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Analysis of Mathematical Models and Their Application to Extreme Events

Avellino I. Mondlane, Karin Hansson, Oliver Popov

Abstract**—**This paper discusses the application of extreme events distribution taking the Limpopo River Basin at Xai-Xai station, in Mozambique, as a case analysis. We analyze the extreme value concepts, namely Gumbel, Fréchet, Weibull and Generalized Extreme Value Distributions and then extrapolate the original data to 1000, 5000 and 10000 figures for further simulations and we compare their outcomes based on these three main distributions.

Keywords**—**Catastrophes, extreme event, disasters, mathematical models, simulation**.**

I. Introduction

THE issue of modeling extremes or rare events covers many areas of life, where events impact negatively on social and economic assets. Floods, earthquakes, snowfalls, hurricanes, heavy rains and extreme temperatures are some of a few examples that can be modeled in the field of natural disasters. Extreme events in the market side extreme events cover fields such as markets crashes, insurance industry and fluctuations in exchange rates that can affect the normal functioning of economic processes. Flood management and other catastrophes are typical events where mathematical approaches play a major role on problem solving and prevention.

Mathematical models illustrated in present paper are just part of the wide range of Decision Support Systems (DSS), extensively discussed in the literature of risk management.

DSS constitute a strong area of technology support to risk management. Risk management including socio-economic problems and environmental issues have benefitted in the last decades from the development of DSS. The following applications are among the most used for problems with flows, Goal Programming, Multiatribute Models, Weighting Matrixes, Pareto Model, ABC Models, Assignment and Fuzzy Models, Linear Programming, GIS modeling, just to mention a few.

Avelino I. Mondlane is a risk management researcher at Stockholm University Department of Computer and Systems Science. DSV; Forum 100 Isafjordsgatan 39, SE-164 40, Kista, Sweden. He is also with the Eduardo Mondlane University for the last 20 years working in the field of project management and risk analysis at the Centre for Informatics "CIUEM", Julius Nyerere Avenue, P.O. Box 257, Maputo Mozambique (e-mail: si-aim@ dsv.su.se).

Karin Hansson is working within the field of decision analysis, and in particular in the disaster risk reduction area. Modeling and simulating both pre and post coping strategies for disasters in a multi criteria fashion, taking into consideration environmental, social and financial consequences for multiple stakeholders. Karin is a member of the research group DECIDEIT. Department of Computer and Systems Science - DSV; Forum 100 Isafjordsgatan 39, SE-164 40, Kista, Sweden (e-mail: karinh@dsv.su.se).

Oliver B. Popov is a Research Scientist at the Department of Computer and Systems Science of Stockholm's University "DSV" where also he is the head of the Systems Analysis and Security Unit. Forum 100 Isafjordsgatan 39, SE-164 40, Kista, Sweden (e-mail: popov@dsv.su.se).

Most mathematical problems are illustrated by equations, but [1] suggest that modeling problems should be included in real world modeling, and then find out the answer back within the model world. This process of converting the real world into a model world is classified as the most important step in the modeling process. Conceptual modeling has been applied in many different fields of study and the example in Fig. 1 illustrates the concept of "Occam's razor" as the starting point for the present research.

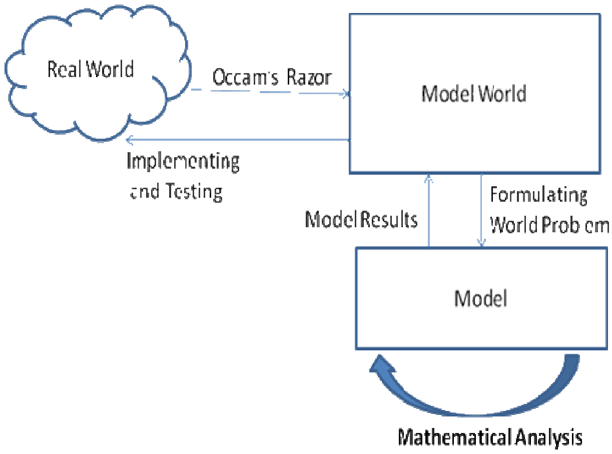

Fig. 1 The Occam's razor Conceptual Model [1]

Fig. 1 shows, according to William of Occam, how "*things should not be multiplied without good reason*." Meaning we shouldn't make things harder than they need to be. In modeling setting, we should exclude the details which are irrelevant given the purpose, or which cannot be handled given the constraints and uncertainties.

II. Purpose and Information Background

A. Motivation and Purpose

The purpose of this paper is to present a set of comparative analyze of mathematical models based on the concept of extreme value distributions concepts, namely Gumbel, Fréchet, Weibull and Generalized Extreme Value Distributions. We also look at application of mathematical models to extreme events such as floods, climate change and global warming. A

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real case is taken into consideration, based on 20 years rainfall data set registered at Xai-Xai precipitation station in Limpopo River Basin, Fig. 2. Moreover using Extreme Events Modeling we extrapolate additional simulations by generating three sets of random numbers of 1000, 5000 and 10000 figures, which are plotted and a comparative analysis is carried out in Section V.

