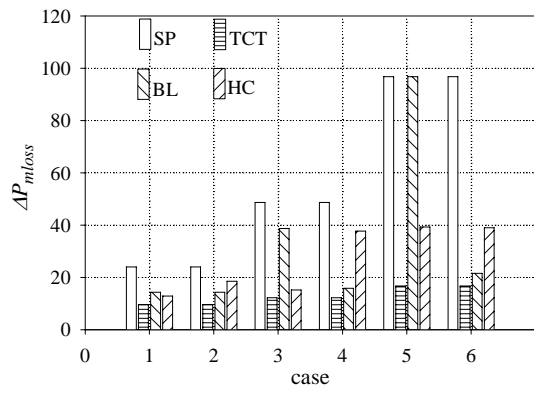


Fig. 18 Comparison of power losses of four configurations

TABLE V  
MAXIMUM POWER AND FILL FACTOR

case	Maximum Power ( $P_M$ )				Fill Factor (F.F)			
	SP	TCT	BL	HC	SP	TCT	BL	HC
1	29.57	35.17	33.32	33.91	0.45	0.54	0.51	0.52
2	29.57	35.17	33.32	31.71	0.45	0.54	0.51	0.49
3	19.97	34.14	23.84	32.99	0.3	0.52	0.36	0.5
4	19.97	34.14	32.75	24.24	0.3	0.52	0.5	0.37
5	1.2	32.40	1.2	23.62	0.02	0.5	0.02	0.36
6	1.2	32.40	30.52	23.73	0.02	0.5	0.47	0.36

IV. CONCLUSION

A novel circuit-oriented PLPB model for PV cell has been presented in this paper. It is based on piecewise linear parallel branches. The number of the branches can be adjusted to meet the required accuracy of simulation. An equal-area algorithm has also been proposed to determine the parameters of the PLPB model. The proposed PLPB model can easily be implemented in circuit simulation software to simulate power systems containing PV arrays. The EMTP has been used to show how the PLPB model is incorporated in the simulation program.

Applications of the PLPB model have been presented in this paper with emphasis placed on the analysis of partially shaded PV modules. Various problems associated with partial shading have been simulated using the PLPB model, including effects of shading percentage, effects of the number of shaded cells, and effects of bypass diodes. Also studied is a comparison of the performance of four connection configurations (*i.e.*, SP,

TCT, BL and HC) of PV cells at the presence of partial shading. All the applications show that the PLPB model is ease to use, flexible and accuracy. Further work on applications of the PLPB model to modeling a grid-interactive PV system is underway. The PLPB model has a great advantage that it allows a PV system to be seamlessly integrated with a large number of built-in power system components (e.g., motors, transformers, transmission lines/cables, switches, etc.) in the simulation software, which is an important feature in modeling and simulating grid-connected PV power systems.

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