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Analysis of Complex Quadrature Mirror Filter Banks

Chimin Tsai

Abstract**—**This work consists of three parts. First, the alias-free condition for the conventional two-channel quadrature mirror filter bank is analyzed using complex arithmetic. Second, the approach developed in the first part is applied to the complex quadrature mirror filter bank. Accordingly, the structure is simplified and the theory is easier to follow. Finally, a new class of complex quadrature mirror filter banks is proposed. Interesting properties of this new structure are also discussed.

Keywords**—**Aliasing cancellation, complex signal processing, polyphase realization, quadrature mirror filter banks.

I. Introduction

In many applications, the input signal $x(n)$ is split into a number of subband signals $y_k(n)$ by an analysis filter bank. After processing, such as in coding and/or transmission, the subband signals near the output $\hat{y}_k(n)$ are combined using a synthesis filter bank to form a reconstructed output signal $\hat{x}(n)$. If decimators and interpolators are inserted before and after the actual process, as shown in Fig. 1, then the overall structure is called the quadrature mirror filter (QMF) bank. In addition to the effect of the actual process on $v_k(n)$, the reconstructed signal $\hat{x}(n)$ differs from $x(n)$ due to aliasing, amplitude distortion, and phase distortion. It is known that all of these distortions can be eliminated [1]. However, this work focuses on the alias-free requirement. Section II reviews the analysis of the conventional two-channel QMF bank. Then, a new method using complex arithmetic is developed for the analysis. The alias-free condition and the corresponding transfer function which describes the amplitude distortion and the phase distortion are expressed in terms of digital filters with complex coefficients.

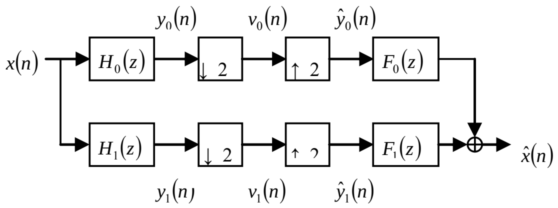

Fig. 1 The QMF bank

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The complex QMF bank was proposed by Nussbaumer and Galand [2], [3]. A slightly different version of the two-channel complex QMF bank is presented in Section III. The input signal $x(n)$ is modulated by two carriers at a frequency of $\pi/2$ with a phase difference of $\pi/2$, four identical real coefficient digital filters are used to implement the analysis and the synthesis banks, and the subband signals near the output are demodulated and combined. The fact that this structure is alias-free has been verified in [2]. The technique developed in Section II is applied to the complex QMF bank. The advantage is obvious. Using digital filters with complex coefficients simplifies the theoretical derivation and the system's structure.

The input signal in the complex QMF bank is modulated, or equivalently, shifted in the frequency domain. In Section IV, the real coefficient digital filter is shifted by $\pi/2$ in the frequency domain, in stead of shifting the complex input signal. The resulting complex coefficient digital filter is adopted to implement the analysis bank of the two-channel QMF bank. The synthesis bank is easily derived for the alias-free system. The overall transfer function has a simple mathematical form. Further properties of this new structure are also discussed in this section.

II. The Two-Channel QMF Bank

The structure of the conventional two-channel QMF bank is illustrated in Fig. 1. In the analysis bank, $H_0(z)$ is typically assumed to be a lowpass filter and $H_1(z)$ to be a highpass filter. However, in practice, the responses overlap, and decimation of the subband signals $y_k(n)$ results in aliasing. The fundamental theory of the QMF bank states that the aliasing in the output signal $\hat{x}(n)$ can be completely canceled by the proper choice of the synthesis bank [1].