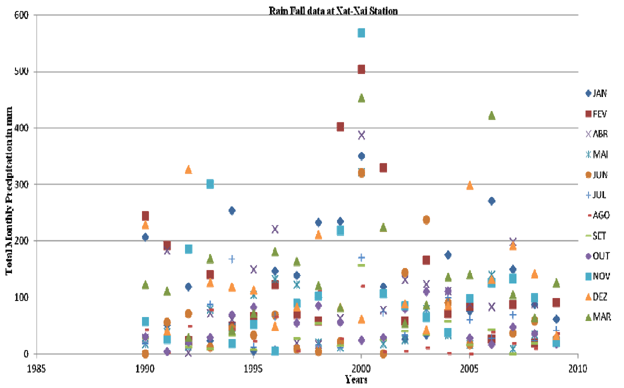

Fig. 2 Twenty years precipitation observed at Xai-Xai
Data Source: INAM "data collected"

Based on [2] there is strong evidence that in most countries where the records of increase and decrease on monthly or seasonal precipitation these countries also experienced changes in the amount of falling during the heavy and extreme precipitation events. Particular motivation for this topic is the magnitude and impact that the 2000 floods in Mozambique, over 40,000 families were resettled to less flood-prone areas, at least 800 people died, 650,000 were displaced, and 4.5 million were affected; this figure corresponds to about a quarter of the country's population [3] nevertheless the scarcity of data related to such events [4] and [5].

B. Back ground Information

"Extreme and non-extreme weather or climate events affect vulnerability to future extreme events by modifying resilience, coping capacity, and adaptive capacity" [6]. The IPCC, 2012 [6] have foreseen unprecedented extreme weather and climate changes as direct consequence of changes in climate and weather anomalies.

Rainfall and water patterns are identified as part of environmental concerns in Africa [7]. Thus, extensive areas experience variability of rainfall and many studies sustain that extreme events will increase or intensify in the future IPCC TAR, IPCC 2001 in [7]. Food security and human lives in Africa are increasingly endangered [8], and the IPCC, 2001b highlights the evidences suggesting the impact of anthropogenic global warming that recently has increased the frequency and magnitude of many extreme climate events such as floods, droughts, tropical and other storms, anomalous temperature and fires. Moreover the IPCC, 2001a [7] points out widespread poverty, recurrent droughts, inequitable land distribution and overdependence on rain-fed agriculture as the main source of vulnerability to climate change in Africa. Figs. 3 (a) and 3(b) highlight the impacts of climate anomalies and extreme events in Africa during the most recent ENSO years [8].

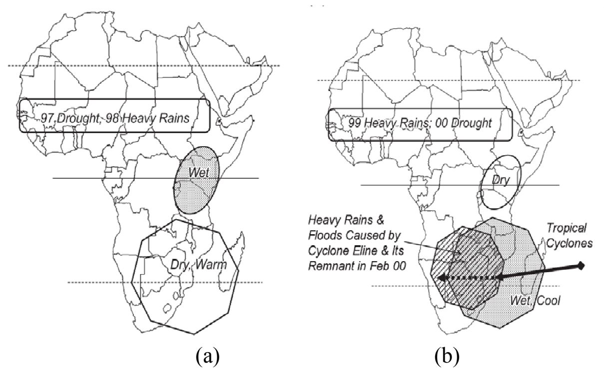

Fig. 3 Climate anomalies and extreme events in Africa [8]

Strategic key areas such as water, agriculture and food security, forestry, health, and tourism, which are closely linked to climate change have been affected extreme events and for the present century the IPCC, 2012 [6], has *medium confidence* that droughts will intensify in some seasons and areas, due to reduced precipitation and/or increased evapotranspiration. This will probably affect some areas in southern Europe and the Mediterranean region, central Europe, central North America, Central America and Mexico, northeast Brazil, and southern Africa.

Social, economic and environmental sustainability can be enhanced by disaster risk management and adaptation approaches. A prerequisite for sustainability in the context of climate change is addressing the underlying causes of vulnerability, including the structural inequalities that create and sustain poverty and constrain access to resources (*medium agreement, robust evidence*). This involves integrating disaster risk management and adaptation into all social, economic, and environmental policy domains [5]-[8].

From the IPCC, 2012 [6] and [8], observations collected since 1950 give clear signs of changes in some extreme events, nevertheless the shortage on quality and quantity of data used for analysis vary across regions and for different magnitude of events and this, impacts the confidence of analysis also differ. Extreme events are subject to uncertainty since it is hard to gather related data, hence to predict and make assessments regarding changes in their frequency or intensity. The management of weather vagaries and extreme hydrological events seems to be beyond the control of human being [9], therefore the rarer the event the more difficult it is to identify long-term changes. Worldwide, the trends on extreme events show different behaviors from global to regional and local scales for specific patterns of hazards as illustrated in Fig. 4. That is; "(a) effects of a simple shift of the entire distribution toward a warmer climate (b) effects of an increase in temperature variability with no shift in the mean and (c) effects of an altered shape of the distribution, in this example a change in asymmetry toward the hotter part of the distribution" [6].