A. The Alias-Free Condition

The signals and filter coefficients in Fig. 1 are assumed to be real. The input signal $x(n)$ is separated into two channels by the analysis bank as

$$Y_k(z) = X(z)H_k(z), \quad k = 0,1. \tag{1}$$

The z-transforms of the decimated signals $v_k(n)$ are

$$V_k(z)=\frac{1}{2}\left[Y_k\left(z^{1/2}\right)+Y_k\left(-z^{1/2}\right)\right],\quad k=0,1. \tag{2}$$

After interpolation,

$$\hat{Y}_k(z)=V_k\left(z^2\right),\quad k=0,1. \tag{3}$$

Then, the output signal is reconstructed through the synthesis bank as

$$\hat{X}(z)=\hat{Y}_0(z)F_0(z)+\hat{Y}_1(z)F_1(z). \tag{4}$$

Substituting (1), (2), and (3) into (4), yields

$$\begin{aligned}\hat{X}(z)&=X(z)\cdot\frac{1}{2}\left[H_0(z)F_0(z)+H_1(z)F_1(z)\right]\\&\quad+X(-z)\cdot\frac{1}{2}\left[H_0(-z)F_0(z)+H_1(-z)F_1(z)\right].\end{aligned} \tag{5}$$

Clearly, the aliasing can be completely canceled by choosing

$$\begin{aligned}F_0(z)&=H_1(-z),\\F_1(z)&=-H_0(-z).\end{aligned} \tag{6}$$

Therefore, the reconstructed output signal has a z-transform of the form $\hat{X}(z)=X(z)T(z)$, in which

$$T(z)=\frac{1}{2}\left[H_0(z)H_1(-z)-H_1(z)H_0(-z)\right] \tag{7}$$

is the overall transfer function of the alias-free system.

B. Analysis of the Two-Channel QMF Bank using Complex Arithmetic

A complex coefficient digital filter with real input is implemented as a one-input/two-output real coefficient system. Similarly, if only the real part of the output of a complex coefficient digital filter is retained, then the filter is equivalent to a two-input/one-output real coefficient system [4], [5].

In this subsection, the analysis of the two-channel QMF bank is simplified using digital filters with complex coefficients. Let

$$\begin{aligned}H(z)&=H_0(z)+jH_1(z),\\F(z)&=F_0(z)+jF_1(z).\end{aligned} \tag{8}$$

Then the upper and the lower paths in Fig. 1 are treated as the real and the imaginary parts of the signal flows. Consequently, the structure is simplified. As shown in Fig. 2, solid and dashed lines represent the real and the imaginary parts of signals. Note

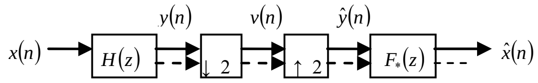

Fig. 2 The QMF bank with complex coefficients

that the synthesis bank is given by $F_*(n)\equiv F_0(z)-jF_1(z)$. The analysis in (1)-(4) is easily replaced by

$$Y(z)=X(z)H(z), \tag{9}$$

$$V(z)=\frac{1}{2}\left[Y\left(z^{1/2}\right)+Y\left(-z^{1/2}\right)\right], \tag{10}$$

$$\hat{Y}(z)=V\left(z^2\right), \tag{11}$$

$$\hat{X}(z)=\mathrm{Re}\left\{\hat{Y}(z)F_*(z)\right\}, \tag{12}$$

where the notation $\mathrm{Re}\{\cdot\}$ means that only the real parts of the coefficients are retained. From (6) and (7), it can be shown that the alias-free system requires

$$F_*(z)=jH_*(-z), \tag{13}$$

and the corresponding transfer function is

$$T(z)=\frac{1}{4j}\left[H_*(z)H(-z)-H(z)H_*(-z)\right]. \tag{14}$$

III. THE COMPLEX QMF BANK

A two-channel structure of the filter banks with modulated signals was proposed in [2] and [3]. The structure is referred to as the complex QMF bank. As shown in Fig. 3, this structure uses modulators, demodulators, and four identical real coefficient digital filters $G(z)$. It was verified in [2] that this

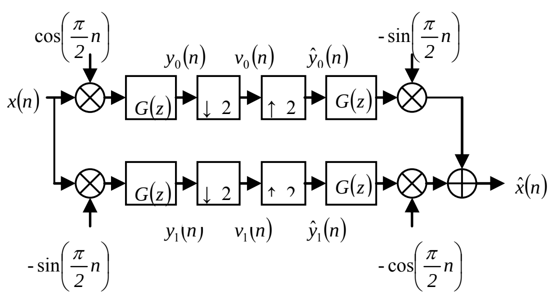

Fig. 3 The complex QMF bank

particular structure automatically satisfies the alias-free requirement.