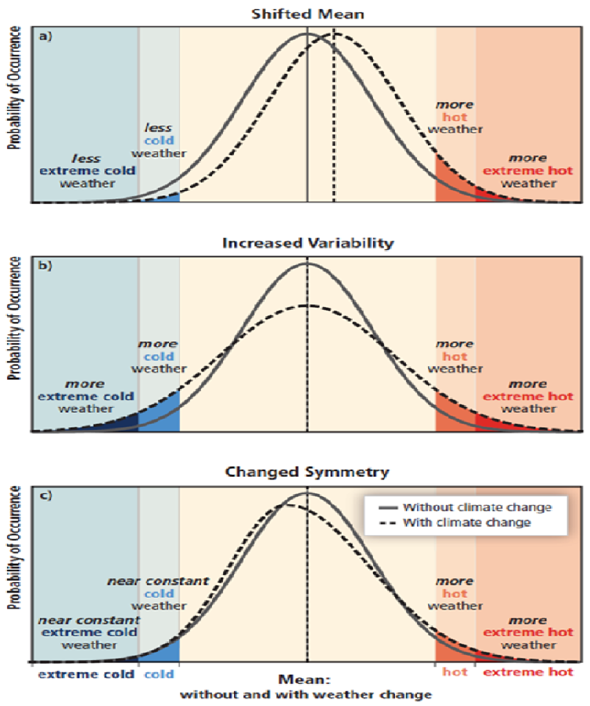

Fig. 4 The effect of changes in temperature distribution on extremes [6]

Fig. 4 illustrates the scenarios changes on mean, variability and symmetry of weather with and without changes that might influence directly the extreme colds and extreme hot weather worldwide.

The (IPCC, 2012) projections [6] on precipitation and temperature changes foreseen, with low confidence, changes in floods, due to *limited evidence* and because the causes of regional changes are complex. The impacts of climate extremes and the potential for disasters result from the climate extremes themselves and from the exposure and vulnerability of human and natural systems. Moreover there are strong evidence of their impacts on both human society and the natural environment. Recent decades witnessed increasing economic damages and loss of lives. The hurricane Mitch in 1998 caused over 10,000 deaths in Central America [2], devastating 1999 December floods in Venezuela with over 10,000 causalities and about $1.8 Bin cost of reconstruction [9] and [10] a flood in Mozambique February year 2000 with about 800 death and about 2% of economic losses [11], the Haiti earthquake in January 2010 and the 11 March devastating earthquake in Japan [12] and [13] are few evidences of how hard the world has been affected by changes in extremes. Both last events have resulted in economic and social asset losses that the world still to deal with. The Haiti is classified as the third most deadly disaster since 1900 [13], while the Japanese was not only the most expensive [12], but also the most complex disaster ever happen, evidencing that human being even with modern technology continues to be exposure to natural disaster [4].

Coping and cost of adapting mechanism and strategies to climate change constitute one of the main priorities and concern for international institutions such as World Bank, the Organization for International Cooperation and Development, and the Secretariat of the United Nations Framework Convention on Climate Change (UNFCCC) whom have assessed and drew cost for the developing countries. Mozambique has been a specific case study from a group of experts [15] from researchers' consortium, developing 50-year scenarios using an empirically derived model of human losses to climate-related extreme events, as an indicator of vulnerability and the need for adaptation assistance. The results of these scenarios were extended based on Mozambique's achievements to 23 (LCDs) using high-resolution climate projections, whose partial results from [15] are illustrated in Fig. 5.

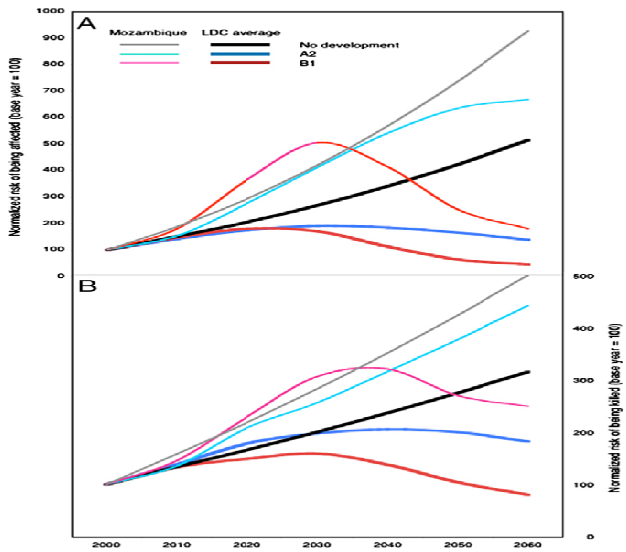

Fig. 5 The effect of changes in temperature distribution on extremes [15]

Comparing the achieved results in [15] from Mozambique with average from a sample of the chosen 23 least developed countries, all scenarios assumed linear continuation of disaster frequencies absorbed over the period 1970-2008. The sample of 23 LDCs comprises Bangladesh, Burkina Faso, Cape Verde Islands, Central African Republic, Comoros Islands, Djibouti, Ethiopia, Gambia, Haiti, Laos, Lesotho, Madagascar, Malawi, Mali, Mauritania, Mozambique, Nepal, Niger, Senegal, Solomon Islands, Tanzania, Vanatu and Zambia.

Notwithstanding, the authors in [15] highlighted the limitations to the applied methods, for instance, like using a cross-sectional regression model to make forward-looking estimates. Thus, the climate exposure scenarios relay on assumption of a linear increase in disaster frequencies over the next 50 years and the uncertainties are large and difficult to estimate. The accuracy and also the degree of difficulty involved in solving the above-mentioned problems, depends critically on the simplifying assumptions applied by the analyst in setting up mathematical model of the real-world situation.

The magnitude of exposure of developing countries to extreme events is discussed by [16] where the scenario in insurance business is commented "*underlying trends show that there is in fact a real risk that climate change could create unsustainable losses in developed countries, and prevent the introduction of insurance to developing countries. The problems could arise from extreme events, such as storms or floods, or more indirectly, from changes in water, air, food quality and quantity, ecosystems, agriculture, and consumer or business behavior brought on by climate change*". Fig. 6 show to what extent climate change might impact the economy.

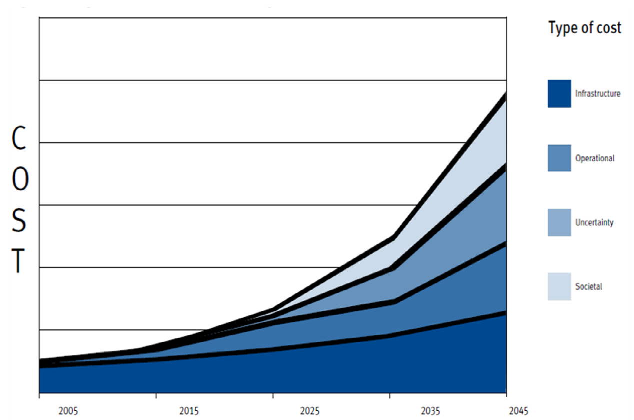

Fig. 6 Progressive onset climate change cost [16]

Historically the three distributions have been considered isolated and their single parameter family unification is a relatively recent development [17], therefore the different options in regard to choice of their application. The Gumbel simplification is based on the existence of many standards distributions of the X_i, in (2) for which Gumbel distribution is the appropriate limit for Mn, where are included normal, exponential, Weibull and gamma families. Fig. 7 illustrates the annual maximum from rainfall data set for the period 1990-2009.

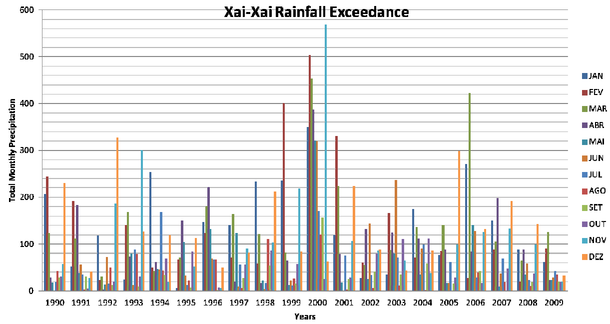

Fig. 7 Annual maximum rainfall data recorded in Xai-Xai

In Fig. 7, we can see that January, February and March are the most critical months of main rainfall. Coincidentally this emphasizes the impact of rains during the period when heavy regional rains and strong depressions and cyclones hit the Mozambique Channel causing the 2000 floods, resulting in thousands of deaths, lost of economic and social assets.

The scenario foreseen in [16] starts with the damage of infrastructures such as building and roads among other predominately. This will be followed by the escalation of prices as direct consequence of increased operating costs of consumers and businesses, derived from changing weather patterns and rising sea level. Thirdly, the "*opportunity cost, will emerge; the deferment of decisions due to uncertainty as the realization grows that climate change is a material issue*" [16], and lastly at societal level the changes in climate will damage the environment, affecting together the non-earning segments of the population and creating socio-economic stresses and escalate social volatility".

III. Extreme Value Theories

A. Mathematical Models in Floods Risk Management

Flood risk management research has been of extreme importance in both hydrology theory and applications related to planning and management of water resources. For this purpose, several models applied from the very simple one such as the stochastic point process [15] to more complex simulation models computer based. The Poisson process although it is not the most simples in the stochastic classes of point process is privileged to be most important process, therefore it is important to refer the Poisson process as one of the first models for flood risk analysis, see Fig. 8, with mean value of Np=50.