A. Aliasing Cancellation

The fact that complex QMF bank is free from aliasing is established by observing that

$$Y_0(z)=\frac{1}{2}\left[X(-jz)+X(jz)\right]G(z),$$
$$Y_1(z)=-\frac{1}{2j}\left[X(-jz)-X(jz)\right]G(z). \tag{15}$$

The decimated signals $v_k(n)$ and the interpolated signals $\hat{y}_k(n)$ are given by (2) and (3). The z-transform of the modulated outputs is

$$Z\left\{\left(\hat{y}_0(n)*g(n)\right)\left(-\sin\frac{\pi n}{2}\right)\right\}=-\frac{1}{2j}\left[\hat{Y}_0(-jz)G(-jz)-\hat{Y}_0(jz)G(jz)\right],$$
$$Z\left\{\left(\hat{y}_1(n)*g(n)\right)\left(-\cos\frac{\pi n}{2}\right)\right\}=-\frac{1}{2}\left[\hat{Y}_1(-jz)G(-jz)+\hat{Y}_1(jz)G(jz)\right]. \tag{16}$$

Adding both sides of (16) results in

$$\hat{X}(z)=X(z)\cdot\frac{1}{4j}\left[G^2(jz)-G^2(-jz)\right]. \tag{17}$$

It is observed from the above equation that the aliasing is canceled and the overall transfer function is

$$T(z)=\frac{1}{4j}\left[G^2(jz)-G^2(-jz)\right]. \tag{18}$$

B. Analysis of the Complex QMF Bank using Complex Arithmetic

The structure in Fig. 3 is easily simplified by using complex arithmetic. The resulting structure is given in Fig. 4, from which

$$Y(z)=X(jz)G(z). \tag{19}$$

The decimated signals $v(n)$ and the interpolated signals $\hat{y}(n)$ are obtained by (10) and (11).Then,

$$\hat{X}(z)=\mathrm{Re}\left\{j\hat{Y}(-jz)G(-jz)\right\} \tag{20}$$

yields the same results as shown in (17) and (18).

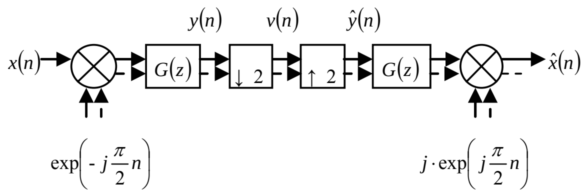

Fig. 4 The complex QMF bank with complex coefficients

IV. The Modified Complex QMF Bank

Fig. 4 clearly shows that the complex modulator left shifts the spectrum of the input signal $X\left(e^{j\omega}\right)$ by $\pi/2$. On the other hand, the spectrum of the output is right shifted by $\pi/2$. This section investigates the effect of shifting the frequency response of $G\left(e^{j\omega}\right)$ in stead of shifting the signals.