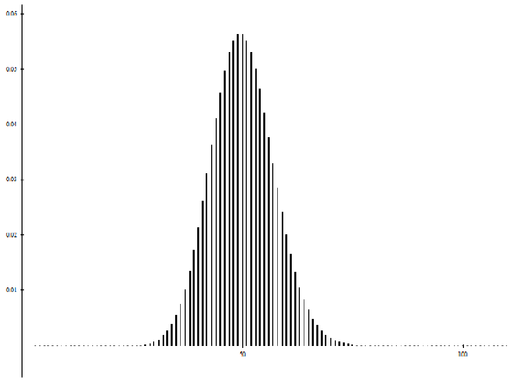

Fig. 8 Poisson distribution with Np=50

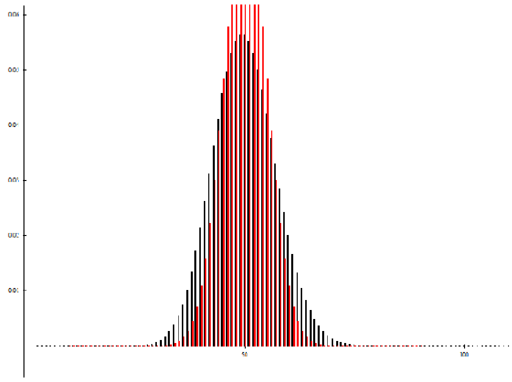

Fig. 9 Fitting the Poisson distribution P(dark) to a binomial distribution B100

In between the rank of the homogeneous stochastic point process model for flood risk analysis [15] discuss, in a detailed manner, the point process models as a classical model suitable to describe the flood peak and analyze flood risk, applying variety of points according to different purposes and cases. From the simplest, Poisson Process Model to flood risk can be assumed as the basic starting point. For example Zongxue (1993) [15] extrapolates two flood risk models using the Bayesian "HSPPB" model where the Bayes reference principle is introduced into "stochastic point process theory" is discussed: the authors have as an outcome: (1) the PBH model risk, which indicates that the exceedance flood should be large than a certain $X_R(\geq R_0)$ during a certain future period of years and (2) the Risk RPG model, which is based on having more information about historical floods prior to distribution of the intensity λ that is considered to be taken as a two parameter gamma distribution. These models are beyond the scope of our discussion, but can serve as part of wide reference on flood risk management and more details on their application, c.f. [15].

On another hand [12] apply different models for flood forecasting in Sri Lanka starting by a hydrodynamic model test and in the following discuss a set of blackbox models and some conceptual models, which help to extract valuable results. Mathematically, our views and beliefs to the real world are translated to mathematical language that can be used for different purposes, such as development of scientific support, experimental and practical tests and decision making support for both management and planning. Fig. 9 shows the same distribution with trend to a binominal B100.

Mathematical models, when we run simulations can lead to both deterministic and stochastic models, according to the different kind of factors that we might take into consideration. Deterministic models give specific type of results, a single number, but if we use random factor then we obtain a distribution of numbers. Given the uncertainty that is bounding our research, the present paper focus on stochastic models. The traditional method of flood risk management based on historical data is extensively discussed on [15], where many author illustrate the results out coming from the advances of hydrological science. One of the most important issue that is taken in consideration in this research, given the uncertainty is the selection criteria of the mathematical equations that lead the research, in order avoid the misleading and ambiguous of the results.

The use of mathematical models has shown how important this discipline contributes to the management of people's real life situations.

Mathematical modeling of physical systems is, by definition, the description of system behavior by means of suitably chosen mathematical relations or equations. Since the mathematical description of real-world situations must always be, to some extent, imperfect, there is never a model for a given system but rather a spectrum of models [18].

According to [19] information systems belong to the most complex artifacts built in today's society. Developing and maintaining system raises a large number of problems, raging the purely technical to organizational and social ones. Moreover, according to [19], most of the problems within the information systems are due to lack of communication and conflicts among the stakeholders with different point of views and structuring framework. The need of negotiation in order to filter the differences is suggested as the main ground floor for problem solving. And this is called conceptual modeling.

Borrowing these ideas from [19] we believe that mathematical models although consists mainly of logical expressions, they also have more to do with logical and conceptual schema in order to solve the complexity of the real world problems. The authors in [1] describe complexity as an association to the real world problem and argue that the problems that modelers wish to solve exist in the real world and classify the real world as a nasty place with all sorts of complications.

B. Extreme Value Theories, Exceedance and Threshold Rainfall

Provided the historical data we can apply the extreme value theory techniques for estimating probabilities for future extreme levels [17]. Probabilistic extreme value theory, first of all deal, with the stochastic behavior of maximum and minimum random variables in group of time-ordered independent and identically distributed (iid) observations of a process that must follow extreme distribution of a generalized form, usually named as a GEV distribution.

$$G(z) = \exp\left\{-\left[1+\xi\left(\frac{z-\mu}{\sigma}\right)\right]_+^{-1/\xi}\right\} \quad (1)$$

for parameter $\mu, \sigma(>0)$ *and* ξ. This family arises [20] by considering limiting approximations to the distribution of

$$M_n = \max(X_1, ..., X_n) \quad (2)$$

Linearly rescaled as $n \to \infty$, where the X_i are iid random variables having possible short-range dependence, but only very restricted form of long-range dependence pertaining to extreme [20] for more details.

By $\xi = 0$ we get the special case, assuming the limit of (1) as $\xi \to 0$, resulting on the Gumbel family discussed in the following sections with the distribution function [20]

$$G(z) = \exp\left[-\exp\left\{-\left(\frac{z-\mu}{\sigma}\right)\right\}\right], \quad -\infty < z < \infty \quad (3)$$

In probability theory and statistics, the generalized extreme value distribution is a family of continuous probability distributions developed within extreme value theory to combine the three separate classes corresponding to $\xi < 0$, $\xi = 0$ and $\xi > 0$.

Climate models improvements are appointed by [9], as sources to enhance abilities to simulate many aspects of climate variability and extremes and as an example they refer to decreased diurnal temperature ranges, warmer mean temperature associated with increased extreme cold days, and increased intensity of rainfall events.

As discussed above the data set under analysis denote exceedance over the threshold (the average) rainfall set of 20 year. According to [20] a model that support to extract upper extremes from a given set of data $x_1, ..., x_n$ to take exceedance over a predetermined, high threshold μ. exceedance y_j over μ (peaks-over-threshold (plotted) are those $x_n > \mu$ taken in the original order of their outcome or in any other order. This number of exceedance will be denoted by k or, occasionally, by K to emphasize the randomness of this figure. zero, otherwise. If X_i are iid random variables with common distribution function (df) df F, then

$$P\{K=k\} = \binom{n}{k} p^k (1-q)^{n-k} = B_{n,p}\{k\}, where k = 0, ..., n, \quad (4)$$

where $B_{n,p}$ is the binomial distribution with parameters n and

$$p = F(\mu) \quad (5)$$

The mean number of exceedance over μ is

$$\psi_{n,F(\mu)} = np - n(1 - F(\mu)) \quad (6)$$

which defines a decreasing mean value function.

Another mathematical model that [20] suggest support the analysis of highest figure of the rainfall data set is the so called annual maxima, blocks or Gumbel method given below.

For iid random variables $X_1, ..., Xm$ with common df F, we can easy compute the df of maxima, as follow:

$$P\left\{\max_{i \le m} X_i \le x\right\} = P\{X_1 \le x, ..., X_m \le x\} = F^m(x) \quad (7)$$

This can be complemented by the minima, generally deduced from corresponding results for maxima as follows:

$$\min_{i \le n} X_i = -\min_{i \le n}(-X_i) \quad (8)$$

that can generate results in terms of the survivor function

$$F = \overline{1} - F \quad (9)$$

The study of extreme value distributions is widely discussed in different text books of this subject and different author such as, [10], [20]-[23] three main types of extreme value name the Gumbel Distribution also called Extreme Value Distribution (EDV) Type I, Fréchet Distribution (EDV Type II), the Weibull Distribution (EDV Type III) and the Generalized Extreme Value Distribution, a flexible three parameter distribution that combines the all three type listed above.

IV. Extreme Events Modeling

A. Generalized Extreme Values Distribution

In the field of statistical analysis the issue of Extreme events modeling is covered by Extreme value theory as a separate branch; the background of this theory is the extreme types of theorem, also known as three types theorem and it supports that there are only three types of distributions that are needed to model the maximum or minimum of the collection of random observation from a data set The standard Extreme values Distributions Functions, given in most statistic text books, mathematically are illustrated by the following equations:

The standard Gumbel distribution also known as Extreme Value Type I distribution, is given by the following probability density function

$$f(x)=e^{-e^{-x}} \tag{10}$$

in case where $\mu=0$ and $\beta=1$.

The Fréchet Distribution, also known as the Extreme Value Type II, is defined by probability distribution function:

$$f(x)=\frac{\alpha}{\beta}\left(\frac{\beta}{x}\right)^{\alpha+1}\exp\left(-\left(\frac{\beta}{x}\right)^{\alpha}\right) \tag{11}$$

where α is shape parameter $\alpha>0$ and β is a scale parameter $\beta>0$,

While the Weibull, also known as Extreme Value Type III, has the following probability distribution function:

$$f(x;\lambda;k)=\begin{cases}\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)k} & x\geq 0\\ 0 & x<0\end{cases} \tag{12}$$

where k>0 is the shape and $\lambda>0$ is the scale parameter of the distribution.

More explanation about the application of the three probability distribution function and the usage of this Extreme Value Distribution is given by [20], where the parameterization of Weibull dfs is different from the standard one, where Weibull dfs with positive shape parameter is considered.

B. Illustrations and Simulations

When running the extreme function based on the row data the models result as shown in Figs. 10 and 11 with normal distribution and Fréchet, Gumbel Max and Gumbel Min respectively with a basic histogram. The Gumbel family as exhibited on Fig. 11 has symmetric behavior, the min to the max since it the generalized extreme value for minima is obtained based on a negative value of x, i.e. by subtitling with (-x). Hence it is important to mention that the results that aim to show the maxima can also be used to illustrate the minima by using a series of $-x_n$ instead of x_n.

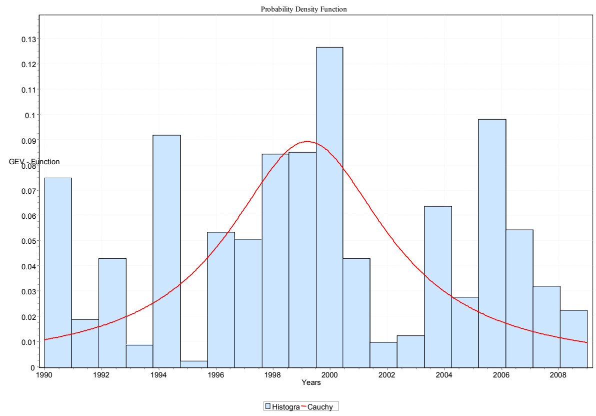

Fig. 10 The histogram and normal distribution as an outcome from the row data

Using the same data, three main distributions can be analyzed, namely the Fréchet , Gumebl maximum and Gumbel minimum, while the Weibull it is not applicable, as shown in Figs. 10 and 11. The Frechet and Gumbel min almost have the same parameters, therefore they illustrate the same behavior.