A. Modification of the Complex QMF Bank

Consider the real coefficient digital filter $G(z)$. A complex coefficient digital filter is defined by

$$h(n)=j^n g(n), \tag{21}$$

or equivalently,

$$H(z)=G(-jz). \tag{22}$$

Therefore, the frequency responses of these two filters are related by

$$H\left(e^{j\omega}\right)=G\left(e^{j(\omega-\pi/2)}\right). \tag{23}$$

Since $g(n)$ is real,

$$H(z)=H^*(-z). \tag{24}$$

Suppose this complex filter $H(z)$ is applied to Fig. 2. The alias-free condition in (13) is rewritten as

$$F_*(z)=jH(z). \tag{25}$$

The resulting structure is given in Fig. 5 and is referred to as the modified complex QMF bank. Moreover, with (24), the transfer function of the alias-free system in (14) is changed to

$$T(z)=\frac{1}{4j}\left[H^2(-z)-H^2(z)\right]. \tag{26}$$

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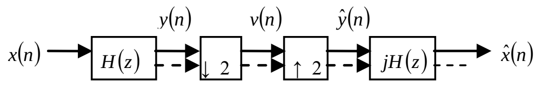

Fig. 5 The modified complex QMF bank

Applying (22) reveals that the above transfer function $T(z)$ is exactly the same as (18). This result explains why the complex QMF bank in Section III automatically satisfies the alias-free requirement.

B. *Realization of the Modified Complex QMF Bank*

Based on the definition in (21), the complex coefficient digital filter $H(z)$ can be written as

$$H(z) = H_0(z) + jH_1(z)\ , \tag{27}$$

in which

$$\begin{aligned} H_0(z) &= g(0) - g(2)z^{-2} + g(4)z^{-4} - g(6)z^{-6} + \ldots \\ H_1(z) &= g(1)z^{-1} - g(3)z^{-3} + g(5)z^{-5} - g(7)z^{-7} + \ldots \end{aligned} \tag{28}$$

have real coefficients. When the filter bank is realized with the above real coefficient digital filters, the alias-free condition in (6) is rewritten as

$$\begin{aligned} F_0(z) &= -H_1(z), \\ F_1(z) &= -H_0(z). \end{aligned} \tag{29}$$

As a consequence, the modified complex QMF using real coefficients is illustrated in Fig. 6. Besides, the transfer function in (26) is rewritten as

$$T(z) = -H_0(z)H_1(z). \tag{30}$$

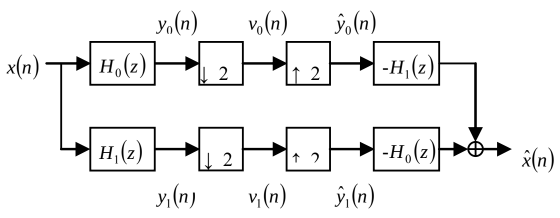

Fig. 6 Realization of the modified complex QMF bank

Furthermore, from (28), it is observed that

$$\begin{aligned} H_0(z) &= E_0(z^2), \\ H_1(z) &= z^{-1}E_1(z^2), \end{aligned} \tag{31}$$

where

$$\begin{aligned} E_0(z) &= g(0) - g(2)z^{-1} + g(4)z^{-2} - g(6)z^{-3} + \ldots \\ E_1(z) &= g(1) - g(3)z^{-1} + g(5)z^{-2} - g(7)z^{-3} + \ldots \end{aligned} \tag{32}$$

Therefore, a polyphase realization of the modified complex QMF bank is obtained and displayed in Fig. 7. The transfer function is in the form of

$$T(z) = -z^{-1}E_0(z^2)E_1(z^2). \tag{33}$$

V. Conclusion

This work analyzed the conventional two-channel QMF bank and the complex QMF bank using complex arithmetic. A new class of complex QMF banks was thus observed. The alias-free condition and several properties of this new structure are presented here. However, many issues remain to be investigated in the future.

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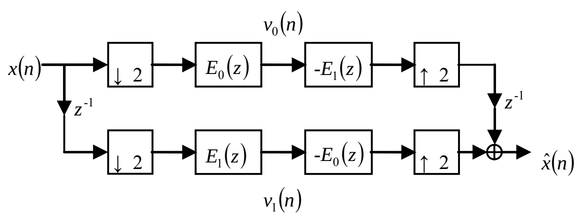

Fig. 7 Polyphase realization of the modified complex QMF bank