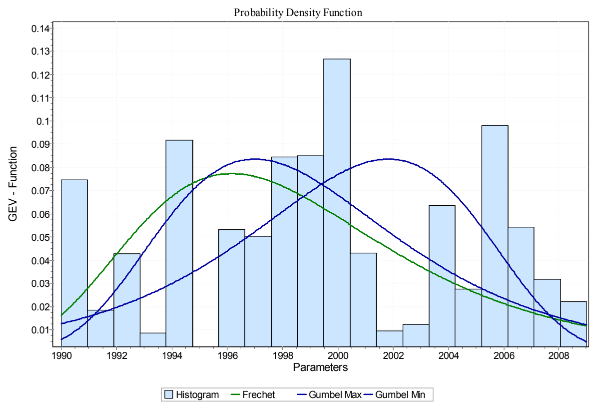

Fig. 11 Extreme distributions, collected data

All graphics can be analyzed using their statistics and Goodness of fit, based on Kolmogorov and Anderson Darling as illustrated in Tables I to VIII.

TABLE I
Fitting Results Based On Collected Data

#	Distribution	Parameters	
1	Fréchet	$\alpha=4.4E+2$	$\beta=2.0E+3$
2	Gumbel Max	$\sigma=4.2$	$\mu=2.0E+3$
3	Gumbel Min	$\sigma=4.2$	$\mu=2.0E+3$
4	Weibull	$\alpha=5.3E+2$	$\beta=8.0E+3$

TABLE II
GOODNESS OF FIT BASED ON COLLECTED DATA

#	Distribution	Kolmogorov Smirnov		Anderson Darlin	
		Statistic	Rank	Statistic	Rank
1	Fréchet	0.17	3	0.75	1
2	Gumbel Max	0,13	1	1.0	2
3	Gumbel Min	0.16	2	1.1	3
4	Weibull	N/A		N/A	

TABLE III
FITTING RESULTS BASED ON 1000 RANDOM NUMBERS

#	Distribution	Parameters	
1	Fréchet	$\alpha = 4.8E+2$	$\beta = 2.0E+3$
2	Gumbel Max	$\sigma = 3.6$	$\mu = 2.0E+3$
3	Gumbel Min	$\sigma = 3.6$	$\mu = 2.0E+3$
4	Weibull	$\alpha = 5.4E+2$	$\beta = 2.0E+3$

In order to reinforce our analysis on extreme values and exceedance, we used the collected data to generate random numbers for more simulations namely with 1000, 5000 and 10000 figures, which outputs are illustrated in graphic and their statistical summary, respectively.

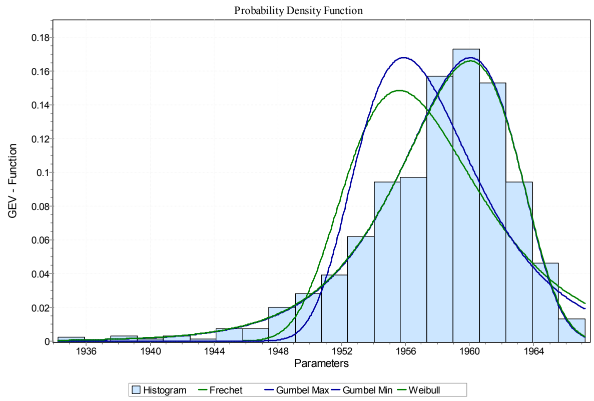

Fig. 12 Extreme distributions for 1000 random number

TABLE IV
GOODNESS OF FIT BASED ON 1000 RANDOM NUMBERS

#	Distribution	Kolmogorov Smirnov		Anderson Darlin		Chi-Square	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Fréchet	0.15	4	48.0	4	2.3E+2	4
2	Gumbel Max	0,15	3	59.0	1	2.1E+2	3
3	Gumbel Min	0.01	1	0.15	5	1.8	1
4	Weibull	0.01	2	3.1E+2	2	2.6	2

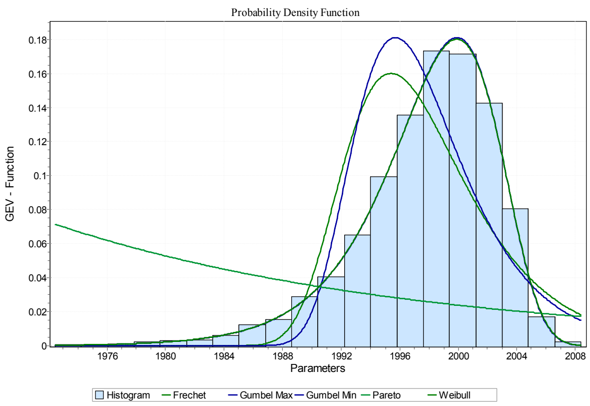

Fig. 13 Extreme distributions for 5000 random number

TABLE V
FITTING RESULTS BASED ON 5000 RANDOM NUMBERS

#	Distribution	Parameters	
1	Fréchet	$\alpha = 4.8E+2$	$\beta = 2.0E+3$
2	Gumbel Max	$\sigma = 3.6$	$\mu = 2.0E+3$
3	Gumbel Min	$\sigma = 3.6$	$\mu = 2.0E+3$
4	Pareto	$\alpha = 78.0$	$\beta = 2.0E+3$
5	Weibull	$\alpha = 5.5E+2$	$\beta = 2.0E+3$

TABLE VI
GOODNESS OF FIT BASED ON 5000 RANDOM NUMBERS

#	Distribution	Kolmogorov Smirnov		Anderson Darlin		Chi-Square	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Fréchet	0.14	4	2.5E+2	3	1.4E+2	4
2	Gumbel Max	0,14	3	3.0E+2	4	1.2E+2	3
3	Gumbel Min	0.01	2	0.29	2	7.6	2
4	Pareto	0.44	5	1.6E+3	5	1.8E+4	5
4	Weibull	0.01	1	0.28	1	6.8	1

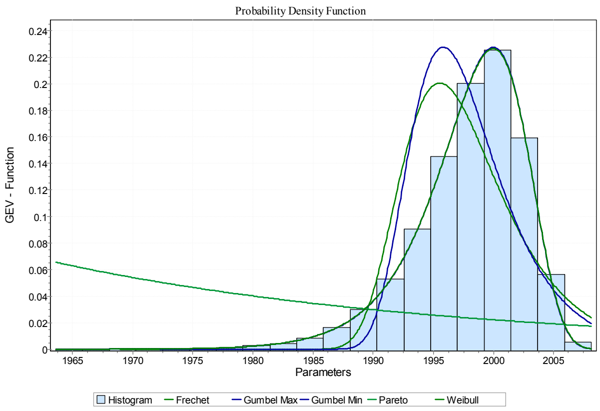

Fig. 14 Extreme distributions for 10000 random number

TABLE VII
FITTING RESULTS BASED ON 10000 RANDOM NUMBERS

#	Distribution	Parameters	
1	Fréchet	$\alpha = 4.9E + 2$	$\beta = 2.0E + 3$
2	Gumbel Max	$\sigma = 3.6$	$\mu = 2.0E + 3$
3	Gumbel Min	$\sigma = 3.6$	$\mu = 2.0E + 3$
4	Pareto	$\alpha = 58.0$	$\beta = 2.0E + 3$
5	Weibull	$\alpha = 5.5E + 2$	$\beta = 2.0E + 3$

TABLE VIII
GOODNESS OF FIT BASED ON 10000 RANDOM NUMBERS

#	Distribution	Kolmogorov Smirnov		Anderson Darlin		Chi-Square	
		Statistic	Rank	Statistic	Rank	Statistic	Rank
1	Fréchet	0.15	4	5.1E+2	3	N/A	
2	Gumbel Max	0,14	3	6.2E+2	4	N/A	
3	Gumbel Min	0.0	1	0.16	1	5.4	1
4	Pareto	0.48	5	3.5E+3	5	4.8E+3	3
4	Weibull	0.0	2	0.23	2	5.7	2

The behavior shown on the distributions seem to be the same as the random numbers go beyond 1000, therefore the same shapes behavior we can see as for 5000 and 10000 random numbers, where the Gumbel minimum and the Weibull grow toward the exceedance over the pick period of floods, while the Gumbel Maximum has the same skewness with Fréchet symmetric shape and the Pareto appears out of range.

V. ANALYSIS AND DISCUSSION

Basically, our contribution is given on the applications of the Generalized Extreme Value distributions to analyze and simulate real data set from rainfall that extremely affected Mozambique, impacting the economy and social assets. Traditional illustrations based on extreme analytical software are shown on Figs. 8 and 9 based on simple Poisson with Np=50, Fig. 8, and additionally with trend to a binominal distribution B100, Fig. 9. Also it plays a major role the comparative analysis that is given based on randomly generated data, where we focused on statistical analysis and the raking process based on Kolmogorov Smirnov, Anderson Darlin and Chi-Squared goodness of fit, which details are shown Tables I-VI.

VI. CONCLUSIONS

Our primary conclusion is that the usage of Extreme Value Models is helpful for data set analysis, where we can assess the limit distribution of the maxima of a set of independent and identically distributed random data. For this specific case we see that the Weibull distribution appears to cover the skewness of data set distribution with trend to maxima, while the Gumbel and Fréchet denote their traditional skewness. Regarding the data under consideration from Mozambique, with Weibull and Pareto distributions there is a trend to follow the threshold is given in the year 2000.

For this specific analysis the application of General Extreme Value that covers the three main Extreme Value Distributions seem to be dominated by both Fréchet and Gumbel Maximum, since their skewness is mostly left while the long tile is right. This is the opposite outcome for the Weibull, Pareto and Gumbel Minimum that gives a trend to the row data given the maximum rainfall of year 2000, expressed in Figs. 8 and 9.

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